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# ICAMΣ'21

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## International Conference on Applied Mathematics in Engineering

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## Book of Abstracts





# ICAMΣ'21

September 1-3, 2021 - Balıkesir, TURKEY

## International Conference on Applied Mathematics in Engineering

### Book of Abstracts

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## Preface

We would like to welcome all participants to join "The Second International Conference on Applied Mathematics in Engineering (ICAME'21)", which will be held September 1 to September 3, 2021 in Burhaniye/Balikesir, Turkey.

This conference allows an ideal academic platform for researchers to present the latest research and evolving findings of applied mathematics on engineering, physics, chemistry, biology, and statistics.

The conference also provides the opportunity of discussing advances in the field of applied mathematics, its effect on engineering and real-life problems. Especially, the conference discusses the most current applied mathematical problems in the world. For example, fractional calculus and its real-life applications, operational research, mathematical modeling in health science and engineering, optimization and control in engineering, non-linear dynamical systems and chaos, optimization and control problems are main topics of the conference.

In this conference, 143 oral presentations will be given to an audience with over 120 participants from 27 countries.

ICAME'21 is an achievement of international cooperation we continuously endeavor to carry out and develop. In this context, on behalf of the chairs of this conference, we would particularly like to thank: plenary speakers Albert C. J. Luo (Southern Illinois University Edwardsville, USA), Sverre Holm (University of Oslo, Norway), Gerhard-Wilhelm Weber (Poznan University of Technology, Poland) and Praveen Agarwal (Anand International College of Engineering, Jaipur, India) invited speakers Carla Pinto (School of Engineering, Polytechnic of Porto, Portugal), Huseyin Merdan (TOBB University of Economy and Technology, Turkey) and Amin Jajarmi (Department of Electrical Engineering, University of Bojnord, Iran) as well as the organizers of special sessions, and the members of the international scientific committee for their contributions and supports.

We would like to extend our best wishes to all of you with a hope that you go back with actual and more powerful ideas, and with new science networks renewed or extended.

Best wishes for an enjoyable and memorable conference.

On behalf of the organization committee,

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*Jordan Hristov*  
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*and Metallurgy,*  
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## Plenary Speakers

### Albert C. J. Luo

Southern Illinois University Edwardsville, USA



#### **Infinite Unstable Periodic Orbits to Infinite Homoclinic Orbits in the Lorenz System**

In nonlinear dynamics, homoclinic orbits are very essential for a better understanding of the corresponding global dynamics. However, it is very difficult to determine homoclinic orbits in 3-D or higher-dimensional nonlinear dynamical systems. In this talk, the routes from unstable periodic orbits to infinite homoclinic orbits are presented in the Lorenz system, and such homoclinic orbits are pertaining to unstable periodic orbits on bifurcation trees. Thus, a semi-analytical method is presented for determining unstable periodic orbits. This is because traditional computational methods cannot obtain unstable periodic orbits in nonlinear dynamical systems due to computational errors and perturbations. For demonstration, a period-doubling bifurcations tree of the period-1, period-2 and period-4 motions are presented. Periodic orbits and homoclinic orbits in the Lorenz system are illustrated through 3-D views, from which one can imagine complex stable and unstable periodic motions and the Lorenz attractors. Further, the corresponding mathematical structures of homoclinic orbits and Lorenz attractors can be further developed.

### J. A. Tenreiro Machado

Institute of Engineering, Polytechnic of Porto, Portugal



#### **The Logical Song**

Data analytics is penetrating in all areas of the human activity. The availability of data measuring the behavior of complex phenomena allows a new quantitative perspective not conceivable with classical mathematical tools. This presentation addresses 3 distinct areas of key impact in society and that reflect the behavior of men kind. We consider music, artistic painting and soccer as manifestations of the human spirit that can be processed and analyzed, since large volumes of data are presently available in digital form. The data records are studied using several mathematical and computational tools, such as, fractional calculus, entropy, multidimensional scaling, hierarchical clustering and scientific visualization. The embedding of mathematical, computational and algorithmic modeling leads to the emergence of patterns that are analyzed and interpreted.

## Sverre Holm

University of Oslo, Norway



### Fractional Wave Equations and Complex Acoustic Media

Wave equations with non-integer derivative operators can describe attenuation which increases with frequency with other powers than two, unlike ordinary wave equations. Such attenuation is found in many complex media. Both shear and compressional waves in media as diverse as biological tissue, rocks, and sub bottom sediments are examples of this. These wave modes are central in applications such as medical ultrasound, diagnostic shear wave imaging in elastography, seismics, and underwater acoustics. These equations can be divided into two classes depending on whether they can be derived from more fundamental principles or not. In the first class one can find the fractional Kelvin-Voigt and fractional Zener wave equations, while several fractional Laplacian wave equations are in the second category. Such examples as well as the properties of their solutions will be presented. In many cases just having such a wave equation is enough to model a phenomenon.

In [Holm, S. (2019). Waves with power-law attenuation. Springer and ASA (Acoustical Society of America) Press] I also wanted to understand what it is about complex media that gives rise to power law behavior. The main attenuation mechanisms of standard acoustics are heat conduction and relaxation, structural relaxation, and chemical relaxation. They have fractional parallels and the first one is heat relaxation described by fractional Newton cooling due to anomalous diffusion. The most important mechanism is however the second one, the fractional parallel to structural relaxation. Instead of one there are multiple relaxation processes with a distribution of relaxation times that follows a power-law distribution, possibly indicating that the material has fractal properties. This distribution also has a relationship to the Arrhenius equation, indicating a link to chemical relaxation, albeit a quite speculative one.

Other sources of power-law behavior can be non-Newtonian rheology with time-varying viscosity and propagation when there is a fractal distribution of scatterers in an otherwise lossless medium. Existing models in sediment acoustics such as the grain shearing model and the Biot poroelastic model can also be reformulated with fractional operators. These approaches are presented in the hope of coming one step closer to answering if fractional wave equations give clues to some deeper reality, or if they are just a compact phenomenological description.



## Dumitru Baleanu

Cankaya University, Turkey



### **On Singular and Non-Singular Fractional Operators and Their Applications in Mathematical Biology**

The fractional calculus and its applications is a hot topic for researchers from many branches of science and engineering. Real world applications started to be investigated with a great success within this very helpful mathematical tool. In my talk I will concentrate on the successful applications of both singular and non-singular fractional operators to the complex dynamics of some mathematical biology systems. Besides, some new aspects of the classification of fractional operators will be presented. Illustrative examples will be provided.

## Gerhard-Wilhelm Weber

Poznan University of Technology, Poland



### **Defined Contribution Pension Funds by Robust Stochastic Optimal Control**

In the present work, we study the problem of optimal management of defined contribution pension funds, during the distribution phase, under the effect of inflation, mortality, and model uncertainty. More precisely, we consider a class of employees, who, at the time of retirement, enter a life assurance contract with the same insurance firm. The fund manager of the firm collects the entry fees to a portfolio savings account and this wealth is to be invested optimally in a Black-Scholes type financial market. As such schemes usually last for many years, we extend our framework, by: (i) augmenting the financial market with an inflation-adjusted bond, and (ii) taking into account mortality of the fund members. Model uncertainty aspects are introduced as the fund manager does not fully trust the model he/she faces. By resorting to robust control and dynamic programming techniques, we provide: (a) closed-form solutions for the case of the exponential utility function, (b) a detailed numerical study of the qualitative features of the problem at hand that elucidates the effect of robustness and inflation on the optimal investment decisions.

## Praveen Agarwal

Anand International College of Engineering, Jaipur, India



### **Certain Generalization of Fractional Derivative Operators**

Many authors have introduced and investigated certain extended fractional derivative operators. The main object of this talk to study extended fractional differential operators (such as the Riemann-Liouville and Caputo type fractional operators) involving generalized hypergeometric functions introduced recently and investigate its various (potentially) useful and (presumably) new properties and formulas, for example, integral representations, Mellin transforms, generating functions, and the extended fractional derivative formulas for some familiar functions.

## Invited Speakers

### Jordan Hristov

University of Chemical Technology and Metallurgy, Bulgaria



### **Fractional Operators with Non-Singular Memories in Viscoelasticity: Basic Concepts Applicable to Linear and Non-Linear Viscoelasticity**

The fractional operators with non-singular memory kernel described by exponential (Caputo-Fabrizio derivative) and generalized Mittag-Leffler function (Atangana-Baleanu derivative) raise many questions about their properties and mainly about their physical relevance and applications.

This lecture focuses on basic principles in description relaxation behaviors of linear and non-linear viscoelastic materials and the adequate selection of the memory kernels of the fractional operators leading to both derivatives with singular or non-singular memories.

The targets are non-aging (linear and non-linear) viscoelastic materials with behaviors beyond the power-law limit related to the Caputo fractional derivative. The stress-strain response functions are the main physical objects allowing selecting the corresponding memories of the fractional operator and their constructions. As a consequence of the memory kernel selection, the causality of both the constitutive equations and the frequently used rheological equations are discussed.

### Carla Pinto

School of Engineering, Polytechnic of Porto, Portugal



#### **Tackling specificities of different diseases using within-host models**

Epidemics make exciting news. They are often presented with dramatic headlines, and the pictures accompanying them are of healthcare workers dressed with protective equipment or working at labs. People often forget about the behind scenes work of mathematicians, who, with more or less simplified models, help on the understanding and prediction of infections spread. In this lecture I will focus on several within-host models useful for a deeper knowledge of virus dynamics with different specificities, namely HIV, HCV, HSV-2, etc.

### Huseyin Merdan

TOBB University of Economy and Technology, Turkey



#### **Nonlinear dynamics of a ratio-dependent prey-predator model: Stability, bifurcations and chaos**

Nonlinear dynamical behaviors of a prey-predator system with Leslie type will be presented. First, the dynamics of its continuous form will be analyzed; the local and global stabilities and bifurcations will be discussed. Second, the dynamical behavior of its discrete form will be analyzed; bifurcations and chaotic behavior will be shown. Numerical simulations will be given to support and extend the theoretical results. Finally, we will compare the results that we obtained.

### Amin Jajarmi

Department of Electrical Engineering, University of Bojnord, Iran



#### **Recent developments in the mathematical modelling and control of biological system**

Recently, the new aspects of fractional calculus have been widely employed to investigate different features of many complex biological systems. In this direction, fractional models help us to understand how the memory of the certain components of a system affects the progress of diseases as a whole, and therefore, it enables us to implement the memory effects into the evolution of considered system together with its environment. This kind of analysis is also important in order to improve the current medications and to explore new ways of quick, effective and low-cost treatments. In this talk, we explore a recent development in the mathematical modelling of biological systems. The complex dynamics of an epidemic are investigated within the use of both classical and a new fractional framework. The obtained results are analyzed by the help of some simulations in a comparative way for both integer- and fractional-order cases. Finally, an efficient control scheme is designed for the purpose of intervention in an appropriate, effective way.

## Special Sessions

### Modelling & Optimization in Engineering

*Ramazan Yaman, Istanbul Atlas University, Turkey*

*Ahmet Sahiner, Suleyman Demirel University, Turkey*

*Firat Evirgen, Balikesir University, Turkey*

#### Theme

The goal of this session is to discuss recent developments in applications of optimization methods by bringing together researchers and practitioners working in the field of optimization theory, methods, software and related areas.

#### Topics

Mathematical programming

Global optimization

Nondifferential optimization

Continuous optimization

Combinatorial optimization

Multicriteria optimization

Equilibrium programming

Game theory

Data mining

Population based algorithms

Artificial intelligence technologies

Applications of optimization in natural sciences

Applications of optimization in engineering

Energy systems modelling and optimization

### Operational Research

*Gerhard-Wilhelm Weber, Poznan University of Technology, Poland*

*Aslan Deniz Karaoglan, Balikesir University, Turkey*

*Ibrahim Kucukkoc, Balikesir University, Turkey*

*Burcu Gurbuz, Uskudar University, Turkey*

#### Theme

This session aims to bring together researchers working on the topics related to operational research to discuss recent developments in the theory and application of operational research techniques.

#### Topics

Business analytics for manufacturing systems

Analytics, optimization and machine learning in manufacturing and supply chains

Intelligent manufacturing systems

Intelligent transportation

Portfolio optimization

Network models

Inventory control, production planning and scheduling

Sustainable manufacturing

Robotics in manufacturing

Modeling, simulation, control and monitoring of manufacturing processes

Logistics, supply chains and networks

Facility planning and materials handling

Energy systems modelling

Design and reconfiguration of manufacturing systems

## **Control Theory & Applications**

*Kemal Leblebicioglu, METU, Turkey*

*Metin Demirtas, Balikesir University, Turkey*

*Beyza Billur Iskender Eroglu, Balikesir University, Turkey*

### **Theme**

This session aims to discuss a broad range of topics including current trends of linear, nonlinear, discrete and fractional control systems as well as new developments in robotics and mechatronics, unmanned systems, energy systems with the goal of strengthening cooperation of control and automation scientists with industry.

### **Topics**

Adaptive control

Linear and nonlinear control systems

Optimal control

Discrete time control systems

Robust control

Fractional order systems and control

Chaotic systems and control

Evolutionary and heuristic control

Robotic control

Energy management and control

Control of unmanned air and undersea vehicles

## **Fractional Calculus with Applications in Biology**

*Dumitru Baleanu, Cankaya University, Turkey*

*Carla Pinto, School of Engineering, Polytechnic of Porto, Portugal*

*Necati Ozdemir, Balıkesir University, Turkey*

### **Theme**

The goal of this session is to bring together creative and active researchers, in theoretical analysis and numerical tools, to discuss recent developments in applications of fractional order models of biological models. Fractional order models have become ubiquitous research topics in the last few decades. Their memory property contributes to a better and profound understanding of the dynamics of real world models, namely of biological population problems. Stochastic and deterministic models and coinfection models, as well as computational models, are welcome for HIV, HCV, Ebola, Zika, etc, in this session.

### **Topics**

New numerical methods to solve fractional differential equations  
Deterministic and stochastic fractional differential equations  
Computational methods for fractional differential equations  
Bifurcation theory  
Stability theory  
Cancer development models: chaos, synchronization  
Applications in bioengineering, medicine, ecology, biology, epidemiology

## **Numerical Methods in Fractional Calculus**

*Zakia Hammouch,, Universite Moulay Ismail FSTE Errachidia, Morocco*

*Ali Konuralp, Celal Bayar University, Turkey*

*Mehmet Yavuz, University of Exeter, UK*

### **Theme**

In the few decades, fractional differential equations have played a very important role in various fields. Based on the wide applications in engineering and sciences such as physics, mechanics, chemistry, and biology, research on fractional ordinary or partial differential equations and other relative topics is active and extensive around the world. In the past few years, the increase of the subject is witnessed by hundreds of research papers, several monographs, and many international conferences. The objective of this special session is to highlight the importance of numerical methods and their applications and let the readers of this journal know about the possibilities of this new tool.

### **Topics**

New methods for solving fractional differential equations  
Controllability of fractional systems of differential equations or numerical methods applied to the solutions of fractional differential equations applications in physics, mechanics, and so forth  
Iteration methods for solving partial and ordinary fractional equations  
Numerical methods for solving fractional integro-differential equations  
Numerical functional analysis and applications  
Local and nonlocal boundary value problems for fractional partial differential equations  
Stochastic partial fractional differential equations and applications  
Computational methods in fractional partial differential equations  
Numerical methods for solving variable order differential equations  
Perturbation methods for fractional differential equations

## New Fractional Derivatives and Their Applications

*Dumitru Baleanu, Cankaya University, Turkey*

*Jordan Hristov, Univ. of Chemical Tech. and Metallurgy, Bulgaria*

*Derya Avci, Balıkesir University, Turkey*

### Theme

Nowadays, there has been an increasing interest to the new types of fractional derivatives. The well-known fractional derivatives such as Riemann-Liouville, Caputo, Riesz are successful for modelling real World problems. In addition, these fractional operators give the memory and hereditary effects in physical phenomena. However, these are non-local operators described by convolution integrals with weakly singular kernels. Due to these structures, some complexities can naturally occur in the mathematical modelling and solution processes. Because of these hardness, many researchers have paid attention to introduce new derivatives with fractional parameter in the last years. Caputo-Fabrizio, Atangana-Baleanu, Beta, Conformable derivatives with fractional parameter are pioneering definitions in this sense.

### Topics

Description of new fractional derivatives  
New properties of new fractional derivatives  
Integral transform techniques in sense of new fractional operators  
New analytical/numerical methods  
Mathematical modelling in terms of new fractional operators  
Foundation of new relations between existing and new fractional operators

## Nonlinear Dynamical Systems and Chaos

*Huseyin Merdan, TOBB ETU, Turkey*

*Songul Kaya Merdan, METU, Turkey*

### Theme

This special session focuses on the dynamics of complex systems, which are one of the most attractive subjects of the modern sciences. The attractiveness of this particular area arises from two different aspects: The first one is that it provides challenges, which are connected with many uncertainties in description of irregular motions. The second one is methods of investigation, which are not yet well developed and established. Applications of complex dynamics investigations are very important and deal with a wide range of problems. They begin with mechanical problems and extend to earthquake prediction and social sciences problems. We are interested in those investigations in electrical and mechanical engineering, physics, biology, economics, finance, neuroscience, computer sciences, fluid dynamics and earthquake monitoring, which urgently need mathematical modeling of their problems and analysis through nonlinear dynamical systems approach.

### Topics

ODE, DDE and PDE based modelling for complex systems	Stochastic complex dynamical systems and randomness
Dynamical systems and chaos	Hybrid systems
Bifurcation theory	Complex networks based-models
Synchronization	Neural Networks
Control theory	Bio-engineering, bio-imaging and bio-fluids
Fluid Dynamics	Population dynamics and conservation biology
	Ecosystems

Evolution and ecology  
Epidemiology and disease modeling  
Neuroscience  
Regulatory networks  
Cell and Tissue biophysics  
Evolution and populations genetics  
Cell and developmental biology

Cancer and immunology  
Environmental sciences  
Social economy systems  
Climate change  
Financial engineering  
Mathematical finance

## **Nonlinear Transport Phenomena and Models**

*Jordan Hristov, Univ. of Chemical Tech. and Metallurgy, Bulgaria*

### **Theme**

The special section focuses on modelling of nonlinear transport phenomena (heat, mass and momentum) as well as models related to real world application. Models with both local and fractional differential operators involved in modelling in such models are welcome. The topics drawn below are the main directions but no restrictive and any new problems outside them are welcome.

### **Topics**

Nonlinear diffusion and heat transfer (conduction)  
Nonlinear viscoelasticity and plasticity  
Modelling rheology of complex fluids, solids and granular systems (hydrodynamics, large strain deformations and mixing)  
Nonlinear kinetic and rate equations and irreversible thermodynamics  
Models of nonlinear biological and medical problems for real-world applications  
Models for treatment of nonlinear signal processing and control

Nonlinear electrical and magnetic phenomena and nonlinear applied models in electrotechnics (nonlinear magnetic circuits, high frequency skin effects, supercapacitors, etc.)  
Inverse problems in nonlinear models of transport phenomena  
New nonlinear models (broad aspect)  
Analytical and numerical methods for solution of nonlinear models  
Scaling and dimensional analysis

## **Computational Methods for Treatment of Linear and Nonlinear Models**

*Murat Sari, Yildiz Technical University, Turkey*

*Elvan Akin, Missouri University of Science, USA*

*Canan Celik Karaaslanli, Yildiz Technical University, Turkey*

### **Theme**

Mathematical modeling is the art of transforming problems from a field of application into traceable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the better understanding the universe. Mathematical modeling is inevitable in many fields of science and gives precision, direction and low-cost for problem solutions. Mathematical modeling yields a way for better understanding or design of a system and leads to the use of modern computing capabilities.

This session will include distinguished works at the interface between applied mathematics and computational methods via linear and nonlinear models occurred in the physical, biological, engineering, and economical sciences. By considering the linear/nonlinear or deterministic/stochastic models with a flexible approach, this session encourages versatile understanding of the computational science. Robust numerical methods or simulation techniques as well as new designs of mathematical models are welcome to this session.



**Topics**

Numerical Solutions of Partial Differential  
Equations

Biological Models and Computational Analysis

Stochastic Models and Applications

Stiff Problems and Their Numerical  
Investigations

Discrete Models and Applications

Computational Fixed-Point Methods

Adaptive Numerical Methods

Computational Fluid Dynamics

Molecular Dynamic Simulations

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## Optimization of Extrusion Process by using Response Surface Methodology

Aslan Deniz Karaoglan<sup>✉</sup>, Gulistan Bicen, Rabia Durak

*Balıkesir University, Industrial Engineering Department, Balıkesir, Turkey*

### Abstract

One of the most widely used methods in the production of plastic products is the extrusion process. Plastic Extrusion method is a manufacturing method used especially in the production of plastic materials such as pipes, hoses, cables, profiles. In this process, an engine rotates the screw in a sleeve coated with a heater to melt the plastic granules under temperature and pressure. The molten plastic is shaped and cooled along the mold and the production takes place. Many parameters affect product quality during the extrusion process. Optimization of these parameters aims to reduce time, labor and energy costs. This study was carried out in a manufacturer that meets the plastic cups requirement of the food sector. The aim of the study is to find the mathematical relationship between the parameters affecting the thickness of the sheet in the extrusion process by the regression equation and to find out the optimum factor levels in order to obtain the target sheet thickness. Response surface methodology (RSM) is one of the widely used modeling and optimization method [1-3]. By using this method, it is possible to model the relations in terms of quadratic and interaction terms in the regression equation. For this purpose, Minitab statistical analysis program was used. In order to determine whether the number of factors constituting the regression equation is sufficient or not,  $R^2$  determination coefficient is calculated and it is seen that it is quite close to 1. Then, ANOVA analysis results were examined and it was concluded that the regression equation was significant at 95% confidence level (which means  $\alpha=5\%=0.05$ ). According to Minitab ANOVA results; p-value is calculated as 0.04 which is lower than  $\alpha=0.05$ . This means the regression equation is significant. After finding an available regression equation, the final step was optimization with the help of Minitab Response Optimizer module. Verification of the optimum result was performed by field tests and it was found that there was no significant difference between the expected output value and the observed and the results were quite successful.

**Keywords:** Response surface methodology, optimization, plastic extrusion method

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<sup>✉</sup> Corresponding Author Email: [deniz@balikesir.edu.tr](mailto:deniz@balikesir.edu.tr)

## A Reversed Fixed-Point Iteration Method for Burgers Equation

Tahir Cosgun<sup>1,2,✉</sup>, Murat Sari<sup>1</sup>

<sup>1</sup>*Yildiz Technical University, Department of Mathematics, Istanbul, Turkey*

<sup>2</sup>*Amasya University, Department of Mathematics, Istanbul, Turkey*

### Abstract

Although the equation

$$u_t + uu_x = \varepsilon u_{xx}$$

was discovered by Bateman [1], it has been called with the name of Burgers because of his considerably important contributions. Undoubtedly, one of the most important contributions is that Burgers discover the relationship between turbulent flows and this equation [2]. Later, this equation has been solved simultaneously by Cole and Hopf via a nonlinear transformation by converting it to a linear heat equation [3,4].

Burgers equation continues to attract the attention of many researchers from various disciplines since it is the simplification of the Navier-Stokes equation to the one-dimensional case without force and pressure terms. Moreover, being a quasilinear partial differential equation, Burgers equation includes both an advection term and a diffusion term, and so, it is also a test problem for a lot of numerical methods.

In this study, the Burgers equation has been discussed by using a newly developed numerical approach that is called the reversed fixed-point iteration method. The proposed method has been constructed to find out the repelling fixed points of a nonlinear mapping and has been observed to be an effective tool to capture the unstable behavior of a nonlinear dynamical system. The current method has been compared fairly with the conventional fixed-point iteration method by considering both advantageous and disadvantageous aspects.

**Keywords:** Nonlinear dynamical systems, fixed point theory, unstable equilibrium

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✉ Corresponding Author Email: [tahircosgun@gmail.com](mailto:tahircosgun@gmail.com)



## Modeling and Analysis of the Relationship with Regression Equation between Heating/Cooling Load Change and Outdoor Meteorological data of Business and Service Buildings

Okan Kon<sup>1,✉</sup>, Bedri Yuksel<sup>2</sup>, Aslan Deniz Karaoglan<sup>3</sup>

<sup>1</sup>Balikesir University, Mechanical Engineering Department, Balikesir, Turkey

<sup>2</sup>Gelisim University, Mechatronics Engineering Department, Istanbul, Turkey

<sup>3</sup>Balikesir University, Industrial Engineering Department, Balikesir, Turkey

### Abstract

In this study, Balikesir University Rectorate Building was chosen as the business and service building. In the winter months, measurements were made in the hot water boiler for the heating load. For the cooling load, measurements were made in the chiller groups. Taking the working hours into consideration, the average daily values were determined. As meteorological data, 8 factors such as outdoor temperature, relative humidity, solar radiation, wind speed, atmospheric pressure, sunshine duration, steam pressure and 1 m underground temperature were taken into account. In the study, the relationship between summer cooling load and meteorological factors affecting this load is modeled with a regression equation that includes linear terms and quadratic terms. Modeling was carried out with the help of the Minitab statistical analysis program and 37 samples were used for this purpose. R<sup>2</sup> (coefficient of determination) was calculated as 96.20% for the model and R<sup>2</sup> - prediction value was calculated as 84.22%. These values show that 8 factors included in the model during the modeling phase are sufficient to explain the change in cooling load. When the Analysis of variance (ANOVA) was examined, the model was found to be significant. Similarly, the relationship between winter heating load and meteorological factors affecting this load is modeled with a regression equation that includes linear terms and interactions. 55 samples were used. For the model, R<sup>2</sup> was calculated as 99.94% and R<sup>2</sup> - prediction value was calculated as 81.95%. When the Analysis of variance (ANOVA) was examined, the model was found to be significant. As a result, the regression equations created for both periods were tested with samples from the relevant periods but not used in the modeling stage and provided high accuracy prediction opportunities for days with very low estimation errors.

**Keywords:** Heating and cooling load, meteorological factors, regression analysis.

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✉ Corresponding Author Email: [okan@balikesir.edu.tr](mailto:okan@balikesir.edu.tr)

## Vibration Monitoring of Coastal and Ocean Structures with Pile Foundations Using Compressive Sensing

Cihan Bayındır<sup>1</sup>✉,

<sup>1</sup> *Istanbul Technical University, Civil Engineering Department, İstanbul, Turkey.*

### Abstract

Structural health monitoring of the maritime structures is a rapidly developing research area. One of the research directions followed within this context is the development of efficient mathematical data and signal processing techniques and their possible usage methods for health monitoring of coastal and ocean structures. In this paper, we investigate the possible usage of one of such methods, namely the compressive sensing technique (CS), for the measurement and reconstruction of the vibration data of the coastal and ocean structures. CS algorithm outperforms the classical sampling theory by using far fewer measurements for the reconstruction of signals having sparse representation in different orthogonal domains [1-3]. The aforementioned maritime structures are continuously subjected to harmonic loads in the marine environment, as well as impact loads such as shiploads, earthquakes [4]. Thus CS algorithm can be used for the reconstruction of vibration velocities, acceleration, and similar parameters under such loadings, which have sparse representations in Fourier or temporal/spatial domains. Implementing a circular cylinder [4] and hollow circular cylinder model [5] for the modeling of the pile foundations, we show that CS can be effectively used for the monitoring and reconstruction of such vibration parameters under cyclic harmonic loads [6-7] and impact loads [8] including shiploads and earthquakes. We discuss our findings and their possible applicability and usage.

**Keywords:** Coastal and ocean structures, structural vibration monitoring, compressive sensing.

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✉ Corresponding Author Email: [cbayindir@itu.edu.tr](mailto:cbayindir@itu.edu.tr)

## Analysis of Tsunami and Tsunami-Structure Interaction Parameters by Compressive Sensing

Cihan Bayındır<sup>1</sup>✉,

<sup>1</sup> *Istanbul Technical University, Civil Engineering Department, İstanbul, Turkey.*

### Abstract

Tsunamis are among the most devastating hazards that can be observed in nature. Observation, sensing, recording, and analysis of the tsunami and tsunami-structure interaction parameters are of crucial importance for the safety of the coastal zone and communities. These parameters include but are not limited to tsunami water surface fluctuations, particle velocities, inundation, runup, sediment deposit, their dynamics pressures on structures. Efficient sensing, data recording, and analysis of these parameters are critically important for the reconnaissance, assessment, early warning, and avoidance of catastrophic consequences of tsunamis. One of the most successful sensing algorithms of the big data era is the compressive sensing technique (CS), which can outperform classical sampling methodologies by using far fewer samples while achieving exact recovery [1-2]. In this paper, we investigate the possible usage of the CS for the effective measurement and reconstruction of the tsunami parameters of water surface fluctuation, particle velocities, and tsunami-induced wave pressures [3]. Using the data sets of the Japanese Tohoku Tsunami occurred in 2011 after a major earthquake of Mw 9.0 [4], provided by the USA's National Oceanic and Atmospheric Administration (NOAA)'s Deep-Ocean Assessment and Reporting of Tsunamis (DART) portal, we show that CS can be used as an effective tool for the measurement, analysis, and reconstruction of the tsunami and tsunami-structure interaction parameters. Although we limit ourselves with the reconstruction of water surface fluctuations, particle velocities and tsunami-induced dynamic pressures, the CS can be applied for monitoring of the tsunami parameters in more general settings including the effects of vortices and shorter waves [5, 6]. We discuss our findings and comment on their possible applicability and usage.

**Keywords:** Tsunamis, tsunami-structure interaction, compressive sensing.

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✉ Corresponding Author Email: [cbayindir@itu.edu.tr](mailto:cbayindir@itu.edu.tr)

## Analysis of Wave Runup, Overtopping and Overwash Parameters via Compressive Sensing

Ali Rıza Alan<sup>1</sup>✉, Cihan Bayındır<sup>1,2</sup>

<sup>1</sup> *Istanbul Technical University, Civil Engineering Department, İstanbul, Turkey.*

<sup>2</sup> *Boğaziçi University, Civil Engineering Department, İstanbul, Turkey.*

### Abstract

The analysis of wave overtopping and overwash is fundamental to prevent damage to coastal structures and zones. There are many studies in the literature on this subject that shed light on today's research [1-2]. Wave overwash modeling methods are principally based on the prediction and generation of overtopping parameters as the essential inputs [3]. Currently, available methods are inefficient for the evaluation of the big field data. Recording and analyzing these data with efficient sensing are fundamentally significant for the observation, appraisal, and prevention of catastrophic results of coastal hazards. For this purpose, new algorithms should be developed, implemented, and tested. Compressive sensing technique (CS) is one of the most efficient algorithms that can beat old-style sensing approaches by utilizing far fewer samples while accomplishing accurate recovery [4-5]. In this paper, we investigate the possible usage of the CS for the viable estimation and analysis of wave runup, overtopping, and overwash for coastal areas. Using the time-series data sets of wave runup, overtopping, and overwash, as well as some other empirical formulas proposed by Mase [6], we show that CS may be utilized as a powerful instrument for the estimation, investigation, and analysis of wave overtopping and overwash in coastal areas and structural health monitoring. We discuss our results and remark on their pertinency and possible usage areas. The results of this study will be useful for the coastal engineering community in implementing wave runup, overtopping, and overwash reduction strategies to mitigate coastal hazards and the associated human and economic losses.

**Keywords:** Wave runup, wave overtopping, wave overwash, compressive sensing.

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✉ Corresponding Author Email: [alan21@itu.edu.tr](mailto:alan21@itu.edu.tr)

## Berezin Symbols and Related Problems

Ulaş Yamancı✉

*Süleyman Demirel University, Department of Statistics, Isparta, Turkey*

### Abstract

The Berezin symbol which was introduced by Berezin (1972) is a bounded function by the norm of the operator. Every bounded operator on the most familiar RKHS is uniquely determined by its Berezin symbol. We discuss some problems in terms of the Berezin symbol of diagonal operator in the reproducing kernel Hilbert space.

**Keywords:** Berezin symbol, reproducing kernel, reproducing kernel Hilbert space, diagonal operator, radial limit.

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✉ Corresponding Author Email: [ulasyamanci@sdu.edu.tr](mailto:ulasyamanci@sdu.edu.tr)

## Mathematical Modelling of Drying Kinetics of Cantaloupe in a Solar Assisted Dryer

Melike Sultan Karasu Asnaz<sup>1</sup>✉

<sup>1</sup> Balikesir University, Industrial Engineering Department, Balikesir, Turkey

### Abstract

Crop drying, which aims to reduce the moisture content to a certain level, is a method used to extend the shelf life, and prevent it from spoiling. One of the oldest food preservation techniques is open sun or shade drying. Even though this technique is the most affordable of all drying methods, there are some drawbacks such as contamination by insects, environmental pollution, windborne dust, and direct exposure to weather conditions such as wind, rain, hail [1]. However, solar dryers that provide a hygienic and controllable environment to preserve food and extend its shelf life have been developed and used to dry agricultural products. Thus, foods can be dried quickly without being affected by weather variables and quality products can be obtained [2].

This research is mainly devoted to investigating the modelling of drying kinetics of cantaloupe in a forced convection solar dryer. Mathematical models for the drying process should be defined to simulate the drying behavior of the foodstuff, which will greatly contribute to the development of solar dryer designs [3]. Thus, drying experiments were conducted and replicated five times, and various data such as temperature, relative humidity, solar irradiation, drying air speed, and weight were instantly monitored and recorded. Moisture content of sliced and pretreated cantaloupe were converted into moisture ratio, and then fitted against drying time for constructing drying curves. Then, 10 quasi-theoretical and empirical drying models were applied to find the best drying curve equation according to the Levenberg-Marquardt nonlinear optimization method. The best fitted mathematical drying model was selected according to the highest coefficient of determination ( $R^2$ ), and the mean square of the deviations ( $\chi^2$ ) and root mean square error (RMSE) criteria [4]. The best fitted model was utilized to simulate a thin layer solar drying of cantaloupe, and the simulation results were compared with the experimental data for validation purposes.

**Keywords:** Solar dryer, mathematical modelling, drying kinetics, cantaloupe drying.

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✉ Corresponding Author Email: [karasu@balikesir.edu.tr](mailto:karasu@balikesir.edu.tr)

### 3-D MHD Flow Over Array of Cubic Ducts

Selçuk Han Aydın

*Karadeniz Technical University, Department of Mathematics, Trabzon, Turkey*

#### Abstract

The magnetohydrodynamic (MHD) flow has many important applications in different area. Therefore, it is one of the most popular and important task to obtain the solution of the MHD equations formed by the coupled partial differential equations. In this study, we have considered the stabilized FEM solutions of the 3-D MHD equations over array of several cubic ducts separated by conducting thin walls. The considered stabilization method called SUPG(Streamline Upwind Petrov-Galerkin) enables to obtain stabilized numerical solutions for the high values of the Hartmann number. Different problem configurations are considered depending on the direction of the applied magnetic field and the number of ducts. Obtained solutions are displayed in terms of figures using the 2-D slices of the 3-D plots at different levels.

**Keywords:** 3D MHD flow, stabilized FEM, Cubic duct.

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✉ Corresponding Author Email: [shaydin@ktu.edu.tr](mailto:shaydin@ktu.edu.tr)

## Stabilization in 3-D FEM and Solution of MHD Duct Equations

Selçuk Han Aydın<sup>✉</sup>, Mahir Ceylan Erdoğan

*Karadeniz Technical University, Department of Mathematics, Trabzon, Turkey*

### Abstract

In this study, we have considered the numerical solution of 3-D convection-diffusion typed equations using the finite element method (FEM). In order to show the accuracy of the FEM, firstly we have tested the numerical procedure on the Laplace equation with known exact solution on the cubic and spherical domains. Then, streamline upwind Petrov-Galerkin (SUPG) type stabilized version of FEM is introduced for the 3-D problems. The proposed algorithm is test on convection dominated convection diffusion problems on L-shaped and cubic domains where both of them contain boundary layers for the small values of the diffusion coefficients. It is seen that even there are some numerical instabilities in the numerical solutions obtained by using standart FEM, the stabilized model eliminates this difficulties and enables to obtain stable and accurate numerical results. Finally, the extended version of the stabilized numerical procedure is applied to the solution of 3-D Magnetohydrodynamic (MHD) duct equations on the cubic duct. The coupled partial differential equations for the velocity and induced magnetic field are solved simultaneously for the different values of the Hartmann number and different externally applied magnetic field directions. All of the test results are displayed in terms of the tables and figures in order to show the accuracy of the proposed algorithm. Also the boundary layer behaviors are visualized using the 2-D slices of the 3-D plots at different levels.

**Keywords:** 3D-FEM, Convection-diffusion equation, Stabilization, MHD duct flow

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<sup>✉</sup>Corresponding Author Email: [shaydin@ktu.edu.tr](mailto:shaydin@ktu.edu.tr)



## Stabilized FEM Solution of Liquid-metal MHD Flow in a Rectangular Duct with Conducting Cracks

Münevver Tezer-Sezgin<sup>1✉</sup>, Selçuk Han Aydın<sup>2</sup>

<sup>1</sup>*Department of Mathematics, Middle East Technical University, 06800 Ankara, Turkey*

<sup>2</sup>*Department of Mathematics, Karadeniz Technical University, 61080 Trabzon, Turkey*

### Abstract

In this study, the numerical solution of the fully developed liquid-metal magnetohydrodynamic (MHD) flow is considered in a rectangular duct with the boundary conditions as no-slip velocity and insulated walls with crack regions [1]. An external oblique magnetic field is applied with an angle  $\alpha$  made with the y-axis. The mathematical model of the considered physical problem is governed by the coupled MHD flow equations in terms of the velocity of the fluid and the induced magnetic field. These coupled equations are transformed first into decoupled convection-diffusion type equations in order to apply the SUPG stabilization in the finite element method (FEM) solution procedure for high values of Hartmann number which determine the convection dominant case [2]. Obtained stable numerical solutions for high values of Hartmann number and several orientation angles of external magnetic field as well as different crack configurations depict the effects of these parameters on the flow and induced current. The velocity of the fluid and the induced magnetic field display also the well-known characteristics of the MHD pipe flow as Hartmann number increases. Also, the flowrates are presented for different crack positions in the walls, and for number and the lengths of the cracks. It is found that, the flowrate drops with an increase in the number and lengths of the cracks which are located on the Hartmann wall. If the crack is located on the side layer, it does not significantly affect the flowrate. The FEM with SUPG stabilization is capable of simulating flow changes for high values of Hartmann number even in the small-sized crack regions.

**Keywords:** Stabilized FEM, MHD pipe flow, Conducting boundary cracks.

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Corresponding Author Email: [munt@metu.edu.tr](mailto:munt@metu.edu.tr)

## Numerical Simulations of the Modified Regularized Long Wave Process

Asuman Zeytinoglu<sup>✉</sup>

*Suleyman Demirel University, Mathematics Department, Isparta, Turkey*

### Abstract

The purpose of this study is to introduce in capturing numerical simulations of the modified regularized long wave (MRLW) equation by obtaining efficient and accurate numerical solutions. The MRLW equation is known as the special nonlinear form of the generalized regularized long wave (GRLW) equation which models a wide range of physical processes in several areas including nonlinear transverse waves in shallow water, ion acoustics and magneto-hydrodynamic waves in plasma, longitudinal dispersive wave in elastics rods, rotating flow down a tube and pressure waves in liquid gas bubble mixtures. To produce the efficient and accurate numerical solutions for the MRLW equation, a compact finite difference scheme, which needs less storage space as opposed to the conventional numerical techniques, is preferred to discretize the spatial derivatives of the model equation without any linearization or transforming procedure, and then MacCormack method is applied to the resulting semi-discrete differential equations system in time. The proposed method is applied for some test problems. To validate the accuracy and efficiency of the method, some error norms, invariants corresponding to conservation of mass, momentum and energy, and the relative percent errors of the invariants are computed. The wave motions for considered test problems are also displayed at several times. The produced results are compared with some earlier studies, and it can be said from presented results that the method is seen to be a strongly advisable alternative to discover both qualitative and quantitative behaviors of similar physical processes.

**Keywords:** Modified regularized long wave equation, solitary wave, compact finite difference scheme, MacCormack method.

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<sup>✉</sup> Corresponding Author Email: [asumanzeytinoglu@sdu.edu.tr](mailto:asumanzeytinoglu@sdu.edu.tr)

## RBF Solution of MHD Flow in a Square Duct

Merve Gürbüz<sup>✉</sup>

<sup>1</sup> *Başkent University, Department of Management, Ankara, Turkey*

### Abstract

This paper considers the numerical solution of the two dimensional, steady, laminar flow of an incompressible, viscous and electrically conducting fluid in a cross-section of square duct under the impact of uniform magnetic field which is applied in the  $y$ -direction. The magnetohydrodynamics (MHD) equations [1-2] are the Navier-Stokes equations of the fluid motion coupled with Maxwell's equations of electromagnetic through Ohm's law. The non-dimensional form of the MHD equations are solved in terms of velocity and electric potential by using radial basis function (RBF) approximation for the computational efficiency and its easy implementation [3]. The boundary conditions for the electric potential are obtained from the RBF coordinate matrix for the space derivatives of the electric potential. The numerical results are simulated for several values of Hartmann number ( $M$ ) and wall conductance ratios ( $c_1, c_r, c_t, c_b$ ) to analyse the influences of the magnetic field and the wall conductivity, respectively, on the flow and the electric potential behaviours. It is found that, an increase in the Hartmann number causes to develop boundary layers on the Hartmann walls (perpendicular to applied magnetic field) and side walls (parallel to magnetic field). This is an expected behaviour for the MHD flow problem. As either intensity of the magnetic field or the wall conductance ratio increases, the magnitude of the flow decreases. When the wall conductance ratio is included to the boundary condition for the electric potential, M shaped flow behaviour is observed for the high Hartmann number. The increase in the wall conductance ratio decreases the magnetic potential value.

**Keywords:** RBF approximation, MHD flow, wall conductance ratio.

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✉ Corresponding Author Email: [mervegurbuz@baskent.edu.tr](mailto:mervegurbuz@baskent.edu.tr)

## The Effects of Problem Parameters on the DRBEM Solution of MHD Flow Subjected to the Time-Varied Magnetic Field

Elif Ebr̈en Kaya<sup>1✉</sup>, M̈unevver Tezer-Sezgin<sup>1</sup>

<sup>1</sup> Department of Mathematics, Middle East Technical University, 06800 Ankara, Turkey

### Abstract

The aim of the present study is to examine transient behavior of the MHD flow of a viscous, incompressible and electrically conducting fluid in a channel influenced by an external magnetic field  $B_0(t)=B_0f(t)$ .  $B_0$  is the intensity of the applied magnetic field at the initial time and  $f(t)$  determines the time variation of the applied magnetic field [1]. The time-varied function  $f(t)$  is chosen as a polynomial, exponential and trigonometric function to depict the effects on the flow at transient levels together with the effects of the problem parameters as  $R_e$  and  $R_m$ . The dual reciprocity boundary element method (DRBEM) [2] combined with the implicit forward finite difference for the time derivatives is used to solve coupled MHD equations in the cross-section of the channel (duct). The main advantage of DRBEM lies in the discretization of the boundary of the duct and obtaining unknown velocity and induced magnetic field both on the boundary and interior region. The study revealed that, the increase in Reynolds number ( $R_e$ ) or magnetic Reynolds number ( $R_m$ ) postpones the time level, that the flow elongates, to a further time level (i.e. the flow elliptical vortex turns to the direction of the applied magnetic field). Also, as  $R_m$  increases the magnitude of the flow increases, on the other hand, as  $R_e$  increases the flow is flattened as in the case of Hartmann number increase. The velocity and induced current profiles are presented at different transient levels by taking Hartmann number ( $Ha$ ) as 20 and when the direction of applied magnetic field is parallel to the  $x$ -axis. For polynomial and exponential type functions,  $f(t)$ , the flow and induced current keep the same behavior after the flow elongates. The trigonometric type  $f(t)$  causes the flow to repeat the original and elongation level behaviors with a period.

**Keywords:** Drbem, unsteady MHD duct flow, time-varied magnetic field

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✉ Corresponding Author Email: [elifebren90@gmail.com](mailto:elifebren90@gmail.com)

## A BEM Approach for Time-dependent Convection-Diffusion Type Equation With Variable Coefficients

Hande Fendođlu<sup>1</sup>✉, Canan Bozkaya<sup>1</sup>, Münevver Tezer-Sezgin<sup>1</sup>

<sup>1</sup> Middle East Technical University, Department of Mathematics, Ankara, Turkey

### Abstract

In this work, a numerical study is carried for solving the time-dependent convection-diffusion type equations with variable coefficients by using two different boundary element approaches. That is, the governing equations are discretized by either domain BEM (DBEM) or dual reciprocity BEM (DRBEM) in the spatial domain while a backward finite difference scheme is adopted for the time integration. DBEM or DRBEM is used to transform the differential equations into equivalent integral equations by employing the fundamental solution of convection-diffusion equation. The resulting BEM integral equations contain a domain integral which is kept in DBEM and treated by numerical integration, while it is transformed into a boundary integral by means of radial basis functions in DRBEM [1,2].

The application of the methods are explained through the convection-diffusion (CD) equation with variable coefficients and the codes are validated by the exact solution of the heat conduction problem which is governed by the CD equation with varying coefficients in a square computational domain. Then, the techniques are implemented for some fluid dynamics problems, namely, lid-driven flow, natural convection flow, channel flow and MHD natural convection flow in a porous medium. All these problems are governed by the NS equations and energy equation in the presence of a heat source, in which the momentum and energy equations can be treated as CD type equations involving variable convective coefficients which are functions of the unknown.

It is observed that, for the heat conduction problems involving variable coefficients of space variable, the use of both DBEM and DRBEM with the fundamental solution of CD equation results in reasonably well compatible results with the exact solutions. It is found that for the fluid dynamics problems governed by NS equations, energy and MHD flow equations, the DBEM with the fundamental solution of CD equation can be performed as an alternative numerical technique which gives quite accurate results for small and moderate values of Reynolds, Rayleigh and Hartmann numbers.

**Keywords:** DRBEM, DBEM, Convection-Diffusion equation, Navier-Stokes equations.

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✉ Corresponding Author Email: [handefendoglu@gmail.com](mailto:handefendoglu@gmail.com)

## Numerical Solutions of Kaup-Kupershmidt and Ito Equations with B-spline Collocation Method

Derya Sucu<sup>1✉</sup>, Seydi Battal Gazi Karakoç<sup>1</sup>

<sup>1</sup> Nevşehir Hacı Bektaş Veli University, Department of Mathematics, Nevşehir, Turkey

### Abstract

In this study, it is aimed to obtain the numerical solutions of two types of fifth-order Korteweg-de Vries (KdV) equations namely Kaup-Kupershmidt (K-K) and Ito. For this purpose, collocation finite element method is used.  $L_2$  and  $L_\infty$  error norms are computed for single soliton solutions to demonstrate the proficiency and accuracy of the present method. The method is shown to be unconditionally stable by performing the von-Neumann stability analysis.

A well-known model of the generalized fifth-order nonlinear evolution equations of the form:

$$u_t + \alpha u^2 u_x + \beta u_x u_{xx} + \gamma u u_{xxx} + u_{xxxx} = 0 \quad (1)$$

where  $\alpha, \beta$  and  $\gamma$  are arbitrary nonzero and real parameters and  $u = u(x, t)$  is a differentiable function, is an important mathematical model with wide applications in quantum mechanics and nonlinear optics [1-3]. As the constants  $\alpha, \beta$  and  $\gamma$  take different values, the properties of equation (1) drastically change. Such as K-K equation with  $\alpha = 20, \beta = 25, \gamma = 10$  is also known to be integrable [4] and to have bilinear representations [5], but the explicit form of its  $N$ -soliton solution is apparently not known. Kaup-Kupershmidt equation has the following form:

$$u_t + 20u^2 u_x + 25u_x u_{xx} + 10u u_{xxx} + u_{xxxx} = 0.$$

Ito equation with  $\alpha = 2, \beta = 6, \gamma = 3$  which is not completely integrable but has a limited number of conservation laws [6] and the equation has the following form:

$$u_t + 2u^2 u_x + 6u_x u_{xx} + 3u u_{xxx} + u_{xxxx} = 0.$$

**Keywords:** Kaup-Kupershmidt equation, ito equation, collocation finite element method.

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✉ Corresponding Author Email: [deryasucu@aksaray.edu.tr](mailto:deryasucu@aksaray.edu.tr)

## Ocean Energy Conversion Analysis by Compressive Sensing

Ali Rıza Alan<sup>1✉</sup>, Cihan Bayındır<sup>1,2</sup>

<sup>1</sup> *İstanbul Technical University, Civil Engineering Department, İstanbul, Turkey.*

<sup>2</sup> *Boğaziçi University, Civil Engineering Department, İstanbul, Turkey.*

### Abstract

Considering that many types of energy resources in the world are limited, it is inevitable to focus on renewable energy sources to meet the needs of the world's increasing population. Ocean energy is undoubtedly one of the most efficient resources among renewable energy sources in terms of its potential. It is created by winds as they blow across the seas, and this movement gives a helpful and characteristic convergence of energy in the ocean environment [1]. There are many studies in the literature that have been conducted about energy conversion and which are guiding for further studies [2-3]. Ocean wave energy conversion modeling methods are principally based on the basic wave parameters with some additional concepts. Although some concepts have been developed so far, with the advancing technology, scientists have sought superior solutions. The methods to be used to analyze ocean wave energy are as significant as obtaining the ocean energy itself, especially in the big data era for electricity generation and grid connection purposes. The compressive sensing (CS) technique, which outperforms the classical techniques since it uses a smaller number of samples [4-5], is one of the algorithms that can be used for such purposes. In this paper, we examine the utilization of the CS for the efficient analysis and assessment of ocean wave energy and ocean energy conversion in general. Using the time series of the wave energy flux given in an experimental study [6], the application of the CS proves to be an advantageous tool for the analysis and assessment of wave energy and ocean energy conversion. We discuss our findings and comment on their possible usage and applications.

**Keywords:** Ocean energy, wave energy, energy converter, compressive sensing

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✉ Corresponding Author Email: [alan21@itu.edu.tr](mailto:alan21@itu.edu.tr)

## Stabilizing the Self-Localized Solitons of the Kundu-Eckhaus Equation by Dissipation

Hazal Yurtbak <sup>1</sup>✉, Cihan Bayındır <sup>2,3</sup>

<sup>1</sup>Czech Technical University, Mathematical Engineering Department, Prague, Czechia

<sup>2</sup>İstanbul Technical University, Civil Engineering Department, İstanbul, Turkey.

<sup>3</sup>Boğaziçi University, Civil Engineering Department, İstanbul, Turkey.

### Abstract

The Kundu-Eckhaus equation (KEE) is a nonlinear partial differential equation in the nonlinear Schrödinger equation (NLSE) class. This equation was introduced to the scientific literature independently by Kundu [1] and Eckhaus [2]. It is well-known that KEE admits many different analytical solutions like the NLSE. Those solutions of the KEE are widely used in fields such as nonlinear optics, fiber optical waveforms, water waves mechanics, and hydraulics, just to name a few. In this study, the effect of loss/gain on the soliton solutions of the KEE has been investigated. With this aim, we study the dissipative Kundu-Eckhaus equation (dKEE) [4-5]. We analyze the effects of dissipation in the form of a loss term on the self-localized solitons of the dKEE. For this purpose, we propose a Petviashvili method (PM) for the numerical construction of the soliton solution of the dKEE [6]. Using PM, we first numerically compute the soliton solutions of the dKEE and discuss their properties. Then, we analyze the effects of dissipation on the dynamics and stabilities of those soliton using a split-step Fourier method (SSFM) implemented for time-stepping purposes. We show that the dKEE equation admits one and two soliton solutions for zero potential and for photorefractive potential ( $V = I_0 \cos^2(x)$ ) cases. Since the solitons under the photorefractive potentials turned out to be unstable during temporal evolution, we introduce and discuss the effects of dissipation on the dynamics and stabilization of those solitons. The effects of dissipation on soliton characteristics and power are also discussed.

**Keywords:** Dissipative Kundu-Eckhaus equation, Petviashvili method, Split-step Fourier scheme, solitons.

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✉ Corresponding Author Email: [yurtbakhazal@gmail.com](mailto:yurtbakhazal@gmail.com)



## Anemia Prediction Based on Logistic Model Tree Method

Arshed A. Ahmad<sup>1✉</sup>, Ihsan Salman<sup>1</sup>, Murat Sari<sup>2</sup>, Mohammed S. Mohammed<sup>1</sup>, Hande Uslu<sup>2</sup>

<sup>1</sup>Diyala University, Computer Department, Diyala, Iraq

<sup>2</sup>Yildiz Technical University, Mathematics Department, Istanbul, Turkey

### Abstract

Healthcare problems require efficient implementation for building a disease prediction system in medical institutes to be helpful for diagnosis or even for treatment process. This study therefore aims to predict anemia from a population through biomedical variables. So, the anemia prediction system are enhanced here by using three different techniques; Hoeffding Tree, Logistic Model Tree (LMT), and Random Tree. To achieve this, a dataset consisting of 539 subjects [1,2] has been considered under the consideration of useful features. Applying attribute selector on the present methods provides lower cost, faster prediction and more accurate than just a traditional method. The LMT has provided higher accuracy comparison to other two techniques due to, more importantly, advantage of combination a tree and logistic regression [3-7].

**Keywords:** Anemia, Logistic Model Tree, Hoeffding Tree, Random Tree, Prediction

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✉ Corresponding Author Email: [arshed980@gmail.com](mailto:arshed980@gmail.com)

## Prediction of Anemia through Particle Swarm Optimization

Arshed A. Ahmad<sup>1✉</sup>, Khalid M. Saffer<sup>2</sup>, Murat Sari<sup>3</sup>, Hande Uslu<sup>3</sup>

<sup>1</sup>*Diyala University, Computer Department, Diyala, Iraq*

<sup>1</sup>*Diyala University, Computer Science Department, Diyala, Iraq*

<sup>2</sup>*Yildiz Technical University, Mathematics Department, Istanbul, Turkey*

### Abstract

Healthcare needs the maintenance or improvement of health via the prevention, diagnosis, treatment, and other physical and mental cares for people. So, anemia is one of the most common health issues at this era. This paper aims at estimating pathological individuals from a population through various biomedical variables by using particle swarm optimization (PSO). Dataset consisting of some blood variables (eight blood variables, sex, and age) and output (anemia types) and conducted in terms of data set consisting of 539 subjects [1,2]. Biomedical variables are used to forecast anemia types as independent parameters. Since the PSO starts randomly and walks all solution space, the produced results in terms of the proposed PSO algorithm seem to be reasonably good for predicting anemia types with large number of the biomedical variables [3-7].

**Keywords:** Anemia, Particle Swarm Optimization, Prediction

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✉ Corresponding Author Email: [arshed980@gmail.com](mailto:arshed980@gmail.com)

## Effects of Anemia Features on Data Mining Performances

Mohammed S. Mohammed<sup>1✉</sup>, Ahmed Kh. Abbas<sup>1</sup>, Murat Sari<sup>2</sup>, Arshed A. Ahmad<sup>1</sup>, Hande Uslu<sup>2</sup>

<sup>1</sup>University of Diyala, Computer Department, Diyala, Iraq

<sup>2</sup>Yildiz Technical University, Mathematics Department, Istanbul, 34220, Turkey

### Abstract

Implementation of a single system of machine learning methods for biomedical problems has been affected by disease features. Anemia is one of the diseases that need a fast and accurate diagnosis to prevent it or just to reduce possible danger in early stages. This talk aims to investigate if there is a relation between the biomedical factors through on developing machine learning prediction by specifying which techniques are mainly affected by blood variables [1,2]. Neglecting the less effective features of the patients leads to the reduction of cost and time and improve system performance. Relatively less amount of data needs accurate techniques to deal with it regardless of the number of features [3-8].

**Keywords:** Anemia, Data Mining, Biomedical variables

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✉ Corresponding Author Email: [mohammed\\_sami9@yahoo.com](mailto:mohammed_sami9@yahoo.com)

## Random Forest Regression Model Extended by Alternative Model Selection Procedure

Mehmet Ali Kaygusuz<sup>✉</sup>, Vilda Purutcuoğlu

<sup>1</sup> Middle East Technical University, Department of Statistics, Ankara, Turkey

<sup>2</sup> Middle East Technical University, Department of Statistics and Department of Bioengineering, Ankara, Turkey

### Abstract

Multiple testing procedures are very popular in recent years due to the huge amount of data in genetics, engineering and finance. In order to construct proper models for those data, different approaches are suggested. More recently, Candès et al. (2018) propose the conditional randomization tests (CRT) while there is no distributional assumption about the model. On the other hand, Tansey et al. (2019) suggest the hold-out-randomization test by using both bootstrap and cross validation algorithms. Previously, the causal additive model for the high dimensional regression is presented by Buehlmann et al. (2014). Then, Heinze-Deml et al. (2018) study independent test for the causal additive model. Currently, Bates et al. (2020) apply the Model-X knockoff filter for these models. Hereby, in this study, we investigate the computational cost and statistical properties of the random forest model which is one of the well-known causal additive regression approaches. In our analyses, we detect its accuracy by using two distinct model selection criteria, namely, consistent Akaike information criterion (AIC) with Fisher information matrix and Information and COMPLEXity criterion (ICOMP). We evaluate the performance of the underlying extended model via protein-protein interaction networks' datasets under different dimensions and compare the results via the original random forest model.


**Keywords:** Causality, nonparametric regression, independent test, model selection.

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✉ Corresponding Author Email: [vpurutcu@metu.edu.tr](mailto:vpurutcu@metu.edu.tr)

## Deep Learning with Multivariate Adaptive Regression Spline with Bagging Methods

Mehmet Ali Kaygusuz<sup>1</sup>, Abdullah Nuri Somuncuoğlu<sup>2</sup>, Vilda Purutcuoğlu <sup>1, 2</sup>

<sup>1</sup> Middle East Technical University, Department of Statistics, Ankara, Turkey

<sup>2</sup> Middle East Technical University, Department of Biomedical Engineering, Ankara, Turkey


### Abstract

Deep learning methods have huge success in application in the recent years. Bauer and Kohler et al. (2020) studied the relationship between deep generative networks and Multivariate adaptive regression spline (MARS) regression in low dimensionality. On the other hand, this problem can be very difficult when the number of parameter is more than the number of samples which is called the overparametrized problem. The main purpose of this study is to investigate the computational and statistical properties of deep learning with the MARS regression in such a way that the classified object via deep learning will be modelled by MARS whose model selection will be further optimized by information complexity approach (Koc and Bozdogan, 2015). By this way the plausible groups in the data can be detected, resulting in better fitting when MARS is implemented within each group separately. Because, MARS has a great flexibility to explain explanatory variables with the help of spline functions (Friedman, 1991). Moreover, to cope with the problem of bias-variance trade-off in the calculation, bagging (Bootstrap aggregating) methods (Breiman, 1991) may be more appropriate for the MARS regression (Buehlmann and Yu (2002)) since bagging approaches are variance reduction techniques and have less computational cost. In our assessment, we examine our proposal approaches in different dimensional protein-protein interaction networks data and distinct biomedical signal datasets.

**Keywords:** Deep learning, Bagging methods, MARS regression.

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 Corresponding Author Email: [vpurutcu@metu.edu.tr](mailto:vpurutcu@metu.edu.tr)

## The Effects of Turbulent Fluctuations on Nonlinear von Kármán Vortex Shedding

Kubilay Ateş<sup>1✉</sup>, Cihan Bayındır<sup>2,3</sup>

<sup>1</sup>*Yıldız Technical University, Civil Engineering Department, İstanbul, Turkey.*

<sup>2</sup>*İstanbul Technical University, Civil Engineering Department, İstanbul, Turkey.*

<sup>3</sup>*Boğaziçi University, Civil Engineering Department, İstanbul, Turkey.*

### Abstract

Flows around bluff bodies generate wakes and vortices downstream of flow. This phenomenon is known as vortex shedding and such vortices are generally named as von Kármán vortices after Theodore von Kármán. Although such a phenomenon is introduced to the scientific literature by the study of fluid flows, it is also observed in other fields such as Bose-Einstein condensation. Due to the complexity of the governing equations and involved complex geometries, such phenomena are generally studied numerically using different software and various turbulent modeling techniques. One of the other commonly utilized models for the study of nonlinear vortex shedding is the complex Ginzburg-Landau (GL) equation [1-4]. This dynamic equation is an equation in the nonlinear Schrödinger class and also appears in various other branches of science. In this paper, we investigate the effects of turbulent fluctuations on the vortex shedding in the frame of the GL equation. With this aim, we solve the GL equation using a spectral scheme with a 4<sup>th</sup> order Runge-Kutta time integrator. For the spectral solution, efficient FFT routines are employed. We analyze the possible modulation instabilities causes by turbulent fluctuations, their effects on the regular stable vortices, and possible rogue vortex formation [5-6]. We also study the dynamics and statistics of such vortices under the effect of turbulent fluctuations. Our findings can be used for controlling, mitigating, or resonating the vortices and wake for many different engineering purposes including but are not limited to structural safety and serviceability considerations, noise reduction, energy harvesting, just to name a few.

**Keywords:** Vortex shedding, Ginzburg-Landau equation, turbulent fluctuations, spectral method

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✉ Corresponding Author Email: [14918015@std.yildiz.edu.tr](mailto:14918015@std.yildiz.edu.tr)

## Numerical Solution Method for Delay Chemical Master Equation

Derya Altıntan✉

*Selcuk University, Mathematics. Department, Konya, Turkey*

### Abstract

The Chemical Master Equation (CME) or Kolmogorov forward equation is a system of Ordinary Differential Equation (ODEs) that explains the random dynamics of different stochastic processes modeled by Continuous-Time Markov Chains (CTMCs). For each state in the system, CME produces a new differential equation. As a result of this fact, the size of the system of the differential equation increases exponentially with the number of states. Therefore, numerical methods are needed to obtain numerical solutions of the CME [4]. The dynamics of some stochastic processes such as cellular processes, biochemical reactions etc. at given specific time can hinge on the dynamics of the earlier times. This issue necessitates the extension of the CME to a delay CME, namely, the Delay Chemical Master Equation (DCME) [3]. Similar to the CME, DCME produces Delay Differential Equation (DDE) system whose size depends on the number of states in the system. Therefore, DCME also suffers from the curse of dimensionality.

In this study, we propose a Runge-Kutta method to obtain numerical solutions of the DCME [1,2] and implement the method to biochemical reaction systems.

**Keywords:** Chemical master equation, delay differential equations, delay chemical master equation

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✉ Corresponding Author Email: [altintan@selcuk.edu.tr](mailto:altintan@selcuk.edu.tr)

## The Interaction of Von Kármán Vortices with the Solitons of the Complex Ginzburg-Landau Equation

Sofi Farazande<sup>1✉</sup>, Cihan Bayındır<sup>1,2</sup>

<sup>1</sup>Boğaziçi University, Civil Engineering Department, İstanbul, Turkey

<sup>2</sup>İstanbul Technical University, Civil Engineering Department, İstanbul, Turkey

### Abstract

The complex Ginzburg-Landau (GL) equation is a well-known equation in various areas of physics that is also widely used to model vortex shedding phenomena occurring around a bluff body in a flow field, which is named as von Kármán vortex street [1-2]. In addition, it describes nonlinear waves, second-order phase transitions, superconductivity, superfluidity, Bose-Einstein condensation, liquid crystals, and strings in field theory, etc. [3]. Moreover, the GL equation can be utilized to find soliton solutions of many nonlinear systems [4]. Solitons are self-localized, solitary, nonlinear wave which emerges from a collision with a similar pulse having unchanged shape and speed [5]. Most of its applications lie in the domains of optics and fluid mechanics, which are attained by solutions of some familiar partial differential equations as Korteweg-de-Vries, modified Korteweg-de-Vries, Sine-Gordon and nonlinear Schrödinger equations [6], apart from GL equation. In the present study, we aim to analyze the interaction of the soliton solutions of the GL equation with von Kármán vortex street. For this purpose, we solve the GL equation to obtain the vortex shedding, by adopting a spectral scheme using FFT routines for the space derivative, and a 4<sup>th</sup> order Runge-Kutta time-stepping method. Subsequently, we construct the soliton solutions of GL either using analytical techniques [4] and/or numerical Petviashvili method, and superpose them with vortex figures to observe their interaction. We investigate how the vortex structure and stability are affected and whether the vortex fluctuations are reduced by the solitons. We analyze the spectral properties of their interactions and the mechanisms that lead to changes in vortex shedding. We discuss our findings and their possible usage in controlling the vortices by solitons for structural damage prevention and resonating for energy harvesting.

**Keywords:** von Kármán vortex, solitons of Ginzburg-Landau equation, numerical model, soliton-vortex interaction

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✉ Corresponding Author Email: [sofi.farazande@boun.edu.tr](mailto:sofi.farazande@boun.edu.tr)



## Inference and Marginalization Algorithms for Jump Diffusion Approximation

Derya Altıntan<sup>1,✉</sup>, Heinz Koepl<sup>2</sup>

<sup>1</sup>*Selcuk University, Mathematics Department, Konya, Turkey*

<sup>2</sup>*Department of Electrical Engineering and Information Technology, Technische Universität,  
Darmstadt, Darmstadt, Germany*

### Abstract

In cellular reaction systems, the abundance of species and the speed of reactions can change in a wide range. To exploit this multi-scale nature, we presented jump diffusion approximation which couples Markov updating algorithm with diffusion (Langevin) approach updating algorithm based on an error bound [3]. We also proved that the joint probability distribution of this hybrid model satisfies the hybrid master equation [1]. It is not always possible to know all states/parameters of cellular reaction systems. Therefore, inference algorithms are needed to estimate the unknown quantities. In this study, we develop bootstrap filtering/smoothing algorithms for reaction systems whose probability density function satisfies the hybrid master equation [2]. Isogenic cells, grown in the same environment, can be very different from each other. This variability is the result of intrinsic noise which refers to the inherent randomness hidden in the particular process and extrinsic noise which refers to the effect of other cellular processes in the reaction system under consideration. In this study, we also propose a mathematical framework that uncouples the reaction network modeled by jump-diffusion approximation from its extrinsic noise by using marginalization [4]. We implement all proposed algorithms to different types of reaction systems.

**Keywords:** Jump diffusion approximation, hybrid master equation, filtering/smoothing algorithms.

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✉ Corresponding Author Email: [altintan@selcuk.edu.tr](mailto:altintan@selcuk.edu.tr)

## Variation of Critical Buckling Load in Beam Structures Depending on Damage Region and Direction

Can Gonenli<sup>1</sup>✉

<sup>1</sup> Ege University, Ege Vocational School, Machine Drawing and Construction Department, Izmir, Turkey

### Abstract

Beam structures are the primary elements of many engineering designs. It is well known that defects lead to different dynamic effects on the structures, and they may cause those structures to fail. In this context, many researchers have investigated the effects of damages on different types of structures [1-3]. Also, some researchers investigated the critical buckling load of the structures, which is one of the dynamic characteristics of engineering designs [4-6]. In this study, the buckling analysis of steel beam structures with two different cross-sectional areas, one square (50x50 mm) and one rectangular (50x25 mm) is performed for the first three critical buckling loads by employing SolidWorks. In the buckling analysis of the square section beam with a length of 1 meter, the fixed dimensions of the cut-out (10x10 mm) are added separately to nine different points, and the buckling analysis is repeated. The buckling analysis is repeated in the rectangular section beam by adding the fixed damage at nine different points in two different directions. It has been determined that the critical buckling load of the structure varies according to the region and direction of the damage. While the effect of the damage on the bending zones of the structure is greater due to the buckling mode, the damage has no effect in the regions that are not affected by the relevant buckling mode. In addition, it seems possible to determine the region of damage by using dynamic characteristics in future studies.

**Keywords:** Buckling, beam, damage

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✉ Corresponding Author Email: [can.gonenli@ege.edu.tr](mailto:can.gonenli@ege.edu.tr)

## A Bidirectional Generalized Synchronization of Nonlinear Advection-Diffusion-Reaction Processes

Shko Ali Tahir<sup>1</sup>✉, Murat Sari<sup>2</sup>

<sup>1</sup> *University of Sulaimani, Department of Mathematics, Sulaimani, Iraq*

<sup>2</sup> *Yildiz Technical University, Department of Mathematics, İstanbul, Turkey*

### Abstract

This research focuses on generalized synchronization (GS) of two dependent chaotic nonlinear advection-diffusion-reaction (ADR) processes with forcing term. Based on the drive-response concept, which bidirectionally coupled is derived. The approach combines backward differentiation formula-Spline (BDFS) scheme with the Lyapunov direct method. To illustrate the proposed approach, simulation examples based on the ADR equations that provide the synchronization and the observer conception are presented.

**Keywords:** Nonlinear ADR equations, generalized synchronization, BDFS method, Lyapunov direct method

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✉ Corresponding Author Email: [shko.tahir@univsul.edu.iq](mailto:shko.tahir@univsul.edu.iq)

## Bivariate Mittag-Leffler Functions and Associations with Fractional Calculus

Arran Fernandez<sup>1✉</sup>, Cemaliye Kürt<sup>1</sup>, Mehmet Ali Özarslan<sup>1</sup>

<sup>1</sup> Department of Mathematics, Eastern Mediterranean University, Famagusta, Northern Cyprus

### Abstract

The classical Mittag-Leffler function  $E_\alpha(z)$ , defined as a power series in one variable  $z$  with one parameter  $\alpha$ , has deep and fundamental connections with fractional calculus. It appears naturally in the solutions of fractional differential equations, and it has been used as a kernel in some modern fractional operators. This function has been generalised to Mittag-Leffler functions with one variable  $z$  and two or more parameters, such as  $E_{\alpha,\beta}(z)$ , and these have also been studied in connection with fractional calculus.

We will discuss the extension of Mittag-Leffler functions to bivariate and multivariate versions, functions of two or more variables defined using double or multiple power series. Several authors have defined such functions and studied their basic mathematical properties [1,2,3], but there are many different aspects to the study of bivariate Mittag-Leffler functions which are only just emerging now.

In particular, we will focus on one bivariate Mittag-Leffler function which arose naturally from applications in bioengineering, and on the associated operators of fractional calculus defined using this function in the kernel [4]. This function appears as the solution to some elementary fractional differential equations with two independent fractional orders of differentiation, and its Laplace transform is in a useful and easily manipulable form. The associated fractional integral operators have a semigroup property which enables corresponding fractional derivative operators to be defined in a natural way. The resulting model of fractional calculus can be used in bioengineering applications.

**Keywords:** fractional calculus, Mittag-Leffler functions, bivariate Mittag-Leffler functions

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✉ Corresponding Author Email: [arran.fernandez@emu.edu.tr](mailto:arran.fernandez@emu.edu.tr)

## Solving Nabla Fractional Partial Difference Equations Using Discrete Homotopy Analysis Method

Figen Özpınar<sup>1</sup>✉

<sup>1</sup> Afyon Kocatepe University, Bolvadin Vocational School, Afyonkarahisar, Turkey

### Abstract

Fractional calculus has been of considerable interest in numerous fields of science and engineering, such as electrical networks, chemical physics, optics and signal processing, diffusion, viscoelasticity, and so on. Nabla fractional is a discretized version of the fractional derivative. Discrete fractional calculus has been recognized as a powerful instrument to explore the secret ways of physical processes and various materials, expressed by discrete versions of integrals and derivatives of arbitrary orders, called fractional sums and differences. We propose the discrete homotopy analysis method to solve linear and nonlinear nabla fractional initial value problems in the present paper. Nabla fractional differences are described by Caputo's sense. To illustrate the applicability of our approach, we apply the discrete homotopy analysis method to nabla fractional partial difference equations with initial value problems. Obtained results show that the approaches are easy to apply and accurate when implemented to time-fractional difference equations. Unlike the other analytical techniques, the discrete homotopy analysis method is independent of small/large physical parameters. Since the discrete homotopy analysis method has many advantages compared to other analytical methods, it is employed to solve linear and nonlinear nabla fractional initial value problems. The discrete homotopy analysis method contains the auxiliary parameter, which provides a simple way to guarantee the convergence region of the solution series. This method is quite powerful in solving wide classes of nabla fractional partial difference equations.

**Keywords:** Discrete homotopy analysis method, nabla fractional sum, Caputo-like nabla fractional difference, nabla difference equations.

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✉ Corresponding Author Email: [fozpınar@aku.edu.tr](mailto:fozpınar@aku.edu.tr)

## Numerical Computation of Measure-Valued Solutions to a Hyperbolic Fokker-Planck Equation Subject to Nonlocal Boundary Conditions

Hidekazu Yoshioka<sup>1✉</sup>, Kunihiko Hamagami<sup>2</sup>, Haruka Tomobe<sup>3</sup>

<sup>1</sup> Shimane University, Graduate School of Natural Science and Technology, Matsue, Japan

<sup>2</sup> Iwate University, Faculty of Agriculture, Morioka, Japan

<sup>3</sup> Tokyo Institute of Technology, Department of Civil and Environmental Engineering, Kanagawa, Japan

### Abstract

Fokker-Planck equations are partial integro-differential equations (PIDEs) governing probability densities of stochastic dynamics. Numerical computation of a Fokker-Planck equation is an important topic in both science and engineering because probability densities characterize macroscopic system dynamics. Stochastic dynamics driven by Lévy processes [1] were extensively studied as they are representative white noise processes; however, dynamics driven by non-standard noises, such as the continuous-state branching processes with immigration (CBI processes in short) [2], have not been studied well. We derive and analyze a Fokker-Planck equation of a nonsmooth stochastic dynamical system governing macroscopic hydrodynamics and sediment storage in aquatic environments, such as rivers, lakes, and seas. The former dynamics is represented by a CBI process, while the latter by simple nonsmooth storage dynamics where the nonsmoothness comes from a physical constraint that there is no sediment transport when the sediment is depleted. We consider an urgent engineering problem where the sediment is replenished randomly if the storage is below a threshold. The resulting Fokker-Planck equation is a unique hyperbolic PIDE having a singular integral subject to nonlocal boundary conditions, and admits measure-valued solutions in a Radon's sense. The equation has not been found in the literature so far. Based on experimental data, we present demonstrative computational examples of the equation using a globally-conservative finite volume scheme and apply it to the evaluation of replenishment schemes to suppress the bloom of nuisance benthic algae in a river. Future research direction coupling micro-macro dynamics and regularity of physical coefficients are also discussed.

**Keywords:** Nonsmooth stochastic dynamics, Fokker-Planck equation, Sediment replenishment

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\* Corresponding Author Email: [yoshih@life.shimane-u.ac.jp](mailto:yoshih@life.shimane-u.ac.jp)

## The use of mathematical modeling to analyse fear factor for a stochastic pre-predator system with linear functional response

Aytül Gökçe<sup>1</sup>✉

<sup>1</sup>Ordu University, Department of Mathematics, Ordu, Turkey

### Abstract

In this presentation, a prey-predator model including the cost of fear in the prey dynamics is considered with a linear functional response [1]. The density of prey species may be affected not only from direct predation, e.g. through killing, but also as a result of indirect predation, e.g. physiological changes in prey species [2,3,4]. Firstly, the mathematical analyses show that change in the level of fear in prey population does not effect the local stability of the system around the equilibria. Numerical simulations given in this presentation is performed to analyse the relationship between prey populations as a function of different system parameters. As a result, only transcritical bifurcation has been observed. Secondly, the model is analysed with noise term incorporated in the prey's death rate. The presence of noise term turns the given prey-predator model into stochastic differential equation and non-periodic noise related oscillations can be observed in both prey and predator densities. The conditions for extinction of the species with noise is also numerically and mathematically analysed.

**Keywords:** Stability analysis, linear functional response, population dynamics, stochastic differential equations

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✉ Corresponding Author Email: [aytulgokce@odu.edu.tr](mailto:aytulgokce@odu.edu.tr); [aytulgokceodu@gmail.com](mailto:aytulgokceodu@gmail.com)

## Dynamics of a Diffusive Oxygen– Plankton Model with Time Lag Effect and its Stability Behaviour

Aytül Gökçe<sup>1✉</sup>, Samire Yazar<sup>1</sup>, Yadigar Sekerci<sup>2</sup>

<sup>1</sup> Ordu University, Department of Mathematics, Ordu, Turkey

<sup>2</sup> Amasya University, Department of Mathematics, Amasya, Turkey

### Abstract

In this presentation, we analyse a generic diffusive model of dissolved oxygen, phytoplankton and zooplankton species, for which constant time delays are incorporated in growth response of phytoplankton and in the gestation time of zooplankton [1, 2]. We mainly focus on the stability analysis of the coexisting states and the existence of Hopf bifurcation through the characteristic equation, where time delay and oxygen production rate are chosen as control parameters for all cases. Although both delays have a destabilising effect, our results show that time delay in gestation may induce sharp irregularity in the spatiotemporal dynamical regimes (leading to chaotic oscillations) whereas time delay in phytoplankton growth lead to more regular but higher frequency oscillations for oxygen-plankton interactions.

**Keywords:** Stability, time delay, prey-predator interactions, oxygen-plankton system, chaos

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✉ Corresponding Author Email: [aytulgokceodu@odu.edu.tr](mailto:aytulgokceodu@odu.edu.tr)



## A Near Wall Model For the Navier-Stokes- $\alpha$ Turbulence Model

Özgül İlhan<sup>1</sup>✉

<sup>1</sup> Muğla Sıtkı Koçman University, Mathematics Department, Muğla, Turkey

### Abstract

The question of finding appropriate boundary conditions when using a constant length is known as Near Wall Modelling and a boundary condition is known as Near Wall Model. Very often, there are difficulties in LES models about predicting turbulence generated by interactions of a flow with a boundary. We used boundary conditions similar to those used by Navier [1] and Maxwell [2] in their work. Firstly, we compute appropriate friction coefficient using power law and then analyze its asymptotic behavior as the averaging radius  $\delta \rightarrow 0$ , and as the Reynolds number  $Re \rightarrow \infty$ . In this study, we don't purpose to develop new theories of laminer boundary layer, we use existing boundary layer theories and improve numerical boundary conditions for flow averages. We consider exactly this problem herein and we apply this boundary condition to Navier-Stokes- $\alpha$  model (NS- $\alpha$ ) [3]. Numerical tests on two dimensional channel flows across a step using this boundary condition on the top and bottom wall as well as on the step are performed [4].

**Keywords:** Boundary Layers, Turbulence, Near Wall Models (NWM), Power Law, Navier-Stokes- $\alpha$  model (NS- $\alpha$ ).

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✉ Corresponding Author Email: [oilhan@mu.edu.tr](mailto:oilhan@mu.edu.tr)

## Comparison of Statistical and Neural Regression Using Activation Functions Derived from Swish Activation Function

Yılmaz Koçak<sup>1</sup>✉, Gülesen Üstündağ Şiray<sup>2</sup>

<sup>1</sup> Cukurova University, Adana Vocational School of Higher Education, Adana, Turkey

<sup>2</sup> Cukurova University, Faculty of Arts and Sciences, Department of Statistics, Adana, Turkey

### Abstract

Artificial Neural Network (ANN) is a kind of artificial intelligence and it has been commonly used by scientists and practitioners. ANNs are computational tools that are widely accepted in many disciplines for modelling complex real-world problems such as function approximation, classification, regression, pattern recognition, and forecasting. The attractiveness of an ANN is due to its nonlinearity, parallelism, robustness, error and fault tolerance, learning, and ability to process. ANN learns from examples through iterations without demanding prior information on the relationships of parameters. The most important parameter of an ANN is the activation function (AF). AFs can significantly affect the performance of an ANN and therefore choosing a well-defined AF is important. In this study, we investigate the effects of AFs on the performance of any ANN for regression. For this purpose, ReLu-swish AF and generalized swish AF that are derived from the swish AF are considered. For the comparison of these AFs with swish AF, mean square error, mean absolute error, and  $R^2$  metrics are utilized. To investigate the performance of these AFs, different data sets which are simulated data and some benchmark data from the University of California Irvine Machine Learning Repository are used.

**Keywords:** Artificial neural network, activation function, swish AF, ReLu-swish AF, generalized swish AF

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✉ Corresponding Author Email: [ykocak@cu.edu.tr](mailto:ykocak@cu.edu.tr)

## Discrete Sturm-Liouville Equation with Point Interaction

Güher Gülçehre Özbey<sup>1✉</sup>, Yelda Aygar<sup>1</sup>, Güler Başak Öznur<sup>2</sup>

<sup>1</sup> Ankara University, Department of Mathematics, Ankara, Turkey

<sup>2</sup> Gazi University, Department of Mathematics, Ankara, Turkey

### Abstract

We present an investigation about scattering analysis of a boundary value problem for a discrete Sturm-Liouville equation with point interaction. Here, we give polynomial type Jost solution and the scattering function of this problem and we find the properties of scattering function by using the scattering solutions. We also give an asymptotic equation for the Jost solution of this problem. We also find the Green function and resolvent operator of this boundary value problem. Finally, we apply the new results on an example.

**Keywords:** Scattering theory, scattering function, discrete Sturm-Liouville equation, point interaction.

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✉ Corresponding Author Email: [ozbeyguher@gmail.com](mailto:ozbeyguher@gmail.com)

## Impulsive Discrete Dirac Equation with Spectral Parameter

Güler Başak Öznur<sup>1✉</sup>, Elgiz Bairamov<sup>2</sup>, Yelda Aygar<sup>2</sup>

<sup>1</sup> Gazi University, Department of Mathematics, Ankara, Turkey

<sup>2</sup> Ankara University, Department of Mathematics, Ankara, Turkey

### Abstract

The aim of this study is to examine the scattering analysis of an impulsive discrete Dirac equation. The novelty in this study is that the boundary condition depends on the spectral parameter. This gives a new perspective to the problem. The first part of this study consists of basic definitions and theorems [1-4]. In the second part, after getting the Jost solution, we find the scattering function and examine the properties of the scattering function. Furthermore, we obtain Green function and resolvent operator of this impulsive discrete Dirac equation. Using resolvent operator, we investigate eigenvalues and spectral singularities of the problem. We also obtain continuous spectrum of this impulsive problem [5,6]. In the last part of this study, we handle an example to demonstrate the application of our results.

**Keywords:** Dirac systems, scattering function, spectral parameter, resolvent operator

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✉ Corresponding Author Email: [basakoznur@gazi.edu.tr](mailto:basakoznur@gazi.edu.tr)

## The Harmonic Response of the Circular Composite Plates Having Various Cut-Outs

Oguzhan Das<sup>1✉</sup>

<sup>1</sup> Dokuz Eylul University, Department of Motor Vehicles and Transportation Technologies, Izmir, Turkey

### Abstract

Harmonic excitation is widely observed in various engineering problems, especially those of which include rotating structures. It is essential to examine the dynamic response of such structures since they may be subjected to dynamic forces that may lead them to operate under close-to-resonance frequencies. These structures may include cut-outs in various forms for several purposes such as riveting, piping, weight reduction, maintenance, or optimizing the dynamic properties of a structure [1,2]. Therefore, the shape of the cut-out becomes a significant parameter to investigate their effects on the harmonic responses of the structure in which it exists. Researchers conducted various studies including the harmonic response of structures considering different parameters [1-5] In this study, the effect of the shape of cut-out on the harmonic response of the circular cross-ply composite plate has been measured. For this purpose, circular cross-ply graphite-epoxy composite plates having triangular, square, pentagon, hexagon, and circular cut-outs have been modeled and analyzed via ANSYS Workbench 18.2. The composite material has been implemented in detail by using the ANSYS Composite Pre-Post (ACP) module. For analysis, the circular plate has been considered as a thin structure. The harmonic response analysis of the structure has been performed under fixed boundary conditions. Besides, the dynamic load has been subjected to the whole surface of the circular plate. The analysis results have been compared in terms of the difference in fundamental frequency, phase angle, maximum deformation, maximum deformation location, maximum stress, and maximum stress location. To understand the effect of the cut-out itself, a circular plate with no cut-outs has been also taken into account. It has been concluded that the cut-outs generally increase the fundamental frequency, maximum displacement, and stress values of the structure. Besides, increasing the number of vertex of the geometric shape of the cut-out decreases the maximum displacement and stress values, while increases the fundamental frequency value of the circular structure.

**Keywords:** Harmonic response, finite element analysis, cut-out, circular composite plates

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✉ Corresponding Author Email: [oguzhan.das@deu.edu.tr](mailto:oguzhan.das@deu.edu.tr)

## **An Integrated Model for Disassembly Line Balancing and Worker Assignment Problem: A Multi-Objective Optimization Approach**

Yildiz Kose<sup>1✉</sup>, Emre Cevikcan<sup>1</sup>

<sup>1</sup> *Istanbul Technical University, Industrial Engineering Department, Istanbul, Turkey*

### **Abstract**

Disassembly line balancing efforts aim to increase the efficiency of product recovery in terms of remanufacturing [1]. Even if advanced technology and autonomous for productivity are used, there is a need for the resource-effective execution of disassembly activities in disassembly lines where the workforce is still actively used. Multi-manned workstations present the potential of increasing disassembly line performance. Moreover, on the condition that workload differences among workers in multi-manned workstations have been focused enhancement of the line efficiency is provided by considering the worker performance. This paper introduces a mixed-integer linear programming (MILP) model and a novel framework heuristic algorithm to minimize the number of workers and stations as well as the workload differences between workstations in the disassembly line. MILP model has been applied to a dishwasher disassembly system. The application results indicate that the line balancing with multi-manned has superiority over the classical disassembly system design. Moreover, the proposed heuristic has been executed on newly generated test problems for DLBP. The results confirm that desired solutions are presented and large-sized problems are solved within a reasonable time. The potential contributions of the study come up with two aspects both the task characteristics and organizational viewpoints. The first one is related to human factors which changing potentials in terms of skill and effort for disassembly tasks. As far as the organizational viewpoint, the synchronous consideration of disassembly line balancing and worker assignment provides a proactive design process for disassembly line since constraints of line balancing and operators' workload are addressed even during line design. The study also gives the opportunity to obtain accordance between flexible working conditions and disassembly line balancing. In other words, the workload of operators is deployed to the line design in a more sensitive manner considering the workload differences.

**Keywords:** Disassembly, line balancing, worker assignment, multi-manned workstations

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✉ Corresponding Author Email: [kose18@itu.edu.tr](mailto:kose18@itu.edu.tr)

## Global Existence of Solutions to a Singular Riemann-Liouville Fractional Differential Equation of Higher Order

Şeyma Ramazan<sup>1✉</sup>, Müfit Şan<sup>1</sup>

<sup>1</sup> Çankırı Karatekin University, Department of Mathematics, Çankırı, Turkey

### Abstract

Initial and boundary value problems (IVPs and BVPs) for fractional differential equations are two of the most investigated subjects in the theory of Fractional Calculus. In the recent literature, some published papers (See [1],[2]) reconsider some IVPs and BVPs previously studied and reveal some inconsistency and incorrectness in converting the problems into the corresponding integral equations. On the basis of these studies besides [3],[4], this work presents an investigation for an initial value problem involving a singular Riemann-Liouville fractional differential equation of higher order when the right-hand side function has a singularity. It is proposed some condition under which the problem can be reduced to a Volterra integral equation having a doubly singular kernel. By a lemma for continuous function we can show that the existence and uniqueness using Grönwall inequality for integral equation with a single (not double) singular kernel given in [5].

**Keywords:** Existence and uniqueness, Grönwall inequality, fractional differential equation.

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✉ Corresponding Author Email: [demirsym@gmail.com](mailto:demirsym@gmail.com)

## A Comparative Study for the Numerical Solution of the Tumor Growth Model

Sıla Övgü Korkut<sup>1</sup>✉

<sup>1</sup> Izmir Katip Celebi University, Department of Engineering Sciences, Izmir, Turkey

### Abstract

The main purpose of this study is to present a comparative work for solving the tumor growth model. To do this, we restrict ourselves to the glioblastomas which is one of the most aggressive forms of cancer with a non-linear heterogeneous diffusion logistic density model. For solving the equation, various discretization techniques on the spatial domain are combined with the 4th order Runge–Kutta Method. The novelty of the study is that some of the techniques are applied to the specified equation for the first time and such a comparative study is also studied for the first time, as well. After the convergence analyses of all hybrid methods are provided theoretically, detailed comparative computational results are presented. All these methods are compared in terms of their efficiencies in varying time-step and mesh-discretization not only to one another but also with the methods given in the literature.

**Keywords:** The tumor growth model, Glioblastomas, mesh-discretization

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✉ Corresponding Author Email: [silaovgu.korkut@ikcu.edu.tr](mailto:silaovgu.korkut@ikcu.edu.tr)



## Spectral Properties of the Finite System of Discrete Sturm-Liouville Operators with Hyperbolic Eigenparameter

Nimet Coskun<sup>1</sup>✉

<sup>1</sup> *Karamanoglu Mehmetbey University, Mathematics Department, Karaman, Turkey*

### Abstract

In this presentation, spectrum and spectral properties of the finite system of Sturm-Liouville type difference operators with hyperbolic eigenparameter have been taken under investigation. The transformation chosen for the eigenparameter affects drastically the representation of Jost solution and analyticity region of the Jost function [1-5]. Hence, determining the sets of eigenvalues and spectral singularities, we generalize the recent results [3,5] to the hyperbolic eigenparameter case.

**Keywords:** Spectral theory, difference operators, Sturm-Liouville operator, spectral singularities

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✉ Corresponding Author Email: [cannimet@kmu.edu.tr](mailto:cannimet@kmu.edu.tr)

## Autonomous Landing of a VTOL UAV on a Stationary Landing Point

Gökhan Gülmez<sup>✉</sup>, Ö. Tolga Altınöz

Ankara University, Electrical and Electronics Engineering Department, Ankara, Turkey

### Abstract

In this study, a control strategy has been developed for autonomous landing of unmanned aerial vehicles that is capable of vertical landing and take-off at a target point. The solution developed for the landing of VTOL UAV's to a fixed point consists of 2 main parts: appointment and landing. When the autonomous landing application is started, there is a long xy-plane distance between the quadrotor which switches to the appointment stage, and the target landing point. At this stage, by producing waypoints for the x, y and z position controllers, tracking of a two-dimensional (xy plane) motion trajectory is provided. In this way, the appointment point  $(x_t, y_t, z_t)$ , which is vertically just above the target landing point  $(x_t, y_t, 0)$ , is reached. When the UAV reaches the appointment point, the position control of the quadrotor at this point is continued for 2s, in other words, before the precise landing the stabilization task is performed  $x_t, y_t$ . Thus, this stage is completed with minimum error. When the precise landing stage is started, the position control task of the x and y axes of the quadrotor (for  $x_t$  and  $y_t$ ) is maintained and errors for these axes is kept at zero or minimized. At this stage, while the position control continues for the x and y axes, the task of landing from  $(x_t, y_t, z_t)$  to  $(x_t, y_t, 0.1)$  along the z axis is performed. In order to land precisely and reach the point  $(x_t, y_t, 0.1)$ , the landing phase is completed by making an altitude control at a speed of 0.1 m/s along the z axis. The main motivation behind choosing the point  $(x_t, y_t, 0.1)$  as the target instead of the point  $(x_t, y_t, 0)$  is to develop a solution both by landing the quadrotor very close to the ground and not to affect from ground effect before motors stops. When the point  $(x_t, y_t, 0.1)$  is reached, all rotors on the quadrotor are stopped and the landing to the target point  $(x_t, y_t, 0)$  is completed with the remaining 0.1m free fall.

**Keywords:** VTOL UAV, autonomous, landing.

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✉ Corresponding Author Email: [ghnglmz2@gmail.com](mailto:ghnglmz2@gmail.com)

## A Numerical Solution to the Heat Transfer in MHD Flow

Canan Bozkaya<sup>✉</sup>

*Middle East Technical University, Department of Mathematics, Ankara, Türkiye*

### Abstract

The steady laminar magnetohydrodynamics (MHD) flow and the heat transfer of an electrically conducting fluid in a rectangular duct subject to an oblique magnetic field is of great interest due to its wide range of applications in design of electrical devices, cooling systems in nuclear fusion reactors, metallurgical and material processing. Present study focuses on the numerical solution of the heat transfer in MHD flow which is fully developed and driven by a constant pressure gradient in the axial direction. The basic equations of the flow and the temperature fields are the combination of the Navier-Stokes equations of fluid dynamics, Maxwell's equations of electromagnetism and the energy equation of thermodynamics. The MHD flow equations, which are convection-diffusion type equations in velocity and induced magnetic field, are discretized by using a direct boundary element approach with the fundamental solution treating them directly in their original coupled form, while an alternative indirect boundary element scheme is employed for the discretization of the energy equation. The resulting system of equations involving the unknown values of velocity and induced magnetic field only on the boundary of the duct, is small in size and is solved at one stroke with no iteration. Once these values are obtained, they are used in the solution of the energy equation. Novel results are presented graphically for several values of Hartmann number and inclination angles of the magnetic field in order to investigate combined effects of these parameters not only on the velocity of the fluid and the induced magnetic field but also on the enhancement of the heat transfer rate and the temperature distribution. The results reveal that characteristic features of MHD flow with heat transfer in ducts are physically well-captured.

**Keywords:** MHD, duct flow, boundary elements, heat transfer

✉ Corresponding Author Email: [bcanan@metu.edu.tr](mailto:bcanan@metu.edu.tr)

## A Smoothing Function Approach for Solving Nonlinear Complementarity Problems

Nurullah Yılmaz<sup>1</sup>✉

<sup>1</sup>*Süleyman Demirel University, Department of Mathematics, Isparta, Turkey*

### Abstract

In this study, we focus on solving nonlinear complementarity problems (NCP). We consider two different types of smoothing functions in a new formulation of the NCP. We investigate the relations between the original and reformulated problems. We develop a new smoothing-type algorithm for solving NCP and demonstrate the efficiency of our algorithm on some numerical examples. Finally, the comparison of the obtained results with the other methods is presented.

**Keywords:** Nonlinear complementarity problems, smoothing function approach, Newton-type algorithms

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✉ Corresponding Author Email: [nurullahyilmaz@sdu.edu.tr](mailto:nurullahyilmaz@sdu.edu.tr)

## A New Smoothing Algorithm for Solving Absolute Value Equations of the Form $Ax + B|x| = b$

Nurullah Yılmaz<sup>1</sup>✉, Ayşegül Kayacan<sup>1</sup>

<sup>1</sup> Suleyman Demirel University, Mathematics Department, Isparta, Turkey

### Abstract

In this study, we concentrate on solving absolute value equations (AVE) of the form,  $Ax + B|x| = b$  where  $A, B$  are  $n \times n$  type real matrices and  $b$  is  $n$  –dimensional real vector. The current form of AVE is described as a system of non-smooth equations. We first transform the AVE into a family of parametrized smooth equations by the help of smoothing techniques. We propose two different smoothing function approaches based on  $S$  –shaped functions. By the help of these smoothing techniques, we develop a new smoothing-type algorithm. The numerical experiments have been carried out on some randomly generated test problems. Finally, the comparison with other methods is illustrated to show the effectiveness of the proposed method.

**Keywords:** Absolute value equations, smoothing technique, nonsmooth equations

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✉ Corresponding Author Email: [nurullahyilmaz@sdu.edu.tr](mailto:nurullahyilmaz@sdu.edu.tr)

## Realization of Fractional Band Stop Filter with Asymmetric Slopes and Optimized Quality Factor

Sunil Narayan , Utkal Mehta, Varian Akwai, Steven Weago and Kajal Kothari

*Electrical and Electrical Engineering, School of Information Technology, Engineering, Mathematics, and Physics, The University of the South Pacific, Laucala Campus, Suva, Fiji*

### Abstract

The paper focuses on realization of fractional band stop filter (FBSF) to improve the quality factor and asymmetric slopes. The concept is based on introducing a fractional calculus to approximate fractional capacitors and inductors of impedance,  $z_\alpha = \frac{1}{s^\alpha c_\alpha}$  and order  $\alpha$  ( $0 < \alpha < 1$ ). The quality factors and the asymmetric slope magnitude response of the FBSF have been optimized using new technique by proposing new transfer function and implementing it using different passive and active realization technique. The result shows a better control of the magnitude slope and quality factor of the filter through optimum coefficient parameter combinations and by tuning the value of order  $\alpha$  ( $0 < \alpha < 1$ ). The quality factors and the asymmetric slope magnitude response of the FBSF have been optimized using genetic algorithm (GA). The filter was verified by plotting MATLAB simulation results for different values of  $\alpha$  and showing its comparison with experimental results.

**Keywords:** Fractional Band Stop Filter (FBSF), Quality Factor, and Particle Swarm Optimization (PSO)

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 Corresponding Author Email: [spnz2014@gmail.com](mailto:spnz2014@gmail.com)

## A New Smoothing Technique for Global Optimization by Auxiliary Function Method

Ahmet Şahiner<sup>1</sup>✉, Güliden Kapusuz<sup>1</sup>

<sup>1</sup>*Süleyman Demirel University, Mathematics. Department, Isparta, Türkiye*

### Abstract

In this study, a new smoothing technique is introduced to solution of min-max optimization problem and its applications by using Cubic Bezier Curve, the effectiveness of the technique is tested on the solutions of nonlinear global optimization problems by auxiliary function method.

**Keywords:** Non-Smooth Optimization, Smoothing Technique, Bezier Curves

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✉Corresponding Author Email: [ahmetsahgulden32@gmail.com](mailto:ahmetsahgulden32@gmail.com)

## Realization and Sensitivity Analysis of Fractional Order Kerwin-Huelsman Newcomb Filter

Sunil Narayan<sup>1</sup> ✉, Utkal Mehta<sup>1</sup>

<sup>1</sup>The University of the South Pacific, School of Engineering and Physics, Suva, Fiji

### Abstract

The paper presents a design, realization and sensitivity analysis of a fractional order Kerwin-Huelsman Newcomb filter. The filter will be realized using two fractional order capacitors of order  $\alpha$  and  $\beta$  ( $0 < \alpha, \beta \leq 1$ ). The paper will focus on the fractional order Kerwin-Huelsman Newcomb filter low-pass, high-pass filters and band pass are investigated based on the coefficient for the transfer function is obtained by optimization technique. The effects of the exponents  $\alpha$  and  $\beta$  of the realized KHN filter has been examined where two fractional order elements of order  $\alpha$  and  $\beta$  ( $0 < \alpha, \beta \leq 1$ ) are used for realization. The sensitivity analysis is carried out to see the performance of the FO filter due to the parameter variations. The responses of the realized filter are obtained by using Matlab simulation and it is seen that the pole frequency and quality factor of FO filter is more sensitive towards component variations as compared to integer order

**Keywords:** Kerwin-Huelsman Newcomb filter, Quality Factor, and optimisation

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✉ Corresponding Author Email: [spnz2014@gmail.com](mailto:spnz2014@gmail.com)



## Effects of a Rotating Cylinder on MHD Forced Convection in an Infinite Channel

F. Sidre Oglakkaya<sup>1</sup>✉, Canan Bozkaya<sup>2</sup>

<sup>1</sup> *Osmaniye Korkut Ata University, Department of Mathematics, Osmaniye, Turkey*

<sup>2</sup> *Middle East Technical University, Department of Mathematics Ankara, Turkey*

### Abstract

The magnetohydrodynamic forced convection in an infinite channel including a rotating cylinder is numerically investigated under the effect of a uniform magnetic field. The fluid flow under consideration is two-dimensional, steady, laminar and obeys the Boussinesq approximation while the effects of the radiation and viscous dissipation are neglected. It is assumed that the working fluid is incompressible and Newtonian. The temperature of input fluid, which is taken smaller than that of horizontal walls, enters the heated duct. Further, isothermal boundary condition for the rotating cylinder is imposed, respectively. A uniform magnetic field of strength  $B_0$  is vertically applied to the whole channel. The governing equations in terms of stream function, vorticity and temperature are numerically solved using the dual reciprocity boundary element method in which the governing equations are transformed into integral equations only on the boundary by using the fundamental solution of the Laplace equation and treating all the other terms as nonhomogeneity through radial basis function approximation. Thus, the resulting discretized system is small in size compared to the ones obtained by domain discretization techniques, and hence the computational cost in obtaining the solution is small. The numerical simulations are carried out to investigate the effects of the physical parameters such as Hartmann number, the cylinder rotation angle on the flow field and heat transfer. The results indicate that increasing the Hartmann number enhances the average heat transfer, while the angular rotational speed of the cylinder results in a decrease or an increase in the heat transfer rate at spatial regions inside the channel. Thus, the cylinder rotation angle under the various combination of physical parameters can be used as a control parameter for the heat transfer inside the channel.

**Keywords:** MHD, DRBEM, Rotating cylinder, Forced convection

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✉ Corresponding Author Email: [fsidreoglakkaya@osmaniye.edu.tr](mailto:fsidreoglakkaya@osmaniye.edu.tr)

## Investigation of a Stabilized Finite Element Method for Navier-Stokes Equations

Fatma G. Eroglu <sup>1</sup>✉

<sup>1</sup> *Bartın University, Department of Mathematics,, Bartın, Turkey*

### Abstract

In this study, an efficient and reliable numerical algorithm for Navier Stokes equations (NSE) is investigated by using streamline upwind Petrov Galerkin-pressure stabilizing Petrov Galerkin (SUPG-PSPG) and grad-div stabilization methods. It is known that instability occurs in the case of small diffusion, lack of mass conservation and violation of the inf-sup stability. These cause contamination of solutions with large spurious oscillations. Therefore, efficient methods are needed to damp oscillations.

SUPG method, as one of the most popular residual-based stabilization methods, has been utilized in many engineering and scientific applications, see [1,2,3]. The main idea of SUPG is to reduce the oscillations and stabilize the convection dominance by adding artificial diffusion to each mesh cell along the streamlines of the solution. However, with the use of piecewise polynomial and discontinuous finite element pressure spaces, an additional term is required to add a control on the lack of the inf-sup stability condition. To address this issue, the PSPG method has been found successful, see [4]. In addition, the grad-div method is used to reduce the negative effect of pressure on velocity error and to get control on the lack of the principle of mass conservation [3,5].

We extend the novel idea of [2,6] to the SUPG-PSPG and grad-div settings for the time-dependent incompressible NSE. First, we establish the existence and uniqueness theorems of the solutions. Then, the stability and convergence analyses of the proposed method are presented with the backward Euler temporal discretization. Finally, to obtain optimal error estimation, the choice of stabilization parameters and time-step restrictions are investigated, and the paper is concluded with a summary.

**Keywords:** streamline upwind Petrov Galerkin (SUPG) method, pressure stabilizing Petrov Galerkin (PSPG) method, grad-div stabilization

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✉ Corresponding Author Email: [fguler@bartin.edu.tr](mailto:fguler@bartin.edu.tr)

## Flow in a Cavity Subjected to Two Variable Magnetic Sources

Pelin Senel<sup>1</sup>✉

<sup>1</sup> Karadeniz Technical University, Mathematics Department, Trabzon, Turkey

### Abstract

In this study, influences of two variable magnetic sources on the steady, fully developed, Ferrohydrodynamics (FHD) flow of magnetizable fluid [1] are investigated. The fluid is pumped within the duct due to a constant pressure gradient in the axial direction. Two semi-cylinders are symmetrically located on the left and the bottom walls of the duct and two thin-wires, carrying electric current, are passing through axes of these semi-cylinders. The fully developed flow is modeled in velocity-pressure form on the 2D cross-section of the duct [2]. On this cross-section (cavity), wires act as point magnetic sources placed at the centers of semi-circles. Governing equations are discretized by the dual reciprocity boundary element method (DRBEM) [3] and solved iteratively. Pressure boundary conditions are obtained through momentum equations by approximating pressure gradients using finite differences and all the space derivatives of the unknowns using the DRBEM coordinate matrix. Velocity and the pressure profiles are obtained for different strengths of magnetic sources. Numerical results show that, when both magnetic sources are uniform and they increase with the same rate, pressure in the cavity increases and the flow in the axial direction decelerates around semi-circles. When sources have different strengths, the flow and the pressure profiles are dominated by the strong magnetic source. The numerical results are achieved with less computational cost due to the boundary only nature of the DRBEM.

**Keywords:** FHD flow, variable magnetic sources, DRBEM

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✉ Corresponding Author Email: [psenel@ktu.edu.tr](mailto:psenel@ktu.edu.tr)

## Detection with Bistatic Sonobuoys: Random vs Coordinated Deployments

Mümtaz Karataş<sup>1</sup>, Levent Erişkin<sup>1</sup>✉

<sup>1</sup> National Defence University, Turkish Naval Academy, Industrial Engineering Department,  
Tuzla, İstanbul, Turkey

### Abstract

The problem of assessing the coverage quality of underwater sensor networks for search, detection and surveillance is an important research topic both for practitioners and researchers. Among different underwater sensor types, multistatic and bistatic sonars are effectively used by navies worldwide for protecting maritime zones and friendly units against hostile submerged targets. Such systems consist of sources and receivers which need not be collocated. If a multistatic underwater surveillance system is composed of single independent source and receiver at different locations, these systems are called bistatic sensors [1,2]. Target detection or area coverage performance of a bistatic sonar is based on the distance between the source and target as well as the distance between the target and receiver. In particular, for a given environmental condition and target type, the sensing zone of a bistatic sonar is characterized by a set of distinct ovals named as Cassini ovals [3]. In this study, we consider comparing the performance of random and coordinated deployment strategies of bistatic sonobuoys against stationary and mobile targets. To achieve this, using underwater acoustic theory, we first model the coverage zone of a bistatic sensor couple as a group of Cassini ovals. Next, using simple analytical geometry, we map the problem to a two dimensional geometric problem. Finally, we approximate the expected coverage area of bistatic sensors by using equations derived from this mapping and analyse the relationship between the dimensions of the search field and coverage. We also use results from previous work obtained for the coordinated deployment of bistatic sensors against mobile targets and compare the results. The results obtained from this work can be utilized by decision-makers and practitioners for back-of-the-envelope analysis to estimate the search performance of bistatic sensors.

**Keywords:** Acoustics, bistatic sonar, random deployment

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✉ Corresponding Author Email: [leriskin@dho.edu.tr](mailto:leriskin@dho.edu.tr)

## An Application of Statistical Design and Analysis of Experiments for System Performance Evaluation

Levent Erişkin<sup>1</sup>✉, Mümtaz Karataş<sup>1</sup>

<sup>1</sup> National Defence University, Turkish Naval Academy, Industrial Engineering Department,  
Tuzla, İstanbul, Turkey

### Abstract

In this paper, we consider evaluating and comparing candidate systems for acquisition using statistical design and analysis of experiments techniques. Due to numerous decision variables inherent in acquisition decisions and the high technology of the candidate systems, enterprises need decision support for making these decisions. In this regard, utilizing quantitative methods is of utmost importance. To address this issue, statistical design and analysis of experiments field provides quantitative techniques for designing appropriate experiments and analyzing the experimental data. These techniques are particularly useful for enterprises that procure systems as Commercial-Off-The-Shelf (COTS). COTS systems are already developed and produced systems that are available for acquisition. Even though procuring a COTS has advantages for ensuring a shorter acquisition lead time, it inherently involves risks. For instance, COTS are developed for the common user, and may not be compatible and interoperable with buyer's systems that are already in the inventory [1]. Moreover, design specifications declared by the provider may not reflect the real performance of the system under various operating conditions. Consequently, these risks oblige the buyers to conduct a rigorous evaluation.

In order to show how these quantitative techniques can be applied to real-life problems, we present an application dealing with the acquisition of a Naval Gun System (NGS) as COTS. The naval guns are directed to surface targets with Fire Control Radars (FCRs). There are two FCRs within each NGS. The FCR used in firing affects the performance of a NGS and both FCRs are used interchangeably in surface warfare. Since FCRs are specific to each NGS, levels of FCR factor are nested under the levels of NGS factor. Therefore, we design an experiment with both nested and factorial factors and conduct the pertaining statistical analysis to determine the significant effects and the superior NGS [2]. The case study shows that the analysis provides adequate decision support for evaluating and comparing candidate system alternatives.

**Keywords:** Design of experiments, nested design, system performance evaluation

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✉ Corresponding Author Email: [leriskin@dho.edu.tr](mailto:leriskin@dho.edu.tr)

## Some Iterative Methods for a Class of Inverse Problems for Semilinear Differential Equations Backward in Time

Ali Ugur Sazaklioglu<sup>1</sup>✉

<sup>1</sup> *University of Turkish Aeronautical Association, Department of Astronautical Engineering, Ankara, Turkey*

### Abstract

It is known that the problems for differential equations backward in time take a major role especially in the natural and engineering sciences in the framework of determination of the initial condition (state). Similarly, the identification problems for differential equations, in which one or more control parameters are determined, are quite significant for controlling a process. Hence, in order to explore the complex dynamical systems and processes in the real life and also to improve the technological achievements related to these, the two problems mentioned above, among the others, are studied extensively by the researchers. In this study, some problems comprising the arguments of those two problems and also nonlinearity are considered.

In this study, some iterative methods involving some finite difference schemes are proposed for the numerical solution of the following abstract inverse problem governed by a semilinear differential equation backward in time:

$$\begin{cases} \frac{du}{dt} - Au = p + f(t, u), 0 < t < T, \\ u(T) = \psi, u(T_1) = \varphi, 0 \leq T_1 < T, \end{cases} \quad (1)$$

where  $(u, p)$  is the solution pair of the problem, and  $A$  is a linear, self-adjoint and positive definite operator in a Hilbert space  $H$ . Since there are several operators satisfying the properties of the operator  $A$ , problem (1) is in fact a class of some source identification problems for some semilinear parabolic equations backward in time. Note also that when  $0 < T_1 < T$ , problem (1) becomes an inverse problem of simultaneous determination of the source  $p$  and the initial condition  $u(0)$ .

The applications of problem (1) on semilinear parabolic equations and the difference schemes proposed for them are carried out. Moreover, the results on the existence and uniqueness of the solution of problem (1), its applications and the difference schemes proposed for the numerical solutions of all these problems are established using some ideas in [1, 2]. Finally, the proposed methods are performed on some model problems to acquire their initial conditions, source terms and solutions, simultaneously. Furthermore, for showing the efficiency of the proposed methods a numerical analysis is given.

**Keywords:** Inverse problem, numerical solution, iterative method

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✉ Corresponding Author Email: [ausazaklioglu@thk.edu.tr](mailto:ausazaklioglu@thk.edu.tr)

## Capturing The van der Pol Oscillatory Behaviors through a Stochastic Approach

Hande Uslu<sup>1</sup>✉, Murat Sari<sup>1</sup>

<sup>1</sup> Yildiz Technical University, Mathematics Department, İstanbul, Turkey

### Abstract

For decades, oscillatory behavior mostly encountered both in biological and physical sciences has been paid high attention [1]. Even though just solving the model equations has a charming effect, capturing the behavior of physical processes by dealing with a suitable method is also a challenging issue [2-4]. There are various conventional methods to solve those equations suffering from difficulties; high computational cost, computational error or mathematical complexities of realistic systems [5,6]. In this respect, this study aims at focusing on nonlinear oscillatory behaviors represented by the van der Pol equation through stochastic approaches. Thus, a Monte Carlo based algorithm is presented for solving the van der Pol equations [7-9]. To properly discuss the simulated behaviors, detailed analysis has been carried out in an illustrative manner.

**Keywords:** Van der Pol Equation, Nonlinear Oscillatory Behavior, Stochastic Approach

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✉ Corresponding Author Email: [usluh@yildiz.edu.tr](mailto:usluh@yildiz.edu.tr)

## A Linear Approximation Model for a Non-Linear Flow Shop Scheduling Problem with Learning Effect

Augusto Ferraro<sup>1</sup>, Daniel Alejandro Rossit<sup>1-2</sup>✉, Adrián Toncovich<sup>1</sup>

<sup>1</sup> Engineering Department, Universidad Nacional del Sur, Bahía Blanca, Argentina

<sup>2</sup> INMABB, CONICET, Bahía Blanca, Argentina

### Abstract

Learning effects have been considered in operations management problems since the early twentieth century [1]. The learning effect has a direct influence on production scheduling problems, since it modifies the use of production machines [2], and for this reason, it has been a problem widely studied by the scheduling community [3]. However, modeling the learning effect in scheduling problems by means of mathematical programming requires the use of non-linear expressions [4], this has limited the majority of works to be focused on single-machine problems [2] [5]. In this work, it is proposed to extend these formulations for the case that the learning effect is exponentially dependent on the previous jobs processed in the sense of [5]. This mathematical model is clearly non-linear, and by having several machines in which the learning process occurs, the probability of getting trapped in poor local optimums is very high. The proposal of this work is a linear approximation scheme, which can be implemented by a standard MIP solver such as CPLEX, in order to obtain very high quality solutions, without requiring sophisticated and tailored methods. The approximation scheme is based on a set of straight lines, which approximate the expected learning effect, generating a convex shell to the problem with expected values, thus avoiding falling into poor quality local optimal points. For creating the convex shell, a least-squares problem must be solved, which is also non-linear, but does not require integer variables, then, it can be solved by simple solvers like the ones provided by spreadsheet software. To evaluate the capability of the solution scheme, the proposed linear model solution was compared with the solution obtained by a proven MINLP solver such as DICOPT [6], in flow shop problems with makespan as the objective function. The results show that the proposed scheme notably improves the solutions obtained by DICOPT, reducing the makespan in up to 12%.

**Keywords:** flow shop, learning effect, non-linear mixed integer programming, linear approximation

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✉ Corresponding Author Email: [daniel.rossit@uns.edu.ar](mailto:daniel.rossit@uns.edu.ar)



## A Neural Network Learning Approach for Solving the Knapsack Problem

Ertan Yakıcı<sup>1</sup>✉, Tolga Önel<sup>2</sup>

<sup>1</sup> National Defence University, Barbaros Naval Sciences and Engineering Institute, İstanbul, Turkey

<sup>2</sup> National Defence University, Computer Engineering Department, İstanbul, Turkey

### Abstract

Among a finite number of items, whose weights and benefits are known, choosing a subset with the maximum benefit without exceeding the maximum weight that can be carried is referred to as 0-1 Knapsack problem (KP01) [1]. KP01 is known as a Non-deterministic polynomial-time complete (NP-complete) problem [2]. Therefore, no algorithm proposed until now can solve KP01 in polynomial time with the exact solution. Dynamic programming, branch and bound, brute force, Lagrangian decomposition based and many heuristic algorithms are proposed in the literature to solve KP01 [3]. Genetic algorithm with  $O(n)$  time complexity [4] is the fastest of all.

Supervised learning algorithms are used for learning a function with an input-output pair. With the learned function, the output for an input that is not seen before, can be approximately predicted. Neural networks are widely used as a supervised learning mechanism to extract information from data. A neural network can be trained using known inputs and outputs related to the phenomenon. With an unprecedented example of the phenomenon, the output corresponding to the input can be determined with the trained neural network in  $O(1)$  time.

In this study, the use of neural networks is proposed for solving the KP01 problem. We train a neural network using exactly solved KP01 examples. Using the trained neural network, we manage to achieve good results which have high similarity with the pre-solved KP01 test set examples. Hence, the proposed method is suitable for the applications where, fast but approximate solutions are required. Our findings also show that the use of neural networks can be a good candidate mechanism for solving optimization problems.

**Keywords:** Knapsack problem, neural networks, optimization

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✉ Corresponding Author Email: [eyakici@dho.edu.tr](mailto:eyakici@dho.edu.tr)

## LEGO ROBOT Setup and Intelligent Programming for Line Following and Obstacle Avoiding Using EV3RSTORM Software

Ashwin Chand<sup>1</sup>✉, Sunil Narayan<sup>1</sup>, Utkal Mehta<sup>1</sup>

<sup>1</sup>*The University of the South Pacific, School of Engineering and Physics, Suva, Fiji*

### Abstract

The paper focus on the progressively essential that up and coming age of understudies must secure critical thinking, basic reasoning and cooperative abilities to prevail in their profession desires in the 21st century. Innovation assumes a critical part in the absorption of these abilities. Among prospering varieties of advances, mechanical technology gives difficulties and chances to the students in creating inventive thoughts, problematic reasoning, and higher-arrange learning abilities. This paper investigates the instructive utilization of mechanical autonomy in schools and how educators can coordinate this new innovation into the educational modules. The paper additionally proposes the powerful techniques in utilizing mechanical technology as an instructive device and how it will affect understudies' interests in Science, Technology, Engineering, and Mathematics related subjects. A few proposals to upgrade learning exercises in the enterprises are given.


**Keywords:** Educational Robotics, EV3RSTORM , Lego

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✉ Corresponding Author Email: [spnz2014@gmail.com](mailto:spnz2014@gmail.com)

## Existence and Hyers-Ulam Stability of Solutions for a Delayed Hyperbolic Partial Differential Equation

Canan Çelik, Faruk Develi 

*Yildiz Technical University, Department of Mathematics, Istanbul, Turkey*

### Abstract

In this paper, we first prove the existence and uniqueness of the solutions for a delayed hyperbolic partial differential equation by applying progressive contraction technique introduced by Burton in [2] and [3] to the corresponding fixed point problem. Then we derive Hyers-Ulam stability result for this differential equation by using Wendorff-type inequality and Abstract Gronwall Lemma.

**Keywords:** Progressive contractions, hyperbolic partial differential equation, Hyers-Ulam stability, fixed point theory

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 Corresponding Author Email: [fdeveli@yildiz.edu.tr](mailto:fdeveli@yildiz.edu.tr)

## Analytical and Numerical Assessments of Boundary Perturbations in Steklov Eigenvalue Problem

Eylem Bahadır<sup>1</sup>✉, Önder Türk<sup>2</sup>✉

<sup>1</sup> Gebze Technical University, Department of Mathematics, Kocaeli, Turkey

<sup>2</sup> Middle East Technical University, Institute of Applied Mathematics, Ankara, Turkey

### Abstract

In this study, we investigate the influence of boundary perturbations on the eigenvalues of the Steklov eigenvalue problem (EVP) both analytically and numerically. Various one- and two- dimensional cases are considered with different families of boundary conditions by introducing a set of differing perturbations. Theoretically, we explore the convergence together with its order of the perturbed eigenvalues to the original eigenvalues by Taylor's series expansion of the errors between the two eigenvalues in terms of a characteristic perturbation parameter. We further investigate numerically the convergence properties of the approximate eigenvalues of the perturbed problem obtained using the finite element method to those of the continuous (unperturbed) problem. These results reveal the fact that for a fixed mesh, the dependence on the perturbation parameter is in coherence with the already mentioned behavior. Moreover, the Steklov EVP is considered on regular polygons, which are assumed to be variations of the unit disc and inscribed in this disc with an increasing number of sides. We numerically show that the multiple eigenvalues on the polygon can be represented in terms of inverse powers of the number of the sides, as given in [1] for the simple eigenvalues of the Laplace EVP. We propose a procedure based on the shape derivative formula given in [2] to analytically investigate the series representation of the eigenvalues of the Steklov EVP on regular polygons. In the numerical investigation, the eigenvalues of the Steklov EVP on regular polygons are obtained numerically using the finite element method. Furthermore, we provide a proof of convergence of the FEM solution to the exact solution for the unperturbed cases, that is based on the spectral theory.

**Keywords:** Steklov EVP, FEM, boundary perturbations.

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✉ Corresponding Author Email: [eylemilke@yandex.com](mailto:eylemilke@yandex.com), [oturk@metu.edu.tr](mailto:oturk@metu.edu.tr)

## Approximation to Fractals by Means of Non-Affine Contraction Mappings

İsmail Aslan<sup>1</sup>✉, Nisa Aslan<sup>2</sup>

<sup>1</sup> Hacettepe University, Department of Mathematics, Ankara, Turkey

<sup>2</sup> Eskişehir Technical University, Department of Mathematics, Eskişehir, Turkey

### Abstract

Fractals are one of the popular topic in the recent years. They defined as the geometry of the nature by Mandelbrot. They also have many applications not also on applied mathematics and geometry [2,3,4], but also on physics, chemistry, biology, engineering and image processing. Many methods exist in obtaining fractals such as iterated function systems, escape time algorithms and etc [2,3,4]. In this presentation, we concentrate on the iterated function systems linking to nonlinear kernels using in approximation theory.

Although different definitions are encountered in the literature, fractals can be named as self-similar sets in general. As it is known, most of the fractals such as Sierpinski triangle, Sierpinski carpet, Sierpinski tetrahedron, Vicsek fractal and Koch curve can be obtained as an attractor of an iterated function systems [3,4]. It is seen that the contraction mappings used in obtaining these fractals are generally affine transformations. In this study, it is aimed to approach fractals which are obtained by affine or non-affine contraction mappings, by using new non-affine transformations. For this purpose firstly, non-affine transform sequences will be obtained with the help of the known contraction mappings using the Lipschitz continuity property of the kernel function sequences used in linearization of non-linear operators [1]. Later, approximations to fractals will be obtained by using iterated function systems. Construction of new self-similar sets which are obtained by new non-affine contraction mappings are also among the targets of this work. Also, approximations for some well-known fractals will be visualized with the help mathematical programs.

**Keywords:** Iterated function systems, non-affine contraction mappings, classical fractals

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✉ Corresponding Author Email: [ismail-aslan@hacettepe.edu.tr](mailto:ismail-aslan@hacettepe.edu.tr)

## Solving Stochastic Differential Equations with Generalized Entropy Optimization Methods and Simulation

Nihal Ince ✉

*Eskisehir Technical University, Department of Statistics, Eskisehir, Turkey*

### Abstract

In recent years, stochastic differential equations (SDEs) have been included in the literature and become an area of interest with the introduction of randomness to the structure of the problem. Stochastic differential equation models play an important role in various application areas such as biology, physics, chemistry, micro-electronics, biomedical sciences, economics, and mathematical finance. Analytical solutions to many differential equations are quite difficult to find. For SDEs, the solution is much more difficult due to the random variable. In this study, Euler-Maruyama (EM) and Milstein methods are used to numerically solve the SDEs. It should be noted that by starting given statistical data using numerical methods it is possible to construct several trajectories of SDE. At fixed time, mentioned trajectories allow attaining approximate random variable of solution of SDE. In addition, the appropriate probability density function (pdf) of the solution mentioned SDE at a fixed time is found by using Generalized Entropy Optimization Methods (GEOM). The reason for using Generalized Entropy Optimization Distributions (GEOD) represented by GEOM is fact that these distributions are more flexible than other distributions. In application, the mentioned method is fulfilled using a simulated dataset. To demonstrate the efficiency of numerical methods, approximation solutions are compared with the exact solution for different sample paths using Monte Carlo (MC) simulation for each method. GEOD evaluated for simulated data is compared with the corresponding solution of MC simulation in the sense of entropy measures and other statistical measures. In our investigations, two positions are considered. Firstly, GEOD is compared with the exact solutions of MC simulation if it exists. Secondly, if the exact solution of MC simulation doesn't exist, then by approximate methods mentioned solution is obtained and is compared with GEOD. Finally, acquired results show that the fields of applications of GEOM in SDEs can be expanded.

**Keywords:** Generalized entropy optimization methods, Euler-Maruyama method, Milstein method, Monte-Carlo simulation

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✉ Corresponding Author Email: [nihalyilmaz@eskisehir.edu.tr](mailto:nihalyilmaz@eskisehir.edu.tr)

## Parabolic Optimal Control Problems Described by Partial Differential Inclusions

Sevilay Demir Sađlam<sup>1</sup>✉, Elimhan N. Mahmudov<sup>2,3</sup>

<sup>1</sup> Istanbul University, Department of Mathematics, Istanbul, Turkey

<sup>2</sup> Istanbul Technical University, Department of Mathematics, Istanbul, Turkey

<sup>3</sup> Azerbaijan National Academy of Sciences, Institute of Control Systems, Baku, Azerbaijan

### Abstract

The paper concerns the optimization of partial differential inclusions of the parabolic type given by polyhedral set-valued mappings. We derive the optimality conditions for the problems under consideration by employing the result of the discrete approximation problem associated with the continuous problem. We formulate the sufficient conditions by passing formally to the limit as the discrete steps tend to zero in the discrete approximation problem. Over the last decade, significant progress has been made in various fields involving optimal control problems described by ordinary and partial differential equations and/or inclusions [1-9]. Variational analysis of partial differential inclusions is performed, and new optimality conditions of the Euler-Lagrange and Hamiltonian types are obtained using the process of discrete approximations and advanced generalized differentiation tools. We consider some linear optimal control problems to demonstrate the above approach.

**Keywords:** Partial differential inclusions, polyhedral optimization, optimality conditions

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✉ Corresponding Author Email: [sevilay.demir@istanbul.edu.tr](mailto:sevilay.demir@istanbul.edu.tr)

## Successive Iterations and Positive Solutions for Hadamard Type Fractional Differential Equations on an Infinite Interval

Fulya Yoruk Deren<sup>1✉</sup>, Tugba Senlik Cerdik<sup>2</sup>

<sup>1</sup> Ege University, Department of Mathematics, Izmir, Turkey

<sup>2</sup> Beykent University, Department of Mathematics, Istanbul, Turkey

### Abstract

Fractional calculus and fractional differential equations are very important subjects for scientists in investigating real world problems such as physics, control theory, chemistry, biology, engineering, economy and other areas. (See [1,2].) So far, various different definitions of fractional derivatives have been given such as Riemann Liouville, Caputo, Hadamard type fractional derivative. Except than commonly studied Riemann Liouville and Caputo fractional differential equations, we investigated Hadamard type fractional differential equations. This type of fractional derivatives involves a logarithmic function of arbitrary exponent. Recently, Thiramanus *et al.* studied the existence of nonnegative multiple solutions for Hadamard fractional differential equations on an unbounded domain by using the Leggett-Williams and Guo-Krasnoselskii's fixed point theorems [3]. Wang *et al.* considered the iterative positive solutions for a nonlocal Hadamard type fractional differential equation supplemented with nonlocal Hadamard integral and discrete boundary conditions. By employing the monotone iterative method, they obtained the twin positive solutions and the unique positive solution of the nonlocal boundary value problem [4].

In this paper, we focused on the existence results for the Hadamard fractional boundary value problem on an unbounded domain. Because of the noncompactness of an infinite interval, the fractional boundary value problems on an infinite interval have little been studied. We used monotone iterative method and the properties of the Green function to obtain the existence results of positive solutions for the fractional boundary value problem. But, we also construct the iterative scheme for approximating the solutions. Moreover, this method does not require the existence of upper and lower solutions.

**Keywords:** Fractional differential equations, fixed point theory, positive solutions.

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✉ Corresponding Author Email: [fulya.yoruk@ege.edu.tr](mailto:fulya.yoruk@ege.edu.tr)



## A Multi-Objective Approach for a Cubic Cell Formation with Quality Index

Yeliz Buruk Sahin<sup>1✉</sup>, Burak Urazel<sup>2</sup>

<sup>1</sup> *Eskisehir Osmangazi University, Industrial Engineering Department, Eskisehir, Turkey*

<sup>2</sup> *Eskisehir Osmangazi University, Electrical and Electronics Engineering Department, Eskisehir, Turkey*

### Abstract

As a popular field, studies on cellular manufacturing systems generally deal with indicators such as grouping efficiency and grouping efficacy that considers voids and exceptional elements. Besides, it is noteworthy that human issues have been dealt with in recent years [1]. The structure, which deals with the worker-machine-part matrix in a multidimensional way, has been called as the cubic cell formation problem in the literature [2,3]. There are many studies on this subject and it can be said that an important aspect of the studies is the quality dimension [4]. Although the aim in cubic CFP studies was to minimize the movement of workers between cells, it became important to transfer skilled workers from outside the cell to produce higher quality products. Due to the importance of employees, considering their skills in part processing and machine capabilities can improve the quality of cellular manufacturing systems. Thus, it was ensured that the quality of the parts produced was increased by selecting the appropriate workers.

In this study, the part processing skills of workers and machines were taken into account, as well as well-known cell performance indicators. The study also revealed a comprehensive literature review of the literature addressing the human factor in the cellular manufacturing system. To solve the problem, a new model was developed based on the basic cell formation constraints. The model was linearized with additional constraints. The mathematical model developed for the solution of the problem was coded in the GAMS 24.2.1 software and the model effectiveness was demonstrated on the numerical examples.

**Keywords:** cubic cell formation, quality index, part-machine-worker matrix

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✉ Corresponding Author Email: [yelizburuk@ogu.edu.tr](mailto:yelizburuk@ogu.edu.tr)

## Ball Balancing Table PID Controller Design with Optimization Algorithms

Mahmut Gökhan Turgut<sup>✉</sup>, Ö. Tolga Altınöz

Ankara University, Electrical-Electronic Engineering Department, Ankara, Turkey

### Abstract

In this research a PID controller has been developed for ball balancing experimental set which is a common example of the dynamical system used to apply control concepts [1]. It aims to control the position of the ball changes according to the angle of the platform. In this study we will work on Acrome Ball Balancing Table [2]. System must be controlled to bring the ball to the desired position. Since the PID (Proportional integral derivative) controller was proposed, it has been a worldwide solution for a control system [3]. For finding the best PID controller values which are necessary to keep the ball balanced, there are several methods to determine them, such as trial and error method, mathematical methods, Ziegler Nichols method. These classical methods have some disadvantages, they need to describe with mathematical functions, and they are not flexible [4]. For that reason, we preferred nature inspired optimization algorithms which are probabilistic search methods that simulate the natural biological evolution or the behavior of biological entities. Such algorithms can be used to obtain near optimal solutions in optimization problems [5]. In optimization algorithms, fitness function, which is performance criterion, is important. Fitness functions generally based on error equations. In this study we used Particle swarm optimization (PSO), firefly optimization (FA), artificial bee colony optimization (ABC), bees optimization (BA) and invasive weed optimization (IWO) with integral absolute error (IAE) fitness function and these algorithms have been compared. PID results shown below on the table. In the research we will use different controllers based on PID, such as PIDA, PIDC, 2 DOF PID, Fuzzy PID. And the all results will be tried on the real platform.

Algorithm	Rise Time	Settling Time	Overshoot	Steady State Error
ABC	0.1268	0.5352	2.3390	0.0260
BA	0.1269	0.5211	2.4839	0.0245
FA	0.1267	0.5352	2.3446	0.0259
IWO	0.1268	0.5366	2.3427	0.0259
PSO	0.1267	0.5352	2.3432	0.0260

**Keywords:** Ball Balancing Table, Controller Algorithms, PID Controller, Optimization, Optimization Algorithms, PSO, ABC, IWO, FA, BA.

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<sup>✉</sup> Corresponding Author Email: [mahmutgokhan@hotmail.com](mailto:mahmutgokhan@hotmail.com)

## A Chaotic Dynamical System on the Box Fractal

Nisa Aslan<sup>1</sup>✉, Mustafa Saltan<sup>1</sup>

<sup>1</sup>Eskişehir Technical University, Mathematics Department, Eskişehir, Türkiye

### Abstract

In the recent years, there are various studies related to fractals in the different areas such as mathematics, engineering, physics, biology, economics etc. One of the subject of these studies is to define a dynamical system on the self-similar sets. For example, a dynamical system is naturally defined by Barnsley on the right-Sierpinski gasket by using the related iterated function system (IFS) in [2]. Furthermore, in [1-5], the dynamical systems are given on the Sierpinski triangle and the Sierpinski tetrahedron which are defined by using different methods. On the other hand, to define intrinsic metrics on the fractals is also an important problem. Because of these metrics, many geometrical and topological properties of the structures can be investigated. Moreover, these metrics are necessary for examining whether a dynamical system is chaotic or not in the sense of Devaney (for details see [3]).

In this study, our aim is to define a chaotic dynamical system on the box fractal by using expanding and folding mappings. In order to express these dynamical systems more apprehensibly, we use the code representations of the points of the box fractal which provides us many conveniences when analyzing the properties of the dynamical system. Therefore, we first define a dynamical system as a composition function of expanding and folding mappings. Then we express this function on the code sets of the Box fractal by using the code representation of the points. We also give an algorithm to compute the periodic points and finally we show that this dynamical system is chaotic in the sense of Devaney by using the intrinsic metric given in [5].

**Keywords:** Box Fractal, Devaney chaos, code representations, dynamical system

### Acknowledgements

This work is supported by the Eskişehir Technical University Research Fund under contract 19ADP160.

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✉ Corresponding Author Email: [nisakucuk@eskisehir.edu.tr](mailto:nisakucuk@eskisehir.edu.tr)

## Multi-Derivative, Multi-Stage and Multi-Step Time Integration Methods

Hüseyin Tunc<sup>✉</sup>, Murat Sari

*Yildiz Technical University, Mathematics Department, Istanbul, Turkey*

### Abstract

The implicit methods are the best options for solving stiff initial value problems. The implicit time-integration algorithms are computationally costly especially for solving large nonlinear systems. Thus, an implicit solver must be optimized in terms of the local degrees of freedom of the algebraic system of equations. The multi-stage implicit algorithms such as the implicit Runge-Kutta methods (IRKM), implicit Lobatto methods or implicit Radau methods have more degrees of freedom than the linear multi-step methods. The linear multi-step methods such as the backward differentiation formulations (BDFs), the Adam-Bashford methods (ABMs) and the Adam-Moulton methods (AMMs) take advantage of the optimized degrees of freedom. However, this group of algorithms have a lack of storage problems and order-preservation drawback for, especially stiff problems. Here we show that multi-step methods require generally fewer time-steps than the multi-stage methods to get the same accuracy for the solution of stiff problems. An inevitable question arises here: can a stiff solver have both optimized degrees of freedom and order preservation? By eliminating the existing drawbacks of the differential transform method (DTM) based explicit solvers, the implicit-explicit local differential transform method (IELDTM) as a multi-derivative method is here proven to satisfy both essential properties.

**Keywords:** Time integration, Taylor series, implicit algorithms, stiff problems, initial value problems

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✉ Corresponding Author Email: [tnchsyn@gmail.com](mailto:tnchsyn@gmail.com)

## The Construction of a Dynamical System on the Sierpinski Propeller

Nisa Aslan<sup>1✉</sup>, Saliha Şeker<sup>1</sup>, Mustafa Saltan<sup>1</sup>

<sup>1</sup>Eskişehir Technical University, Department of Mathematics, Eskişehir, Türkiye

### Abstract

Fractals, which are also known as self-similar sets, are one of the popular topics of recent years due to their relationship with nature and they have also been researched in many different fields. The self-similarity is the main common property of these sets and it comes to the fore in many studies [2]. Self-similarity can be divided into different classes such as strictly self-similar, not strictly self-similar, and random self-similarity. Sierpinski triangle, Sierpinski tetrahedron, Box (Vicsek) are important examples of strong self-similar sets. In recent years, many different studies have been carried out on these strong self-similar sets by formulating the intrinsic metric [1,4].

On the other hand, there is no known dynamical system which is expressed by using code representations of the points on a not strictly self-similar set in the literature. Our main aim is to define a dynamical system on a not strictly self-similar fractal model, Sierpinski propeller. This set is actually formed by selecting one of the corner points of the Sierpinski triangle and combining different numbers of Sierpinski triangles at this point. Each neighborhood of this corner point contains the Sierpinski propeller. The intrinsic metric formulas are defined on the Sierpinski propellers with the help of code representations of the points in [3].

In this study, in order to define a dynamical system on the Sierpinski propeller, we use some transformations in accordance with the structure of this fractal, such as expanding, folding, translation and rotation mapping. Finally, we express this dynamical system by using the code representations of the points.

**Keywords:** Sierpinski Propeller, dynamical system, intrinsic metric.

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✉ Corresponding Author Email: [nisakucuk@eskisehir.edu.tr](mailto:nisakucuk@eskisehir.edu.tr)

## An Application of Double Stranded Smoothing Technique in Image Processing

Ahmet Şahiner <sup>1</sup>✉, Nurullah Yılmaz<sup>1</sup>

<sup>1</sup>*Süleyman Demirel University, Mathematics Department, Isparta, Turkey*

### Abstract

In this study, using Bezier curves a double stranded smoothing function is used to smooth total variation norm (TV). Experimental results are given by comparing the results obtained by hyperbolic smoothing, and global single stranded smoothing functions.

**Keywords:** Smoothing technique, total variation, regularization problems

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✉ Corresponding Author Email: [ahmetsahiner@sdu.edu.tr](mailto:ahmetsahiner@sdu.edu.tr)

## Fractional Mathematical Model Created to Prevent Cancer Cells from Escaping the Immune system

Esmehan Uçar<sup>1✉</sup>, Necati Özdemir<sup>1</sup>, Eren Altun<sup>2</sup>

<sup>1</sup> Balıkesir University, Mathematics Department, Balıkesir, Turkey

<sup>2</sup> Balıkesir University, Medical Pathology Department, Balıkesir, Turkey

### Abstract

In this article, we studied the new model created with the help of IL-10 cytokine and anti-PD-L1 inhibitor to increase the number and yield of CD8+T cells. This new model was created upon the destruction of cancer cells by the immune system, which have gained the ability to hide from the immune system. In addition, this new model is expressed with the Caputo fractional derivative because of hereditary and memory effect. We analyze stability analysis by finding eigenvalues for the this model described in this study. Finally, the graphics related to the mathematical model obtained with the fractional derivative is obtained and interpreted.

**Keywords:** CD8+T cells, immune system, fractional derivative

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✉ Corresponding Author Email: [esucarr@gmail.com](mailto:esucarr@gmail.com)

## Solution of First-Order Hyperbolic Partial Differential Equation Using Neural Networks

Pelin Çelenk<sup>1✉</sup>, Seda Gülen<sup>2</sup>, Murat Sari<sup>3</sup>

<sup>1</sup> *Yıldız Technical University, Department of Mathematics, Istanbul, Turkey*

<sup>2</sup> *Tekirdag Namik Kemal University, Department of Mathematics, Tekirdag, Turkey*

<sup>3</sup> *Yıldız Technical University, Department of Mathematics, Istanbul, Turkey*

### Abstract

In this study, an artificial neural network method is proposed to solve a first-order hyperbolic partial differential equation. This technique uses a trial function that depends on a neural network and satisfies the initial and boundary conditions to approach the solution of the problem. The unknown parameters of the neural network are adjusted by the gradient-descent optimization method that is a method to minimize the lost function. The numerical results of the problem are compared with the exact and available literature solutions. It is seen that the present method solution is closer to the exact solution than the literature method. Thus, it has been found that the proposed method that can capture the behavior of the problem is reliable and accurate.

**Keywords:** Neural Networks, Gradient Descent, First-Order Hyperbolic Partial Differential Equation

### Acknowledgements

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✉ Corresponding Author Email: [pepin.emecann@gmail.com](mailto:pepin.emecann@gmail.com)



## Household Lockdowns on Weekends can Marginally Reduce the Need for Contact Isolation and Social Distancing to Protect Economic Activity

Ramin Nashebi<sup>1</sup>, Murat Sari<sup>1</sup>, Seyfullah Enes Kotil<sup>2</sup>✉

<sup>1</sup> *Yildiz Technical University, Department of Mathematics, Istanbul, Turkey*

<sup>2</sup> *Bahcesehir University, Department of Medicinal Microbiology, Istanbul, Turkey*

### Abstract

Human behaviour, number of work hours, household lockdowns and economic activity are inseparably entangled in epidemic management. The majority of models are phenomenological. Complex phenomena are represented by simple mathematical expressions, treated with well mixed and mean-field approximations. To address this issue, we further augmented a stochastic, time discrete agent-based model by categorizing the nodes into three categories: household, workplaces, and social environment. By doing so, we track the place of infections. The data that we used is obtained from a real-world social network. We developed methods to simulate household lockdowns explicitly. We found that to stabilize the COVID-19 ( $R_0 = 1$ ), an 91.67% reduction of transmission probability in non-home environments is needed when agents can work 9 hours during the weekdays and act freely on weekends. A minor decrease to 86.16% is required when agents are locked down on weekends. The household lockdown at weekends can also be regarded as a successful staying-at-home policy at weekends. Also, reduction of work hours, from 9 to 6 hours, is as effective as weekend household lockdown. The epidemic can stabilize at higher hidden  $R_0$  depending on the success of contact-tracing/isolation. The impact of the short household lockdowns increases with higher targeted  $R_0$ . If the hidden  $R_0$  is 1.5, at the same conditions, the reduction is 79.34% and 65.20% for no household lockdown and household lockdown at weekends. Overall, if the public is moderately more successful in social distancing measures and policy dictates successful contact-tracing/isolation, then the household lockdowns may not be necessary. In the long run, it shows the importance of approaching pandemic management by keeping human compliance as high as possible. Additionally, our modeling allowed us to understand the contribution of non-home and home environments on overall dissemination of the epidemic. We found that although most of infections occur in homes when  $R_0 < 1$ , dissemination ultimately depends on non-home transmission.

**Keywords:** COVID-19, Agent-based model, Household effect, Social network

### Acknowledgements

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✉ Corresponding Author Email: [enesseyfullah.kotil@med.bau.edu.tr](mailto:enesseyfullah.kotil@med.bau.edu.tr)

## Hermite-Bell Based Bernoulli Polynomials

Uğur Duran<sup>✉</sup>, Mehmet Açıkgöz<sup>2</sup>

<sup>1</sup> *İskenderun Technical University, Department of Basic Sciences of Engineering, Hatay, Turkey*

<sup>2</sup> *University of Gaziantep, Department of Mathematics, Gaziantep, Turkey*

### Abstract

In this study, we introduce Hermite-Bell based Bernoulli polynomials and then investigate multifarious relations and formulas including some implicit summation formulas, identities and derivative properties.

**Keywords:** Bernoulli polynomials, Bell polynomials, Hermite polynomials, Generating functions

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✉ Corresponding Author Email: [ugur.duran@iste.edu.tr](mailto:ugur.duran@iste.edu.tr)

## Degenerate Poisson-Charlier Polynomials

Uğur Duran<sup>✉</sup>, Mehmet Açıkgöz<sup>2</sup>

<sup>1</sup> *İskenderun Technical University, Department of Basic Sciences of Engineering, Hatay, Turkey*

<sup>2</sup> *University of Gaziantep, Department of Mathematics, Gaziantep, Turkey*

### Abstract

In this work, we consider degenerate form of the Poisson-Charlier polynomials and then investigate some of their properties and identities. Also, we derive some implicit summation formulas, integral representation and derivative property for the degenerate Poisson-Charlier polynomials.

**Keywords:** Degenerate exponential function, Poisson-Charlier polynomials, Special polynomials, Generating functions

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✉ Corresponding Author Email: [ugur.duran@iste.edu.tr](mailto:ugur.duran@iste.edu.tr)

## An Elliptically Shaped Ice Particle with Non-Uniform Density and Air Interplay Between Parallel Side Walls

Samire Yazar<sup>✉</sup>

*Ordu University, Department of Mathematics, Ordu, Turkey*

### Abstract

A relatively quite dense particle moving within a fluid especially in the case of air being the fluid is one of the most challenging problem in the aviation industry [1,2]. In the past few years, quite a few different dynamic fluid-particle interactions have been investigated [3-6]. Applications arise in aerodynamic safety context such as the movement of the ice particles into an aircraft engine intake. This phenomenon has potential risk for aircraft passengers [1,2]. In this study, we focus on a single elliptically shaped ice particle's motion between parallel side walls filled with air. The configuration valid for the case of an ice particle surrounded by air, corresponds to the density of the elliptically shaped ice particle being remarkably larger than the density of the fluid. The kinematic relations between the unknown mass fluxes are found owing to the pressure boundary conditions. Unsteady particle motion is accompanied by a quasi-steady air flow, presented by the core equations. The gravitational effects on the ice particle are omitted in this set-up. The effects of each parameter on the system are also examined. The pressure difference creates a positive lift-force and negative moment for a uniform density distributed particle which therefore moves upwards and clockwise. Comparisons between analysis and numerical work show a good agreement. Finally, a rapid time scale is defined and it is found that the particle with non-uniform density (particularly a front-loaded ice particle) free to travel inside parallel side walls has unusual behaviour, giving antiperiodic and irregular oscillations when interacting with the air, which is a very powerful result.

**Keywords:** Nonlinear dynamics, air-particle interaction, computation, direct numerical simulation

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<sup>✉</sup> Corresponding Author Email: [samirebalta@odu.edu.tr](mailto:samirebalta@odu.edu.tr)

## Stability of Common Research Lab with Asymmetric Firms: Effects of an Exclusive Membership Rule Versus an Open Membership Rule Approach

Razika Sait <sup>1</sup>✉, Abdelhakim Hammoudi <sup>2</sup>, Mohammed Said Radjef <sup>1</sup>

<sup>1</sup> LaMOS Research Unit, Operational Research Department, University of Bejaia, Bejaia, Algeria

<sup>2</sup> University of Paris-Saclay, INRAE, UR ALISS, 94205 Ivry-sur-Seine, France

### Abstract

This paper analyses stability conditions of R&D coalition within Common Research Lab agreements where asymmetric firms engage in cost-reducing R&D in the presence of technological spillovers. We determine endogenously and compare the size of stable coalition and the social welfare associated using D'Aspremont et al [1] approach based on Open Membership Rule [2], and the Exclusive Membership Rule [2]. We show that depending on spillover level, the blocking coalition of mid-sized groups allows to obtain an optimal coalition in the sense of Pareto, without requiring an aggregation of profits between the firms, compared to that one formed by Nash stability concept. This blocking coalition generates also a greater welfare compared to that one generated by the industry-wide coalition.

**Keywords:** Endogenous formation of research coalitions, R&D spillovers, Exclusive membership Rule, Open membership rule, Common Research Lab, Internal and External Stability of coalitions, Social welfare.

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✉Corresponding Author Email: [razika.sait@gmail.com](mailto:razika.sait@gmail.com)

## Genetic Algorithm Responses of Advection-Diffusion Processes

Murat Sari<sup>✉</sup>, Esra Yaganoglu, Hande Uslu

*Yildiz Technical University, Department of Mathematics, Istanbul, Turkey*

### Abstract

In this study, we propose an evolutionary approach based on genetic algorithm to obtain the advection diffusion responses of natural processes faced in many branches of science. Evolutionary processes such as crossover, mutation and selection yield the survival of the fittest so by using the algorithm, the optimum candidate solution of the advection-diffusion model is found. In the last part, two test problems are given and the method are compared with the analytic solution of the model equation.

**Keywords:** Advection-diffusion process, Genetic algorithm, Optimum solution

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✉ Corresponding Author Email: [sarim@yildiz.edu.tr](mailto:sarim@yildiz.edu.tr)

## New Algorithms for Two-Sided Disassembly Line Balancing Problem

Zixiang Li<sup>1</sup>, Ibrahim Kucukkoc<sup>2</sup>✉

<sup>1</sup> Key Laboratory of Metallurgical Equipment and Control Technology of Ministry of Education, Wuhan University of Science and Technology, Wuhan, China

<sup>2</sup> Department of Industrial Engineering, Balıkesir University, Balıkesir, Turkey

### Abstract

The recovery of end-of-life products become essential for economic and environmental benefits, where the disassembly of these products is the first and vital step [1]. Two-sided disassembly lines are utilized to disassemble the large-size products, whereas the developed methodologies on two-sided disassembly line balancing problem (TDLBP) are limited [2-3]. Hence, this study contributes to literature on TDLBP by introducing several recent metaheuristic algorithms to solve this NP-hard problem effectively. These metaheuristics are teaching-learning-based optimization algorithm, grey wolf optimizer, and whale optimization algorithm. Computational study is conducted to evaluate the performances of these methods and several new upper bounds are achieved by these methods in solving the large-size instances.

**Keywords:** Two-sided disassembly line, disassembly line balancing, meta-heuristics, manufacturing systems

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✉ Corresponding Author Email: [ikucukkoc@balikesir.edu.tr](mailto:ikucukkoc@balikesir.edu.tr)

## Mathematical Behaviour of Solutions for Kirchhoff-Type System with Logarithmic Nonlinearity

Tuğrul Cömert<sup>✉</sup>, Erhan Pişkin

*Dicle University, Mathematics Department, Diyarbakır, Turkey*

### Abstract

This paper deals with the initial-boundary value problem for Kirchhoff-type system with logarithmic nonlinearity. We discuss the existence, uniqueness and exponential energy decay estimates of weak solutions under some conditions by employing potential method. Moreover, by concavity method, we derive the finite time blow-up results of weak solutions. Studies of logarithmic nonlinearity have a long history in physics as it occurs naturally in different areas of physics such as supersymmetric field theories, optics, quantum mechanics and inflationary cosmology [1-2]. Also, Kirchhoff model is very important for many applications in mechanics, elastic theory and other areas of mathematical physics. Recently some authors studied the parabolic and hyperbolic type equation with logarithmic source term [3-9].

**Keywords:** Kirchhoff-type system, Mathematical behaviour, Logarithmic nonlinearity

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✉ Corresponding Author Email: [tugrulcomert@gmail.com](mailto:tugrulcomert@gmail.com)



## Two Modular Equations Close to the Discrete Logarithm Problem

Sohaib Moussaid El Idrissi<sup>1✉</sup>, Ismail Aounil<sup>2</sup>, Omar Khadir<sup>1</sup>

<sup>1</sup> Hassan II University of Casablanca, Fstm, Laboratory of Mathematics, Cryptography, Mechanics and  
Numérique Analysis, Mohammedia, Morocco

<sup>2</sup> Hassan II University of Casablanca, Fstm, Departement of Mathematics, Mohammedia, Morocco

### Abstract

Let  $p$  be a large prime number and fix any natural integers  $a, b, c$  smaller than  $p$ . Consider the three independent modular equations:

$$a^x \equiv b \pmod{p} \quad (1)$$

$$a^x x^y \equiv b \pmod{p} \quad (2)$$

$$a^x x^b \equiv c^y \pmod{p} \quad (3)$$

where  $x, y$  are the unknown variables.

In the mathematical and computer sciences literature, there is no efficient algorithm for solving equation (1) known as the discrete logarithm problem and widely used in the public key cryptography.

It is easy to show that if we can find the solutions of equivalence (1) for any  $a, b \in \mathbb{N}$ , then we can also solve equations (2) and (3). But conversely, does knowing how to solve equations (2) and (3) allow us to find the solutions of the discrete logarithm problem (1)? In this communication we discuss some points about this open problem.

**Keywords:** Modular equation, discrete logarithm problem, public key cryptography.

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\*Corresponding Author Email: [moussaid.i.sohaib@gmail.com](mailto:moussaid.i.sohaib@gmail.com)

## Design of UHF band Yagi-Uda TV Antenna

Sunil Narayan <sup>✉</sup>, Ashwin Chand and Utkal Mehta

*Electrical and Electrical Engineering, School of Information Technology, Engineering, Mathematics, and Physics, The University of the South Pacific, Laucala Campus, Suva, Fiji*

### Abstract

Yagi-Uda antennas are known to be difficult to design and optimize due to their sensitivity at high gain, and the inclusion of numerous parasitic elements. Still, this antenna is familiar as the commonest kind of terrestrial TV antenna to be found on the rooftops of houses and have a wide application in other communication industries. It can be used at frequencies between about 30 MHz and 3 GHz. This paper presents simulated results of a UHF band Yagi-Uda antenna designed to operate in the UHF TV band ranging from 470 MHz to 890 MHz, using YO 6.5 Yagi Optimizer software. The designed antenna radiates an end-fire fan beam pattern with bandwidth of about 57% for voltage standing wave ratio (VSWR) less than 2. The simulated result shows that the antenna exhibits good bandwidth and moderate gain properties with good impedance characteristics.

**Keywords:** UHF, Yagi – Uda, Antenna and Optimizer

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<sup>✉</sup> Corresponding Author Email: [spnz2014@gmail.com](mailto:spnz2014@gmail.com)

## Bipolar Fuzzy Soft Filter

İzzettin Demir<sup>1✉</sup>, Murat Saldamlı<sup>1</sup>, Merve Okurer<sup>1</sup>

<sup>1</sup> *Düzce University, Department of Mathematics, Düzce, Turkey*

### Abstract

In this paper, we introduce the notion of a bipolar fuzzy soft filter (BFS-filter) by using bipolar fuzzy soft sets. Also, we give the concept of a BFS-filter base and establish the image of the BFS-filter under the bipolar fuzzy soft mappings. Finally, we define the concept of an ultra BFS-filter and investigate some of its properties.

**Keywords:** Bipolar fuzzy soft set, bipolar fuzzy soft mapping, BFS-filter, ultra BFS-filter

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✉ Corresponding Author Email: [izzettindemir@duzce.edu.tr](mailto:izzettindemir@duzce.edu.tr)

## Vibration Controls of a Pier Using Deep Learning LSTM Network

Barış Namlı<sup>1</sup>, Cihan Bayındır<sup>2,3</sup>

<sup>1</sup> *Istanbul Technical University, Civil Engineering Department, İstanbul, Turkey*

<sup>2</sup> *Istanbul Technical University, Civil Engineering Department, İstanbul, Turkey*

<sup>3</sup> *Boğaziçi University, Civil Engineering Department, İstanbul, Turkey*

### Abstract

Balancing the offshore structures in the ocean or sea against the forces created by seismic movements and waves is extremely important for the serviceability and the safety of the structure. In order to achieve this, various approaches are currently being considered [1-2]. Region-specific parameters should be recorded and used when needed to assist these approaches. In addition, the use of artificial intelligence methods, which have been in demand recently, to reduce the oscillation in buildings, allows more effective results in the application that can be done in this field [3-4]. Thanks to the long-short-time memory (LSTM) algorithm [5], one of the deep learning methods, time series prediction can be performed. As a result of the prediction, better approaches can be developed for the future. This study shows that the vibration control of offshore platforms can be achieved by the deep learning techniques, which is a branch of artificial intelligence, against various types of loadings. For this purpose, a long pile is analyzed using the Morison equation [6]. Then, realistic wave and earthquake loads are applied to analyze system behavior in a more realistic setting. The applied earthquake and wave loads are predicted using the LSTM deep learning network and applied to the system as feedback. It is shown that significant reduction in the vibration amplitudes can be achieved by this approach. In order to better examine the findings, the system is examined both in the time domain and Fourier spectral domain. The results of the study are compared with the related literature and their usage and applicability are discussed.

**Keywords:** Vibration control, Offshore structures, Deep learning

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✉ Corresponding Author Email: [barisnamli17@gmail.com](mailto:barisnamli17@gmail.com)

## Analysis Methods and FPAA Implementation of Hyperchaotic Systems

Gülnur Yılmaz<sup>✉</sup>, Enis Günay

*Erciyes University, Department of Electrical and Electronics Engineering, Kayseri, Turkey*

### Abstract

In this study, hyperchaotic systems are researched, analyzed and implemented on Field Programmable Analog Arrays (FPAA) by literature review. To begin with, different structured systems in the literature are specified. Following this, time-series and phase diagrams are created by obtaining the behavior of state variables of dynamic systems. For instance, 2 and 3-dimensional phase diagrams of Rossler [1] hyperchaotic model are observed and time-series graphics of this system are provided.

In the second stage of the study, the methods used in the analysis of hyperchaotic systems are considered. At this point, the Lyapunov exponents are calculated first and it is demonstrated that the Lorenz [2] hyperchaotic system has 2 positive Lyapunov exponents and is in a hyperchaotic structure. Then, bifurcation diagrams are formed and it is interpreted at what values the Lorenz [2] hyperchaotic system entered into chaos and has a periodic structure. An additional method used for analysis is the Poincare maps. These maps are obtained by converting an n-dimensional continuous-time system to an n-1-dimensional discrete-time system and examining them in cross-sections. In this context, the chaotic structure of the Wang-Cang [3] system is shown by creating the Poincare maps.

Power spectrum analysis and 0-1 Test are other analysis methods performed by using the time series obtained in the first part of this study. For chaotic systems, a noise-like power spectrum graph spanning a wide frequency band is acquired. As an example of this method, the 5D Sprott B [4] hyperchaotic model is examined. The 0-1 Test, on the other hand, is a binary-based method which is a more practical way of analysis compared to the others. The chaotic structure of the Sprott B [4] system is also demonstrated using the 0-1 Test.

After all analysis methods are completed, FPAA implementation is performed with the Anadigm Designer 2. The implementation result of the Lorenz [2] hyperchaotic system is given as an example for this part. In the end, accurate graphics similar to the phase space diagrams are obtained.

**Keywords:** Hyperchaos, Lyapunov exponents, bifurcation diagrams, Poincare maps, power spectrum, 0-1 tests, FPAA

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✉ Corresponding Author Email: [gulnur.yilmaz@erciyes.edu.tr](mailto:gulnur.yilmaz@erciyes.edu.tr)

## Machine Learning-Based Profit Analysis of Aviation Sector During The Covid-19 Pandemic

İpek Sırma<sup>1</sup>, Sevgi Şengül Ayan<sup>1✉</sup>

<sup>1</sup> Department of Engineering, Industrial Engineering, Antalya Bilim University, Döşemealtı, Antalya, Turkey

### Abstract

The coronavirus pandemic has deeply affected the whole world. There have been radical changes in many sectors such as logistics, aviation, and health [1]. Moreover, flight suspensions in the first months of the pandemic have long-term effects [2,3]. One of the best ways to deal with the uncertainty created by the pandemic is to make predictions and set the right strategic goals. In this study, profit analysis was made in the aviation sector with the data obtained from the Turkish Airlines Investor website by applying machine learning regression models. Seven methods including Ridge Regression (RR), Lasso Regression (LR), Elastic Net Regression (ENR), Multiple Linear Regression (MLR), Support Vector Regression (SVR), Decision Tree Regression (DTR), and Random Forest Regression (RFR) algorithms are used for the profit modeling. A number of Covid-19 cases, number of passengers, passenger RASK and Net Profit-Loss are considered as features in the models. We observe that Ridge, Lasso, and Elastic Net regressions can solve the overfitting problem. The best fit is found with the Ordinary Least Squares method [4]. In the SVR training phase, an optimal solution is found because it uses convex quadratic programming [5]. The comparisons of these algorithms are done using the R-Squared, Adjusted R-squared, Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) metrics. The aim is to try to understand how the pandemic affects the profit and loss due to a pandemic. The comparative analysis results indicate that SVR, DTR, and RFR regressions outperform the RR, LR, ENR, and MLR models. Forecasting before and after pandemics are also checked for specific case studies that provide meaningful insights about the effect of the Covid-19 on the aviation sector.

**Keywords:** Profit analysis, regression models, aviation sector, machine learning, forecasting

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✉ Corresponding Author Email: [sevgi.sengul@antalya.edu.tr](mailto:sevgi.sengul@antalya.edu.tr)

## Blow up of Solutions for a Wave Equation with Delay

Hazal Yüksekaya<sup>1</sup>✉, Erhan Pişkin<sup>1</sup>

<sup>1</sup> Dicle University, Mathematics Department, Diyarbakır, Turkey

### Abstract

This work deals with a wave equation with delay term. Under appropriate conditions, we prove the blow up of solutions in a finite time. Generally, time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena. Delay effects can be a source of instability. Moreover, it is well known that delay effects may destroy the stabilizing properties of a well-behaved system. In the literature, there are several examples that illustrate how time delays destabilize some internal or boundary control system.

**Keywords:** Blow up, Delay, Wave equation.

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✉ Corresponding Author Email: [hazally.kaya@gmail.com](mailto:hazally.kaya@gmail.com)

## Nonexistence of Global Solutions for a Hyperbolic-Type Equation with Delay Term

Hazal Yüksekaya<sup>1</sup>✉, Erhan Pişkin<sup>1</sup>

<sup>1</sup> Dicle University, Mathematics Department, Diyarbakır, Turkey

### Abstract

In this work, we deal with a hyperbolic-type equation with delay term. Under suitable conditions, we establish the nonexistence of global solutions in a finite time. Generally, time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena. Delay effects can be a source of instability. Moreover, it is well known that delay effects may destroy the stabilizing properties of a well-behaved system. In the literature, there are several examples that illustrate how time delays destabilize some internal or boundary control system.

**Keywords:** Delay term, Hyperbolic-type equation, Nonexistence of solutions.

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✉ Corresponding Author Email: [hazally.kaya@gmail.com](mailto:hazally.kaya@gmail.com)



## Measuring the Service Quality Performance of Hospitals in Managing the COVID-19 Vaccine Process

Melike Erdoğan<sup>1</sup>, Ertuğrul Ayyıldız<sup>2</sup>✉

<sup>1</sup> *Düzce University, Industrial Engineering Department, Düzce, Turkey*

<sup>2</sup> *Karadeniz Technical University, Industrial Engineering Department, Trabzon, Turkey*

### Abstract

The vaccination process has finally started for the pandemic that the whole world has been trying to cope with in recent years. In this context, the vaccination process has started in many countries. Most of the people in Turkey have been vaccinated since January in order of priority by age or occupational group (healthcare workers, teachers, .. etc). People can get an appointment from the online system of The Ministry of Health and be vaccinated at specified times they select. At this point, different service quality is revealed in the vaccination process for different hospitals. In this study, we try to determine the priorities of the criteria that hospitals should consider for the vaccine service quality measurement process. For this purpose, we apply the SERVPERF (Service Performance) and multi-criteria decision-making (MCDM) approach to weighting the service performance attributes of hospitals in the COVID-19 vaccination process. Using the dimensions of the SERVPERF model, MCDM analysis is developed to deal with all the qualitative and quantitative criteria in the decision process is obtained. In addition, fuzzy sets are adopted to reflect the uncertainty to the decision-making process in the best way. As a result of this paper, the most important performance criteria on the vaccination process will be determined for the people who will be vaccinated, and it will be determined which criteria should be given more importance by hospitals in providing vaccination services. With this study, a quantitative analysis of the service quality of hospitals in the vaccination process will be presented for the first time..

**Keywords:** COVID-19, hospital, MCDM, SERVPERF, vaccination, fuzzy logic

✉ Corresponding Author Email: [ertugrulayyildiz@ktu.edu.tr](mailto:ertugrulayyildiz@ktu.edu.tr)

## Convolutional Neural Network for Arabic Word Recognition

Benbakreti Samir<sup>1</sup>✉, Benouis Mohamed<sup>2</sup>, Tlemsani Redouane<sup>3</sup>, Benbakreti Soumia<sup>4</sup>, Roumane Ahmed<sup>1</sup>

<sup>1</sup> National Institute of Telecommunication and ICT, Department of speciality, Oran, Algeria

<sup>2</sup> University of M'sila, Computer Science Department, M'sila, Algeria

<sup>3</sup> University of USTO-MB, Computer Science Department, Oran, Algeria

<sup>4</sup> Djillali Liabes University, Laboratory of Mathematics, Sidi Bel Abbès, Algeria

### Abstract

In this paper, we present CNN architectures for unconstrained Arabic word recognition using the offline writing signal i.e. images [1]. The proposed approach represents a particular algorithm belonging to the family of deep neural networks [2-4]. The advent of deep neural networks has made it possible to get rid of several blocks, such as the pre-processing and extraction of relevant features from the image. The used database is NOUN v3 is a hybrid database, it's contained 4800 images represented the Algerian cities [5]. For this task, we used a deep approach to classify the images, in our case, CNN trained and tested the database. CNN's architecture is based on research on the visual cortex of the cat [6]. The visual cortex contains an arrangement of cells, which act as input space filters. The number of layers and their sizes are determined by the complexity and type of the problem to be treated. The output of a CNN is a probability for each class that categorizes the image (48 in this case). There is a succession of convolutional layers immediately followed by a pooling layer, which is characteristic of CNNs. The advantage of a CNN is to extract characteristics specific to each image by compressing them so as to reduce their initial size. Our experimental study, shows that the optimal parameters (training parameter, learning rate, frequency validation) for our system were (85 %, 0.015, 25). We show that the use of this approach produces a satisfactory word recognition rate of 97.96%.

**Keywords:** Arabic handwriting, offline recognition, deep learning, CNN.

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✉ Corresponding Author Email: [Benbakretisamir@gmail.com](mailto:Benbakretisamir@gmail.com)

## Constraint Programming Model for Rich Electric Vehicle Routing Problems

Damla Kizilay<sup>1✉</sup>, Hande Öztop<sup>1</sup>, Zeynel Abidin Çil<sup>1</sup>

<sup>1</sup>*Izmir Democracy University, Industrial Engineering Department, Izmir, Turkey*

### Abstract

Traditional vehicle routing problems have several applications in the industry; several sectors, including production to service, need to transport the products. Since product distribution takes place in many sectors, transportation becomes an indispensable part for them. Nowadays, online shopping frequency has increased due to the pandemic conditions that have emerged recently and have affected all sectors around the world. This situation has revealed the need for people to reach their orders at any time without any problems. Within the framework of these needs, the demand for transportation problems and the search for fast and quality solutions have gained momentum over time. Therefore, inter-sectoral competition is increasing in this regard. This challenge requires high-quality solutions for the routes as well as on-time delivery in logistic distribution processes. Therefore, the efficient management of the delivery fleet becomes the key indicator for the companies to show themselves among their competitors. Today, logistics companies pay higher fines for each gram/km of emissions due to new policies and regulations regarding greenhouse gas emissions in the transportation sector [1]. With electric vehicle market penetration, many companies are considering the integration of electric vehicles into their own fleets, which produce little noise, have no local greenhouse gas emissions, and are independent of fluctuating oil prices. In this study, the VRP, which has been extensively researched and studied to date, is extended to the electric vehicle routing problem (E-VRP), which takes into account certain features of electric vehicles as well as the vehicle capacities and the customer availabilities, resulting in rich E-VRP problems. The study aims to decrease the total cost while increasing customer satisfaction. A constraint programming (CP) model is developed to solve the indicated rich E-VRP problems through this aim. Since the experimental results of the previous studies state the superiority of the CP results over the mixed-integer linear programming models for the majority of medium-to-large problem instances [2], we only presented the CP model. Unlike the previous models, the study considers the rich E-VRP problems, including several constraints under different scenarios.

**Keywords:** Rich Electric Vehicle Routing, Constraint Programing, Optimization, Green Logistics

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✉ Corresponding Author Email: [damla.kizilay@idu.edu.tr](mailto:damla.kizilay@idu.edu.tr)

## A DSS for Assessing the Health Performance: The Case of City Hospitals

Hilal Demirel<sup>1</sup>, Yeliz Buruk Sahin<sup>1</sup>✉

<sup>1</sup> *Eskisehir Osmangazi University, Industrial Engineering Department, Eskisehir, Turkey*

### Abstract

Performance evaluation in health units is important in the process of improving service quality and decision-making decisions for the future [1]. Decision support systems are management information systems that support organizational decision making processes [2]. Numerous criteria are taken into account to compare health institutions. In Multi-Criteria Decision Making (MCDM) problems, it is possible to obtain fast results for scenarios consisting of different criteria and options with the use of Decision Support System (DSS).

In this study, a Microsoft Excel VBA based DSS was developed to compare the performances of City hospitals in Turkey. A database with criteria and alternatives was created. The developed system presents the bed capacity, the number of operating rooms, the number of intensive care beds, the number of burn units, the number of polyclinic rooms and the number of delivery beds as evaluation criteria. The DSS provides flexibility to the user for the determination of criterion weights. Weight determination is possible by manual entry. Additionally, the proposed MCDM based DSS presents SWARA (Step-Wise Weight Assessment Ratio Analysis) which is developed by Keršuliene et al. [3] and BWM (Best Worst Method) which is proposed by Rezaei [4] to determine criteria weights. As an MCDM approach, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has been integrated into the developed system. An illustrative application is presented to show the features of the proposed system and analysis of different scenarios is also possible. As a result, performance evaluation results and visual comparison results are presented to the decision maker.

**Keywords:** SWARA, BWM, health performance, decision support system

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✉ Corresponding Author Email: [yelizburuk@ogu.edu.tr](mailto:yelizburuk@ogu.edu.tr)

## Optimizing Seasonal Grain Intakes with Non-Linear Programming: An Application In The Feed Industry

Alperen Ekrem Çelikdin<sup>✉</sup>

*Planning & Logistics Manager, Süttaş Inc., Aksaray, Turkey*

### Abstract

In the feed sector, 95% of the input costs are due to the supply of raw materials used in feed production. Competition determines the selling price. Due to the use of similar technologies and the fact that the share of production costs in total costs is very small, it is not considered very likely to provide competitive advantage with innovations in production. Grain products are used between 30% and 50% in the solution of the feed ration. Rather than being agricultural products that continue to be produced at all times of the year, grains are produced only seasonally. Due to this limited time period, feed production enterprises have to balance their financial burdens and operational requirements while making their annual stocks. The study was carried out in enterprises with feed factories in four regions of Turkey. Based on the season data of the year 2020-2021, the grain purchase planning for the year 2021-2022 was tried to be optimized with non-linear programming. While creating the mathematical model, grain prices, interest rates, production needs according to production planning, sales according to sales forecasts, factory stocking capacities, licensed warehousing rental, transportation and handling costs were taken into account.

With the paper that is unique in the cattle feed production sector, storage, transportation and handling costs will be minimized, and a cost return will be achieved with the positioning to be taken in the season. The model, which can provide a cost advantage of 0.7% according to the grain pricing forecast and market data for the 2021-2022 season, will also provide insight to the managers for additional storage space investments.

**Keywords:** Financial optimization, Non-linear programming, Purchase planning

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<sup>✉</sup> Corresponding Author Email: [a.celikdin@gmail.com](mailto:a.celikdin@gmail.com)

## Verhulst Lotka Volterra fractional differential SEIRS model: Analysis of SARS-CoV-2 pandemic disease

Calin-Adrian Comes<sup>1,2✉</sup>

<sup>1</sup>George Emil Palade University of Medicine, Pharmacy, Science and Technology, Targu-Mures, Romania

<sup>2</sup>Institute of National Economy, Romanian Academy, Bucharest, Romania

### Abstract

We proposed a fractional differential SEIRS pandemic spread disease model with population migration between two spatial entities in the framework of fractional Atangana-Baleanu (AB) derivative approach [1], to determine the state of disease-free equilibrium. These results are important for pandemic disease control. An epidemic of an infectious disease that has spread across a large region, continents, or worldwide, affecting a great number of people, but a widespread endemic disease with a stable number of infected people is not a pandemic. The basic reproductive rate  $R_0$  investigation for diffusion models is presented in the following research's [2-5] with roots in classical papers of Verhulst-Lotka-Volterra (VLV) [6-9]. Our contribution to this paper is reflected in the modeling of the pandemic through migration from one location to another location of both susceptible and infected people by determining the precise definition of the basic reproduction number  $R_0$  for each of the hypostases.

**Keywords:** Verhulst–Lotka–Volterra model, Atangana–Baleanu fractional derivative, Laplace transformation, Mittag-Leffler function, SARS-CoV-2 diseases, SEIRS model

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✉ Corresponding Author Email: [calin.comes@umfst.ro](mailto:calin.comes@umfst.ro)

## Analysis and Dynamics Behavior of $\Psi$ -Hilfer Fractional Order Three Dimensional Model

Zakia Hammouch<sup>1,2</sup>, Rando R.Q. Rasul<sup>3</sup>, Abdelouahed Allahamou<sup>✉4</sup>

<sup>1</sup> Department of Medical Research, China Medical University Hospital, Taichung 40402, Taiwan,

<sup>2</sup> Ecole Normale Supérieure, Moulay Ismail University of Meknès 50000, Morocco

<sup>3</sup> Department of Mathematical Sciences, College of Basic Education, University of Sulaimani, Sulaimani, Iraq.

<sup>4</sup>Laboratory of Mathematical Analysis and Applications, Faculty of Sciences Dhar Al Mahraz, Sidi Mohammed Ben Abdellah University B.P. 1796, 30000, Fez, Morocco

### Abstract

Dynamics behavior of three dimensional spatial lattice (especially Thomas's model and its extensions) of integer order were investigated by many authors. In this study, we examine a non-integer order of the three dimensional spatial lattice based on the sine function. We use a new class of fractional derivative (which it is known as  $\Psi$ -Hilfer derivative), to construct a novel fractional model. Existence and uniqueness theorem for the solution were provide by applying Picard's Lendilof xed point theorem. The main purpose of this study is to investigate the dynamics behavior of the constructed model. We provide that under a certain condition the phases of the system are stable around the origin besides some values of the parameters that possess a hopf bifurcation.

**Keywords:** Stability, Hopf bifurcation,  $\Psi$ -Hilfer derivative, Picard's Lendilof fxed point, Equilibrium point,

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✉ Corresponding Author Email: [abdelouahed.allahamou@gmail.com](mailto:abdelouahed.allahamou@gmail.com)

## Efficient Solution of Fractional-Order SIR Epidemic Model of Childhood Diseases with Optimal Homotopy Asymptotic Method

Oluwaseun Olumide Okundalaye <sup>1</sup>✉, Necati Özdemir <sup>2</sup>, Wan Ainun Mior Othman <sup>3</sup>

<sup>1</sup> Faculty of Science, Department of Mathematical Sciences, Adekunle Ajasin University, Akungba-Akoko, Nigeria

<sup>2</sup> Faculty of Science and Arts, Department of Mathematics, Balikesir University, Balikesir, Turkey

<sup>3</sup> Institute of Mathematical Sciences, University of Malaya, 50603, Kuala Lumpur, Malaysia

### Abstract

In providing an accurate approximate analytical solution to the non-linear system of fractional-order susceptible-infected-recovered epidemic model (FOSIREM) of childhood disease has been a challenge because no criteria to guarantee the convergence of the infinite series solution [1, 2]. We compute an accurate approximate analytical solution of the problem by optimal homotopy asymptotic method (OHAM). The fractional-order operator is in the conformable fractional derivative operator sense [3, 4]. The OHAM approach series solution speedily converges to the exact solution as the order of the fractional-order derivative approaches one, which proved OHAM as an excellent tool for this model.

**Keywords:** Infectious disease, fractional mathematical modeling, approximate analytical method, conformable derivative operator, SIR model, reproduction number.

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✉ Corresponding Author Email: [okundalaye.oluwaseun@aau.edu.ng](mailto:okundalaye.oluwaseun@aau.edu.ng)



## dsPIC-based sensorless control of induction motor and real-time monitoring on Simulink

Metin Demirtas<sup>1✉</sup>, Erdem Ilten<sup>1</sup>, Haris Calgan<sup>1</sup>

<sup>1</sup> Balikesir University, Electrical Electronics Engineering Department, Balikesir, Turkey

### Abstract

Induction motor (IM) is utilized mostly in industry due to its low cost, easy maintenance, superior performance and robust structure. Despite the advantages, it requires a complex control system. Scalar control is commonly employed via controlling the ratio between voltage and frequency (V/f) and provides easily application. It has simple algorithm and low steady-state error [1]. However, in order to control the IM it is essential to know the position or the speed of machine. A shaft mounted encoder is generally preferred for this purpose. Considering the noise and cost, many studies focus on the sensorless control of the IM via estimating the speed of motor.

In this study, a new test bench to control IM sensorless is proposed. A classical sliding mode observer (SMO) is used in the experimental set-up to make speed estimation [2]. A tilt integral derivative (TID) controller is designed to track the speed reference. The main advantage of proposed test bench is that the system contains dsPIC33FJ128MC804 which can be operated with Matlab/Simulink in External Mode [3]. Therefore, instead of complex coding platforms, simple Simulink blocks are used to form the sensorless control algorithm. After compiling the program, it is uploaded to the chip by means of the PicKit v3.0. Hereby, real-time monitoring of the rotor speed, stator voltages and currents can be carried out as on-line. Experimental results show that the proposed sensorless control algorithm is well performed in cases of load disturbances and sensor noises. The accuracy and simplicity of proposed test bench make it reliable for further researches.

**Keywords:** sensorless control, sliding mode observer, TID control, digital signal processor, induction motor

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✉ Corresponding Author Email: [mdtas@balikesir.edu.tr](mailto:mdtas@balikesir.edu.tr)

## Optimal TID controller design for dsPIC-based induction motor drive

Metin Demirtas<sup>✉</sup>, Erdem Ilten, Haris Calgan

*Balıkesir University, Electrical Electronics Engineering Department, Balıkesir, Turkey*

### Abstract

Induction motor (IM) is used commonly in several areas in terms of advantages of being low cost, effective and robust. Despite the superiorities, the speed of motor is generally controlled via adjusting the ratio between voltage and frequency (V/f). This control method is called as scalar control. It has simple algorithm and low steady-state error [1]. Besides the scalar method, a specific controller has to be implemented in order to increase the operational performance and to decrease speed error.

In this study, an induction motor is driven by using dsPIC33FJ128MC804 which can be operated with Matlab/Simulink in External Mode [2]. Hereby, instead of complex coding platforms, simple Simulink blocks are used to obtain input output data of the induction motor. A small signal machine model is formed by means of input-output dataset [3]. Thereafter integral of time-weighted absolute error (ITAE) based optimization of Tilt Integral Derivative (TID) controller is carried out. Simulation and experimental results show the validity of the optimal TID controller.

**Keywords:** TID control, digital signal processor, induction motor, ITAE

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<sup>✉</sup> Corresponding Author Email: [mdtas@balikesir.edu.tr](mailto:mdtas@balikesir.edu.tr)

## A Novel Analytical study of Boussinesq-type equations

Zehra Pinar<sup>✉</sup>

*Tekirdağ Namık Kemal University, Mathematic Department, Tekirdağ, Turkey*

### Abstract

In recent years, the works related to models have been concentrated on obtaining the exact solutions. This study focuses on the models which are arising in the ocean sciences where Boussinesq-type equations are most used models. The exact solutions of generalized Boussinesq-type equations are investigated by the view of the modified simplest equation method (MSEM). By using MSEM, the bisoliton solutions of Boussinesq-type equations according to parameters is obtained. As a result, it is thought that the results obtained will lead to new developments in the improvement and application of the model since the application area of the model under consideration is wide.

**Keywords:** Boussinesq-type equations, analytical solutions, symbolic computation

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<sup>✉</sup> Corresponding Author Email: [zpinar@nku.edu.tr](mailto:zpinar@nku.edu.tr)

## Cubic Cell Formation Problem Considering Identical Parallel Machines

Gulcin Bektur

*Iskenderun Technical University, Department of Industrial Engineering, Hatay, Turkey*

### Abstract

Cellular manufacturing is an important production method in today's competitive conditions. There are many problems that need to be solved for success of cellular manufacturing. One of the most important of these problems is the cell formation problem [1]. In the cell formation problem, parts and machines are allocated to cells [2]. Employees are also taken into account in this study, so there are three dimensions as employees, machines and parts. Machine groups have also been taken into account in the study, and there are identical machines in each machine group. The presence of identical machines is an important advantage in minimizing inter-cell operations [3], Although identical machines are not taken into account in many studies, identical machines for cell formation problem is frequently encountered in practice As a result of considering identical machines, parts can be processed on different machines from the same machine group. Demand and capacity constraints are also taken into account. In the study, a mathematical model is proposed for the solution of the problem. The obtained results are analyzed over test problems with different sizes.

**Keywords:** Cubic cell formation, Identical machines, Mathematical model

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Corresponding Author Email: [gulcin.bektur@iste.edu.tr](mailto:gulcin.bektur@iste.edu.tr)

## Errors of the Smoothing Techniques

Ovgu Cidar İyikal<sup>✉</sup>

*Eastern Mediterranean University, Department of Mathematics, Famagusta, North Cyprus, via Mersin 10,  
Turkey*

### Abstract

In this study, in place of traditional error measuring tools such as Mean Square Error(MSE), Mean Absolute Error (MAE) and Root Mean Square Deviation (RMSD) a more efficient way of measuring the error is proposed. This is based on calculating the area between the estimated smoothed function and trajectory represented by raw data. This area named as error area (EA). Minimization of the EA in turn will depend heavily on the density of data, magnitude of fluctuations in the data values, frequency of the missing data, and the number of data values that falls into the lag interval. Two methods of estimation, the stretched interpolated moving average (SIMA) method and the Kernel smoothing method are used to estimate the data. Those are applied on two data sets, the 20x30 shares and 30x30 elevation-coordinates and EAs are calculated and compared with the raw ones. Thus, good performance of the SIMA can be seen by this EA method.

**Keywords:** Stretched moving average, Kernel smoothing, trapezoidal error area.

### Acknowledgements

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<sup>✉</sup> Corresponding Author Email: [ovgu.cidar@emu.edu.tr](mailto:ovgu.cidar@emu.edu.tr)

## An Effective Numerical Approach for RLW Equation

Melike Karta<sup>✉</sup>

*Ağrı İbrahim Çeçen University, Mathematics Department, Ağrı, Turkey*

### Abstract

The purpose of this article is to obtain the numerical solution of the RLW equation, one of the most important nonlinear wave equations used to model physical phenomena such as shallow water waves and plasma waves using a splitting technique combined with the B-spline collocation method by the help of the fourth order Runge–Kutta (RK-4) method. For this purpose, After the proposed problem is transformed into a partial differential equation in the form of two equations in time, these equations are reduced to the ordinary differential equations system (ODEs) using finite element collocation method. After all these procedures, a splitting technique via the fourth order Runge–Kutta (RK-4) method is utilized to solve the ODEs obtained with these equations. Three test problem are suggested to check the accuracy of the present scheme. The error norms  $L_2$  and  $L_\infty$  are calculated to compare the results found in the literature with our results. Also, The conservation properties of the RLW equation are indicated to be preserved as a result of the numerical algorithm. As a result, it can be clearly seen that the present method gives compatible results with previous work.

**Keywords:** RLW equation, B-spline, Collocation method, Splitting technique

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<sup>✉</sup> Corresponding Author Email: [mkarta@agri.edu.tr](mailto:mkarta@agri.edu.tr)

## A Goal Programming Approach for Resource Dependent Assembly Line Balancing Problem

Yakup Atasagun<sup>✉</sup>, Alper Döyen

<sup>1</sup> Konya Technical University, Industrial Engineering Department, Konya, Türkiye

### Abstract

Assembly Line Balancing (ALB) is an important decision problem in mass production systems. One of the main assumptions in most of the ALB studies is that processing times of the tasks are fixed. However, this situation could not always be practical especially when different resource alternatives such as particular equipment or an assistant worker are available to process a task with different durations. Some tasks in practice cannot be processed by only one worker and may necessarily need to have additional assistant worker or particular equipment. Alternatively, assistance of another worker can reduce the processing time of a task even though he/she is not necessary. In this case, different processing alternatives (resource combinations) have to be used in the line [1].

Faaland et al. [2] have defined this problem as resource dependent assembly line balancing problem (RDALB).

Kara et al. [1] have addressed the problem from a wide point of view and have adapted the RDALB approach to U-shaped assembly lines (RDULB) with some new practice-oriented assumptions.

On the other hand, the concept of Goal Programming (GP) was introduced by Charnes and Cooper [3] and has been widely used as an important modelling technique for multi-criteria decision making problems. There are two essential GP approaches in the literature [4]: (i) weighted GP; and (ii) pre-emptive GP.

In this study, a pre-emptive GP model for RDALB is proposed in order to provide flexibility for decision makers based on their decision environment and preferred priorities. The proposed model is structured on the mathematical formulation of Kara et al. [1]. Three conflicting goals namely total cost of workstation utilization (total number of utilized workstations), cycle time and total cost of additional resources such as equipment and assistant workers are considered. The proposed model is validated on an illustrative example and a scenario analysis is performed with different priority levels of the goals. The results show that the proposed goal programming formulation is valid and useful for balancing resource dependent assembly lines.

**Keywords:** Goal programming, line balancing, resource dependent assembly lines

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✉ Corresponding Author Email: [yatasagun@ktun.edu.tr](mailto:yatasagun@ktun.edu.tr)

## Organizational Configurations Boosting Enterprise Performance and Job Satisfaction\*

Alperen Ekrem Çelikdin✉

*Planning & Logistics Manager, Sıtaş Inc., Aksaray, Turkey*

### Abstract

It is known that organizations consist of a combination of several causal factors. Organizational Configurations theory had started with the contingency theory. Contingency theory is known as, an organization cannot be configured with the best single structure and Organizational Configurations theory has developed by the equifinality approach. Equifinality is defined as, “a system can reach the same final situation from different and original starting points”. It has been stated that businesses with similar practices and strategies can be seen as clustered groups and these decisive organizational characteristics and strategies shape the performance of the enterprise. Therefore, the importance of Organizational Configurations approach is increasing. However, since the sustainable performance of the enterprises can be supported with the efforts of the satisfied employees, the Organizational Configurations that are desired to be formed should be shaped accordingly. With this study, the automotive spare parts sector of Central Anatolia Region was selected as the main sample and high performing Organizational Configurations was determined based on the Miles and Snow typologies theory. The relationship between the targeted output and the set of independent variables that make up this output was determined by Fuzzy Set Qualitative Comparative Analysis, which aims explaining with the set theoretical approach. While providing high enterprise performance, the arrangements to be made in order to have high job satisfaction at the same time according to Decision Makers choices, are modeled by the help of Compromise Programming which is one of the Operations Research methods. The results of the study support the Miles and Snow typologies. Finally the results show that, enterprises that manage their entrepreneurial, engineering and administrative problems with solutions suitable for environment and competition conditions, authorizing the employees and encouraging them to participate in the management processes, reach high performance and job satisfaction results.

**Keywords:** Organizational Configurations, Performance, Job Satisfaction, fsQCA, Compromise Programming, Multi Criteria Decision Making.

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✉ Corresponding Author Email: [a.celikdin@gmail.com](mailto:a.celikdin@gmail.com)

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## Combining Fuzzy Full Consistency Method and Fuzzy Axiomatic Design for Facility Layout Selection

Esra Duygu Durmaz<sup>1</sup>✉, İlker Gölcük<sup>2</sup>, Ramazan Şahin<sup>1</sup>

<sup>1</sup> Gazi University, Industrial Engineering Department, Ankara, Turkey

<sup>2</sup> İzmir Bakırçay University, Industrial Engineering Department, İzmir, Turkey

### Abstract

Facility layout evaluation is a critical and strategic decision-making problem that significantly impacts the efficiency of operations. Although various optimization algorithms have been used to tackle facility layout problems, real-world applications require consideration of both qualitative and quantitative attributes. In practice, several qualitative attributes should be considered, which are characterized by their contradictory and conflicting nature. Moreover, intangible attributes are hard to measure in precise terms, so that the decision makers' perceptions have a pivotal role in the outcome. Therefore, fuzzy multiple attribute decision-making (MADM) methods offer a wide variety of approaches to tackle imprecise and uncertain decision makers' evaluations. One obstacle of the fuzzy MADM methods is that the relative importance of attributes is generally obtained by using pairwise comparisons, which is time-consuming and cognitively demanding. The Full Consistency Method (FUCOM) method has recently been proposed to deal with pairwise comparisons with few evaluation vectors. The present work utilizes fuzzy FUCOM method to cope with uncertain and imprecise judgments of the decision-makers. Additionally, facility layout selection is a crucial design evaluation problem, so that axiomatic design is a suitable tool for evaluating design options. The axiomatic design approach sets out two design axioms: independence and information axiom. The independence axiom states that functional requirements must be independent. On the other hand, the information axiom states that design with the smallest information content is the best design among those designs that satisfy the independence axiom. In this study, a new integrated fuzzy MADM model is proposed by integrating fuzzy FUCOM and fuzzy axiomatic design methods. The proposed method has been implemented in a real-life study to demonstrate the effectiveness and applicability of the proposed model.

**Keywords:** Facility layout, fuzzy logic, full consistency method, axiomatic design

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✉ Corresponding Author Email: [esradurmaz@gazi.edu.tr](mailto:esradurmaz@gazi.edu.tr)

## Quantum Analog of Some Simpson and Bullen Type Inequalities for Convex Functions

Abdul Wakil Baidar<sup>1,2</sup>, Mehmet Kunt<sup>2✉</sup>

<sup>1</sup>Kabul University, Department of Mathematics, Kabul, Afghanistan

<sup>2</sup>Karadeniz Technical University, Department of Mathematics, Trabzon, Turkey

### Abstract

In this paper based on a new quantum analog of Hermite-Hadamard inequality given in [2], the quantum analog of some Simpson and Bullen type inequalities are established. The results obtained will generalize the results given in [6].

**Keywords:** Simpson inequality, Bullen inequality, quantum calculus

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✉Corresponding Author Email:[mkunt@ktu.edu.tr](mailto:mkunt@ktu.edu.tr)

## A Simulated Annealing Based Fix-and-Optimize Algorithm for the Assembly Line Worker Assignment and Balancing Problem

Anıl Akpunar<sup>✉</sup>, Şener Akpınar

Dokuz Eylül University, Industrial Engineering Department, İzmir, Turkey

### Abstract

This study handles the type-2 assembly line worker assignment and balancing problem (ALWABP-2). ALWABP-2 is about to allocate the assembly tasks among a predefined number of workstations and identify which operator will work at which workstation. The main goal of this problem is to increase the production rate by minimizing the cycle time. Due to the NP-hard nature of problem and the problem size limitation of exact solution procedures, approximation algorithms are required to be designed to solve the ALWABP-2. The difficulty to optimize the problem through the mathematical programming formulation mainly springs from the abundance of the binary variables in use. At this point, it is projected that the fix-and-optimize algorithm (FOA) as a decomposition approach has a potential to identify optimal or a near optimal solutions for the ALWABP-2. FOA decomposes the binary variables into two disjoint sets X and Y, and forms two subproblems. The first subproblem is about to fix the binary variables set X, while the Y and all other variables, if any, are optimized in an iterative fashion. In other words, a sub-problem is the reduced form of the original problem where a set of decision variables are fixed in value whereas the remaining set are left to be optimized. As such, fix-and-optimize heuristics often provide high quality solutions especially when the binary decision variables are vast. In this study, the operator assignments to workstations constitutes the first binary variable set X, while the allocation of the assembly tasks among the workstations constitutes the second binary variable set Y. This study presents a simulated annealing (SA) based FOA to tackle the ALWABP-2. The SA algorithm matches the operators and workstations firstly, and then the reduced problem optimizes the allocation of the assembly tasks among the workstations. The performance evaluation tests of the proposed SA based FOA are performed on a set of 320 benchmark instances. Computational results indicate that the proposed algorithm is a promised competitive algorithm for the ALWAP-2.

**Keywords:** ALWABP-2, decomposition, fix and optimize heuristic, simulated annealing

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<sup>✉</sup> Corresponding Author Email: [anilakpunar@gmail.com](mailto:anilakpunar@gmail.com)

## A Unique Hamilton–Jacobi–Bellman Equation Having Periodic Solutions and their Computation Using Higher-Order Finite Difference Schemes

Hidekazu Yoshioka✉

*Shimane University, Graduate School of Natural Science and Technology, Matsue, Japan*

### Abstract

Many environmental management problems involve optimization of seasonal, namely time-periodic dynamics under uncertainty. These problems can be effectively formulated from a viewpoint of the stochastic control under uncertainty [1]; however, such approaches are still rare possibly due to the difficulties in solving the optimality equation, called Hamilton–Jacobi–Bellman (HJB) equation. HJB equations are highly nonlinear in general, and finding their time-periodic solutions can be a more difficult task from both theoretical and computational standpoints. Problems considering the risk-aversion nature of the environmental manager lead to HJB equations that are more important in applications but having stronger nonlinearity. We approach these mathematical and computational issues on the modeling and control of environmental management problems based on a dynamic programming principle to control stochastic differential equations (SDEs) [2]. A particular emphasis is put on a seasonal management of aquatic vegetation in a shallow water body whose population dynamics follows a jump-driven SDE having a rational drift coefficient with time-periodic parameters. The HJB equation for a discounted infinite-horizon control problem of the vegetation based on an Erlangization [3] as an adaptive discrete observation/harvesting policy of a risk-averse manager is then heuristically derived. The HJB equation is subject to a periodic boundary condition and has an exponential nonlinearity. We demonstrate that finite difference schemes based on total variation diminishing Runge–Kutta and Weighted essentially non-oscillatory reconstruction methods works well and generate numerical solutions converging toward continuous manufactured solutions. Impacts of the drift of the vegetation dynamic and management cost are discussed numerically as well.

**Keywords:** Stochastic control, Erlangization, Higher-order finite difference schemes

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\* Corresponding Author Email: [yoshih@life.shimane-u.ac.jp](mailto:yoshih@life.shimane-u.ac.jp)

## Optimization versus metaheuristics in forecasting: A comparative study for energy demand forecast of Turkey

Alper Döyen<sup>✉</sup>, Yakup Atasagun

*Konya Technical University, Industrial Engineering Department, Konya, Turkey*

### Abstract

Energy is probably the most important economic factor for countries, particularly for those, that have high energy dependency rates. Those countries need to make accurate estimations of energy demand in order to lower their energy import costs. To estimate the energy demand of Turkey, different methodologies are proposed in the literature, where statistical approaches [1,2], artificial neural network [3], grey prediction [4] and metaheuristics [5,6] are the most commonly used forecasting tools. Regression models constitute an important part of the literature for the statistical approaches. If there are more than one independent variable that affect the value of a dependent variable, the regression model is called as multiple regression model. Since the amount of energy demand depends on numerous predictor variables, fitting a multiple linear regression model is a good choice as a forecasting method. The prediction quality of a regression model is directly related to the parameters of predictor variables. Metaheuristic algorithms [5,6] are used widely to determine the parameters of regression models for energy demand estimation. However, one can find regression parameters in a cleverer way by using optimization instead of searching by metaheuristics. For that purpose, in this study we use Goal Programming (GP), which is a well-known multi-objective mathematical programming technique. The performance of goal programming is compared to the performance of some metaheuristic algorithms proposed in the literature for the energy demand prediction of Turkey. It is observed that, goal programming outperforms metaheuristics in different performance measures and provides comparable results to the well-known least squares method.

**Keywords:** Energy demand forecasting, goal programming, multiple linear regression

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<sup>✉</sup> Corresponding Author Email: [alperdoyen@ktun.edu.tr](mailto:alperdoyen@ktun.edu.tr)

## A Novel Finite Volume Scheme for the Numerical Investigation of Bacterial Communication Model

Burcu Gürbüz<sup>✉</sup>

*Johannes Gutenberg-University Mainz, Institute of Mathematics, Mainz, Germany*

### Abstract

Bacteria populations are considered to be grown by dividing themselves which is explained by diffusion process. Separately, these organisms have communication skills for controlling and coordination of their behavior with regard to biochemical reactions. Thus, this phenomena is modeled by the partial differential equations, in particular, reaction-diffusion equations. On the other hand, solutions of such equations together with initial and boundary conditions are difficult to obtain.

In this study, a novel finite volume method approach for the solutions of reaction-diffusion equations with Diriclet boundary conditions is investigated. The robust numerical scheme is introduced based on the divergence theorem and the finite difference method is evaluated with regard to Laguerre series expansion. Additionally, a discretization process is explained and a local balance is written on each discretization as a control volume. Besides, the structured meshes on geometries are used and the numerical flux is conserved from one discretization cell to its neighbor which gives a convenience for the solution since the flux is of importance particularly in our model. The method is modified based on the series approximation and efficiency of the numerical scheme is investigated by using error analysis and performed on the model examples. Some simulations are obtained and the results are discussed.

**Keywords:** Reaction-diffusion equations, bacterial communication, finite volume method, divergence theorem, numerical simulation.

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✉ Corresponding Author Email: [burcu.gurbuz@uni-mainz.de](mailto:burcu.gurbuz@uni-mainz.de)

## A Numerical Approach on Fitzhugh-Nagumo Model

Burcu Gürbüz<sup>1✉</sup>, Aytül Gökçe<sup>2</sup>, Mahmut Modanlı<sup>3</sup>

<sup>1</sup> *Johannes Gutenberg- University Mainz, Institute of Mathematics, Mainz, Germany*

<sup>2</sup> *Ordu University, Department of Mathematics, Ordu, Turkey*

<sup>3</sup> *Harran University, Department of Mathematics, Şanlıurfa, Turkey*

### Abstract

In this study, we work on a numerical approach for a coupled FitzHugh-Nagumo model. First, we introduce the model which is the base of tunnel-diode nerve model. The nonlinear system of ordinary differential equations with the parameters are given. Then a numerical technique based on Taylor series and matrix representations are described together with the collocation points. Then we apply the method to obtain a system of nonlinear algebraic equations. Besides we apply a finite difference scheme together with its stability and compare the results to show important relations in dynamics. On the other hand, stability analysis of the dynamic system in positive equilibrium condition is explained. In addition, an error analysis, some simulations with numerical solutions and the results of the qualitative analysis of the system are given. The behavior of the model is explained.

**Keywords:** FitzHugh-Nagumo model, Taylor series, finite difference scheme, stability analysis, numerical simulation.

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✉ Corresponding Author Email: [burcu.gurbuz@uni-mainz.de](mailto:burcu.gurbuz@uni-mainz.de)

## An Application of Multi-Objective Scheduling with Fuzzy Measure

Esra Duygu Durmaz<sup>✉</sup>, Sena Aydoğan, Ramazan Şahin

*Gazi University, Industrial Engineering Department, Ankara, Turkey*

### Abstract

In this paper, we study multi-objective scheduling problem with consideration of interactions between the objectives. In the area of production management, the single-machine scheduling (SMS) problem is a major decision issue having more than one objective that needs to be considered simultaneously. Because compliance with deadlines and avoidance of delay penalties have become a necessity to survive, minimum total weighted tardiness is seen to be one of the most common objective functions in SMS literature. Besides, additional objectives need to be addressed to have competitive advantage. Therefore, measures of maximum tardiness and maximum earliness have also been taken into account. Maximum tardiness is associated to meet customer due dates, while maximum earliness is associated with the inventory cost of finished goods. The weighted arithmetic mean operator is typically used to aggregate the objective functions when developing a multi-objective optimization algorithm. However, if there is a positive relationship between some of the objectives, the related ones may prevent the achievement of good solutions by dominating others. The use of a different operator will eliminate this problem in the aggregation process. In this study, a tri-criteria SMS problem with the objectives of minimizing total weighted tardiness, maximum tardiness and maximum earliness is discussed. SMS has been proved to be an NP-hard problem in terms of computational complexity. Since exact methods are insufficient to solve the large-scale problems, a genetic algorithm is proposed to ensure near-optimal solutions. To aggregate the criteria by considering interactions, the Choquet integral operator is implemented as a tool to each individual solution's fitness function. Performance of the proposed algorithm is evaluated on small, medium and large sized test problems. Through this study, fuzzy measure is indicated as an effective tool to aggregate the criteria, especially when they are correlated.

**Keywords:** Single-machine scheduling problem, fuzzy measure, multi-objective optimization, genetic algorithm

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<sup>✉</sup> Corresponding Author Email: [esradurmaz@gazi.edu.tr](mailto:esradurmaz@gazi.edu.tr)



## Pairwise comparison scale's with Analytic Hierarchy Process: Gsm operator preference of university students

Esma Canhasi Kasemi<sup>1✉</sup>, Luan Vardari<sup>2</sup>

<sup>1</sup> Prizren University, Education Department, Prizren, Kosovo

<sup>2</sup> Universum College, Business and Management Department, Prishtina, Kosovo

### Abstract

The Analytical Hierarchy Principle (AHP) [1], which has elegant mathematical properties, is very useful in complex decisions where the data used as input can often be easily provided, and especially in complex decisions that require comparison of decision elements that are difficult to measure. In this study, the explanation of the theory of AHP, which has a multi-purpose and multi-criteria decision making technique, and the traditional judgment scale used in binary comparison, which is the stage of the method, namely the balanced scale [2] are introduced. The authors discuss the use of these two judgment scales in a real problem and their impact on priority estimation in AHP. The aim of the study is to compare the results using two different judgment scales included in the AHP method of the GSM operator preference of students in Kosovo. In practice, the survey was conducted in three languages with University of Prizren students, which are existing in Kosovo with VALA and IPKO Gsm operators. The questionnaire was applied to students studying in 3 languages (Bosnian, Turkish and Albanian) at the Faculty of Education at the University of Prizren. According to the data obtained from the surveys, the importance weights of the decision criteria were calculated separately for each scale and compared, and thus, the preference ranking of the GSM operators was made according to each decision criterion and all criteria.

**Keywords:** Analytic Hierarchy Process (AHP), judgment scales, GSM Operator, Mobile Phone, Students

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✉ Corresponding Author Email: [esmacanhas@hotmail.com](mailto:esmacanhas@hotmail.com)

## Financial Efficiency of Companies Operating in the Kosovo Food Sector: DEA and DEAHP

Esma Canhasi Kasemi<sup>1✉</sup>, Necati Özdemir<sup>2</sup>

<sup>1</sup> Prizren University, Education Department, Prizren, Kosovo

<sup>2</sup> Balikesir University, Department of Mathematics, Faculty of Science and Arts, Balikesir, Turkey

### Abstract

In this study companies operating in the food sector in Kosovo were evaluated with the weight-restricted DEA (Data Envelopment Analysis)[1] model created using the unweighted DEA model and then the AHP (the Analytical Hierarchy Process)[2] model, and the two models were compared. DEA is a nonparametric method that is used frequently in operations research and management sciences. DEA evaluates a large number of input and output variables using mathematical programming techniques and analyzes the effectiveness of similar Decision Making Units (DMU). Unlike traditional methods, the most important advantage of DEA is that the weights of input and output variables can be determined by the analyzer. In this study, the limitations of the weights of DMUs in DEA were determined by using the AHP, which takes into account the expert opinion. In addition, an alternative scale was used for the Saaty scale, which is used as a standard in AHP method, and thus a more sensitive analysis was performed. As a result of the first model that was run, 6 companies were found effective, and only 3 of them were found effective as a result of the second model, and the remaining unethical companies were determined and made necessary suggestions to improve the efficiency of the companies.

**Keywords:** Analytic Hierarchy Process (AHP), Data Envelope Analysis (DEA), AHP-DEA, Decision Making Criteria, Efficiency Measurement, Scale.

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✉ Corresponding Author Email: [esmacanhas@hotmail.com](mailto:esmacanhas@hotmail.com)

## A Caputo Fractional Order Model for Tumor-Immune System-Host Cells Interaction: A Lung Cancer Application

Fatma Özköse<sup>1✉</sup>, İlhan Öztürk<sup>2</sup>, Seçil Yılmaz<sup>3</sup>, Tamer Şenel<sup>4</sup>, Mehmet Yavuz<sup>5</sup>, Burcu Şen Bağcı<sup>6</sup>,  
Medine Doğan<sup>7</sup>

<sup>1</sup> Erciyes University, Departments of Mathematics, Faculty of Science, 38039, Kayseri, Turkey

<sup>2</sup> Erciyes University, Genome and Stem Cell Center (GENKOK), 38039, Kayseri, Turkey

<sup>3</sup> Necmettin Erbakan University, Department of Mathematics and Computer Sciences, Faculty of Science, 42090 Konya, Turkey

### Abstract

In this manuscript, a fractional-order model of tumor-immune system-host cells interaction has been considered. In modelling dynamics, the total population of the model is divided into four subpopulations: macrophages, activated macrophages, tumor cells and host cells. The Caputo fractional order derivative [1] is used in the modelling of tumor-immune system. In this study, the effects of fractional derivative on the stability and dynamical behaviors of the solutions are investigated by using the Caputo fractional operator that provides convenience for initial conditions of the differential equations. The existence and uniqueness of the solutions for the fractional derivative is examined and numerical simulations are presented to verify the analytical results. In addition, our model is used to describe the kinetics of growth and regression of the tumor cells in the lung tissues of five lung cancer patients in Erciyes University hospitals. Numerical simulations are given for different choices of fractional order and the obtained results are compared with the experimental data. One can conclude that fractional model best fit experimental data better than the integer order model [2,3,4,5].

**Keywords:** Tumor-macrophage and host cells, existence and uniqueness, equilibrium points, real data, fractional calculus

### Acknowledgements

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✉ Corresponding Author Email: [fpoker@erciyes.edu.tr](mailto:fpoker@erciyes.edu.tr)

## Design of Clamps for Use in Flexible Pipes and Development of Production System

Engin Demir

*Tork Valve & Automation, Istanbul, Turkey*

### Abstract

Today, it has become a necessity to make continuous improvements in machines and factors affecting production, and to keep production integrity up-to-date and efficient. The whole of the improvements made to meet the requirements is called innovation. In this context, the production integrity of the clamp systems used in flexible pipes is starting to lose its effectiveness with the developing technologies. In this study, the important parts of an improved clamping machine, the current data obtained and its advantages compared to the previous version are included.

Examining the subject is important in terms of the results to be obtained and the evaluation of the gains in clamp production. Innovation covers all the processes carried out to develop a new or improved product, service or production method and to make it commercially profitable. Developing a new or improved product, service or production method arises from new ideas [1]. R&D has a vital importance in revealing technological innovation [2]. However, the role of R&D differs from various perspectives. For some, R&D is the development of new products or processes, while others see R&D as merely conducting scientific studies [3]. By a generally accepted definition, R&D is any creative systematic activity undertaken to increase knowledge, including knowledge of people, culture and society, and the use of this knowledge to develop new applications [4]. R&D involves researching initially, then discovering what has not yet been discovered, and using this knowledge to develop or improve a product [5-6].

**Keywords:** Clamp, flexible conduits, machine design

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✉ Corresponding Author Email: [demir201@hotmail.com](mailto:demir201@hotmail.com)

## Suggestion of Standard and Optimized Stages in LOC (Lab On A Chip), LOD (Lab On A Disc), POC (Point Of Care) Development Process for Biomedical Applications

Gülşen Yaman ✉

*Balıkesir University, Balıkesir, Turkey*

### Abstract

Molecular imaging, sensing and diagnostic systems have recently produced new microanalysis technologies (LOC - Lab on a Chip, LOD - Lab on a Disk) such as biosensors for point of care (POC - Point of Care) testing systems and their benefits in medical applications has been recognized as a promising area of research for further development. Biosensors have evolved over the last decade towards paradigms of fast and real-time sensing, ease of use, and low cost. However, it has difficulties in meeting sensitivity requirements and detecting low concentration for early-stage diagnosis while maintaining efficiency and speed. Typically, a simple point-of-care (POC) diagnosis can test for blood, saliva, or other biological material. They must be able to directly measure a parameter in a sample with little intervention from other fluid components or the need for a series of preparatory procedures prior to analysis. In this study, the whole process is evaluated by examining optimization, Computational Fluid Dynamics (CFD) and simulation stages to design and build microfluidic-based chips by increasing the detection probability of biological materials for diagnostic systems (LOC). Here we provide a glimpse into the stages of a standard and successful manufacturing process, taking into account laboratory bio-analysis on the design and fabrication of a new chip for rapid and early detection of viruses or bacteria used in cellular materials.

**Keywords:** Biosensors, molecular diagnostics, CFD, LOC, LOaD, POC

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✉ Corresponding Author Email: [yamangulsen@gmail.com](mailto:yamangulsen@gmail.com)

## Heat Waves Due To Cattaneo-Hristov Heat Diffusion Occurring on the Half-Real Axis

Derya Avci<sup>1✉</sup>, Beyza Billur İskender Erođlu<sup>1</sup>, Jordan Hristov<sup>2</sup>

<sup>1</sup>Balıkesir University, Department of Mathematics, Balıkesir, Turkey

<sup>2</sup>University of Chemical Technology and Metallurgy, Department of Chemical Engineering, Sofia, Bulgaria

### Abstract

This study aims to investigate the heat waves observed in the Cattaneo-Hristov heat diffusion model [1] under different harmonic effects. In this model, the fading regular memory effect is described with Caputo-Fabrizio fractional derivative [2]. We assume that this phenomenon is occurring on the semi-infinite real axis. Wave-like temperature curves are obtained analytically. For this purpose, the Laplace and the sine-Fourier integral transforms are used [3]. The effects of problem parameters such as angular frequency, the velocity of the moving harmonic heat source, and fractional order on heat waves [4] are simulated with Matlab software.

**Keywords:** Cattaneo-Hristov heat diffusion, Caputo-Fabrizio fractional derivative, heat waves, analytical solution

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✉ Corresponding Author Email: [dkaradeniz@balikesir.edu.tr](mailto:dkaradeniz@balikesir.edu.tr)

## Optimal Control of a Fractional Computer Virus Propagation Model

Derya Avci , Fatma Soytürk

*Balıkesir University, Department of Mathematics, Balıkesir, Turkey*


### Abstract

In this study, we aim to examine the optimal control of computer viruses spreading in a network [1]. The main goal is to reduce both the number of infected computers and the cost of installed programs. For this purpose, an antivirus program or firewall is adapted to the model as a control function [2]. The system discussed is the Caputo fractional derivative SEIR model [3]. The Euler-Lagrange equations describing the optimality conditions are calculated using the Hamiltonian formalism [4,5]. To solve the optimal system numerically, we apply forward-backward sweep method coupled with the Adams-type predictor-corrector method [6]. Numerical results are simulated for the variation of problem parameters using MATLAB software.

**Keywords:** Computer virus, SEIR model, fractional optimal control, Caputo fractional derivative, Adams-type predictor-corrector method, forward-backward sweep method

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 Corresponding Author Email: [dkaradeniz@balikesir.edu.tr](mailto:dkaradeniz@balikesir.edu.tr)

## Fractional Optimal Control Problem For A Delayed Computer Virus Propagation

Beyza Billur İskender Erođlu <sup>✉</sup>, Dilara Yapıřkan

*Balıkesir University, Mathematics Department, Balıkesir, Turkey*

### Abstract

In this study, the fractional optimal control problem (FOCP) is proposed to control the spread of computer viruses generated with a delayed SIR model. For this purpose, the optimal control problem, in which a trade-off is made between the control cost and the control effect, previously presented with the integer derivative in [1], is reconsidered with the Caputo fractional derivative. Using Hamiltonian formulation, the fractional optimality conditions are derived by Euler-Lagrange equations. Then, the optimal control function is achieved with the help of the Pontryagin Maximum Principle. The numerical solutions of the optimality conditions are obtained by using the Adams-type predictor-corrector algorithm for state equations, and the Forward-Backward Sweep method combining with Adams-type predictor-corrector algorithm for costate equations, [2,3]. Finally, the solutions drawn by MATLAB program illustrate that the FOCP minimizes the number of the infected computers.

**Keywords:** Fractional optimal control, delayed computer virus model, Caputo fractional derivative, Adams-type predictor-corrector algorithm, forward-backward sweep method.

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<sup>✉</sup> Corresponding Author Email: [biskender@balikesir.edu.tr](mailto:biskender@balikesir.edu.tr)



## Could Stem Cells' Behaviors be Modeled as an Optimization Algorithm?

Ramazan Yaman✉

*Istanbul Atlas University, Istanbul, Turkey*

### Abstract

Many optimization approaches mimic natural behaviors. These approaches are used for optimizing different engineering and decision problems. The dilemma has occurred from the understanding of natural behaviors to optimize the problems or conventional optimization approaches used for solving natural behaviors. The first is called biomimetic or biomimicry.

This study is on examination for an understanding of the stem cells' mechanism for optimizing their performances. It could also be used for stem cell modeling to optimize systems and decision problems. This study could be assessed as an early point of view of the stem cell algorithm approach for optimization.

**Keywords:** Biomimicry, stem cell mechanisms, stem cell optimization algorithm

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✉ Corresponding Author Email: [ramazan.yaman@gmail.com](mailto:ramazan.yaman@gmail.com)

## Exact Solutions of Lienard II-type oscillator equation by group classification

Zehra Pınar<sup>1</sup>✉, Özlem Orhan<sup>2</sup>

<sup>1</sup> Tekirdağ Namık Kemal University, Faculty of Arts and Science, Department of Mathematics, 59030 Merkez-Tekirdağ, Turkey

<sup>2</sup> Bandırma Onyedi Eylül University, Faculty of Engineering and Natural Sciences, Department of Basic Engineering Sciences, Bandırma, Balıkesir, Turkey

### Abstract

In this study, we show that the Lienard II-type oscillator equation is linearizable for a special parametric choice. We obtain an exact solution for these special choices. Therefore, we identify a method to yield symmetries. This new generalized method is used and they obtain integrals from a single integral. The first integral  $I(t, x, \dot{x})$  of the Lienard II-type oscillator equation is found and obtain an analytical solution by these first integral. The time-dependent first integral can be derived using symmetries and the exact solutions can be found by these time-dependent first integrals. The procedure will be explained then this procedure will be applied to the Lienard II-type oscillator equation. Moreover, this equation can be classified according to arbitrary coefficients.

**Keywords:** First integral, symmetries, Lienard II-type oscillator equation.

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✉ Corresponding Author Email: [zpınar@nku.edu.tr](mailto:zpınar@nku.edu.tr)

## Finite Element Method with Crank-Nicolson Scheme for the Nonlinear Klein-Gordon Equation in de Sitter Spacetime

Harun Selvitopi<sup>1✉</sup>, Muhammet Yazıcı<sup>2</sup>

<sup>1</sup> Erzurum Technical University, Department of Mathematic, Erzurum, Turkey

<sup>2</sup> Karadeniz Technical University, Department of Mathematic, Trabzon, Turkey

### Abstract

It is already known that the Klein-Gordon equation is an important physical phenomena so that it arises in quantum field theory and cosmology. In this article, we consider the initial value problem for the nonlinear Klein-Gordon equation in de Sitter spacetime,

$$u_{tt} + Hu_t - e^{-2Ht} \Delta u + m^2 u = |u|^{p-1} u, \quad (x, t) \in \mathbb{R} \times [0, \infty), \quad (1)$$

$$u(x, t) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \mathbb{R},$$

where  $m > 0$  is the physical mass and  $p > 1$ . Here,  $H$  denotes the Hubble constant. In Minkowski spacetime, the existence of global weak solutions of the initial value problem for the Klein-Gordon equation

$$u_{tt} - \Delta u + m^2 u = |u|^\alpha u, \quad (2)$$

has been extensively studied (see eg., [1]).

The existence of small global solutions of the initial value problem for the following Klein-Gordon equation with the power type nonlinear term

$$u_{tt} + dHu_t - e^{-2Ht} \Delta u + m^2 u = |u|^\alpha u, \quad (x, t) \in \mathbb{R}^d \times [0, \infty), \quad (3)$$

is shown in [2] in the Sobolev space  $H^s(\mathbb{R}^d)$  for  $s > d/2$  when  $m \in (0, \sqrt{d^2 - 1}/2) \cup [d/2, \infty)$ .

Numerical solutions of the nonlinear Klein-Gordon equation in de Sitter spacetime is considered. The mathematical model of the physical problem is a hyperbolic type equation. Therefore in the solution procedure, the temporal variable is discretized by finite difference method and the spatial variable is discretized by implicit Galerkin finite element method. The numerical error analysis is shown with different values of time step. Finite difference method is also used for discretization of spatial variable to confirm the reliability of the numerical results. Several simulations are performed to show the behavior of the numerical solutions for the equation in de Sitter spacetime. The finite element method with the finite difference scheme is used to obtain the numerical solution of the linear form of the considered initial value problem in [3].

**Keywords:** De sitter spacetime, Klein-Gordon equation, Finite element method, Finite difference method

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✉ Corresponding Author Email: [harun.selvitopi@erzurum.edu.tr](mailto:harun.selvitopi@erzurum.edu.tr)

## Reliable Fast Algorithm of Taylor Wavelet Method for Some Fractional Delay Differential Equations

Gökçe Özaltun<sup>1</sup>, Ali Konuralp<sup>1</sup>✉, Sevin Gümgüm<sup>2</sup>

<sup>1</sup> Manisa Celal Bayar University, Department of Mathematics, Manisa, Turkey

<sup>2</sup> İzmir University of Economics, Department of Mathematics, İzmir, Turkey

### Abstract

This paper presents a reliable fast algorithm of Taylor wavelet method to solve some linear and nonlinear fractional differential equations. The fractional derivative is considered in the Caputo sense. This method transforms the differential equation into a system of algebraic equations. Then the approximate solution is obtained from approximated function whose coefficients are calculated from the system of equations. Several types of differential equations are solved by using that algorithm and their numerical data are obtained. It is observed that the proposed algorithm is applicable getting either the exact or the approximate solution to these types of differential equations.

**Keywords:** Taylor Wavelet, fractional differential equation, Riemann-Liouville fractional integral, function approximation

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✉ Corresponding Author Email: [ali.konuralp@cbu.edu.tr](mailto:ali.konuralp@cbu.edu.tr)

## A New Coding/Decoding Algorithm Based on $k$ -Fibonacci Numbers

Öznur Öztunç Kaymak

Balıkesir University, Balıkesir, Turkey

### Abstract

As the mass media has become widespread day by day, it has also brought the problem of information security along with its many advantages. It is well known that the insufficient provision of information security conditions may cause unauthorized alteration, disclosure, recording and damage of data. The technology developed for preventing these problems is called cryptology. "Provable security" is the foundation of cryptology and this security is based on mathematical problems. Therefore, before developing cryptological hardware or software (high speed, low memory track, etc.), an applicable mathematical theory is required. In the coding theory, which is one of these theoretical fields of study, methods are created for the transfer of information from one place to another. These methods focus solely on the encryption and decryption process. Moreover, it is considered that the environment in which the data is exchanged is noisy. For this reason, the main purpose of the coding theory is to provide fast encryption and decryption of information. Apart from that, the relations are also developed in order to correct the errors that are likely to occur in the channel. As a result, the developed model can be said to be successful, if it makes fast encryption and has high error verification capability. Although there are many studies in this field, one of the best known is the Fibonacci method. This method is based on Fibonacci  $p$ -numbers and  $Q_p$  is a square  $(p + 1) \times (p + 1)$  matrix and the Cassini formula well known in the literature in [1]. The  $k$ -Fibonacci sequence, which is the more general form of Fibonacci numbers, is defined by

$$F_{k,n} = k F_{k,n-1} + F_{k,n-2}$$

with initial conditions  $F_{k,0} = 0, F_{k,1} = 1$  for any positive integer  $k$ . These numbers are found by the recursive application of two geometrical transformations named as 4-triangle longest-edge (4TLE) partition in [2]. Many features of the  $k$ -Fibonacci numbers, which are known as the extension of Fibonacci, Pell numbers etc., have been found in related studies in [3]-[8]. In this paper we present a new method of coding/decoding algorithms using Fibonacci  $Q_k^n$ -matrices. Asymmetric encryption method, which uses a mathematically related pair of keys for encryption and decryption, constitutes the basis of our model. In doing so, this algorithm contributes to authentication problem in cryptology.

**Keywords:** Coding Theory,  $k$ -Fibonacci Numbers, Cryptology

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✉ Corresponding Author Email: [oztunc@balikesir.edu.tr](mailto:oztunc@balikesir.edu.tr)

## Analysis of Artificial Intelligence in the Web of Science Database via Topic Modeling

Özlem Aydın ✉, Özlem Tülek, Nimet Aksoy, Öznur Öztunç Kaymak

Balıkesir University, Turkey

### Abstract

As the mass media increased, the prevalence of information increased more and more. As this information presented through different platforms (phones, tablets, computers, etc.) has grown, the analysis, classification and comparison of this information has also become important. Topic modeling, a machine learning method, is used to reveal hidden structures within a single document. In this method, the words contained in the documents are automatically grouped into clusters to characterize similar expressions. [1] Since the proposed algorithms for this model are statistical methods, it is aimed to reach a conclusion by analyzing the words that make up the document. [2] The Latent Dirichlet Allocation(LDA), which is used as a topic modeling method, was first developed by Blei et al. [3] In this method, without the need for any prior knowledge, the distribution of words in subjects and subjects in documents is obtained from the Dirichlet distribution. [4] Although the LDA is a very popular method, its tendency to learn excessively and its inability to generalize on newly seen documents are among the disadvantages of the model. [5] The Non-Negative Matrix Factorization (NMF) method [6], which gives better results compared to the LDA method, does not contain negative values in topic modeling and a vector space is created for the representation of the data. [7]

In this study, it is aimed to determine which studies have been carried out in the field of ‘artificial intelligence’, especially in Turkey, published in Web of Science (WOS) with the Non-Negative Matrix Factorization (NMF) method used in topic modeling and to reach results related to interdisciplinary studies in this context. In the last 15 years, 1284 abstracts extracted from WOS have been analyzed. Word clouds and LDAvis graph [8] are used to visualize the results. In addition, it is aimed to answer three basic questions about topic modeling. These questions are as follows:

1. What is the meaning of each topic?
2. How common is each topic?
3. How are the subjects related to each other?

**Keywords:** Artificial Intelligence, Topic Modeling, Non-Negative Matrix Factorization (NMF)

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✉ Corresponding Author Email: [ozlem.aydin@balikesir.edu.tr](mailto:ozlem.aydin@balikesir.edu.tr)

## The center problem for some biochemical systems

Ilknur Kusbeyzi Aybar<sup>1</sup>✉, Brigita Ferčec<sup>2,3</sup>, Orhan Ozgur Aybar<sup>4</sup>, Masa Dukaric<sup>3</sup>

<sup>1</sup> *Yeditepe University, Department of Mathematics and Science Education, Istanbul, Turkey*

<sup>2</sup> *University of Maribor, Faculty of Energy Technology, Krško, Slovenia*

<sup>3</sup> *University of Maribor, Center for Applied Mathematics and Theoretical Physics, Maribor, Slovenia*

<sup>4</sup> *Piri Reis University, Department of Management Information Systems, Istanbul, Turkey*

### Abstract

The center-focus problem of polynomial differential equations known as the Poincaré center problem is one of the crucial problems in the qualitative theory of dynamical systems. The limit cycles, which are isolated periodic solutions that bifurcate from a center or a focus when the parameters are perturbed by an arbitrarily small amount in particular families of polynomial systems of ordinary differential equations representing biochemical reactions, can be investigated by studying the center problem. The studies of the dynamical systems arising in biochemistry generally involve searching for positive steady states or periodic positive trajectories for only some given values of the coefficients. The center-focus problem is related to the theory of the algebraic invariants of differential equations and the commutative algebra, where there are some new results in the dynamical systems arising in biochemistry. In this work, we develop efficient computational approaches for studying bifurcations of limit cycles and apply them to the investigation of the center problem for some biochemical systems. The methods of computational algebra based on Groebner's base theory are developed with the help of the SINGULAR system to make the complex calculations of the polynomial ideals. We find necessary and sufficient conditions for the existence of a center by using methods of computational algebra. We investigate the center problem with an efficient computational algorithm for computing the focus quantities, which are the polynomials that define the center variety. We study Darboux integrability for proving the existence of a center in a biochemical system applying algorithms of computational commutative algebra. Finally, we perform numerical simulations to visualize the limit cycles of biochemical oscillatory behavior.

**Keywords:** Center problem, biochemical systems, limit cycle

### Acknowledgements

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✉ Corresponding Author Email: [ikusbeyzi@yeditepe.edu.tr](mailto:ikusbeyzi@yeditepe.edu.tr)

## Algebraic and Numerical Analysis of Chaos Transition Mechanisms in Electronic Circuits

Orhan Ozgur Aybar <sup>1,2</sup>✉

<sup>1</sup> Piri Reis University, Department of Management Information Systems, İstanbul, Turkey

<sup>2</sup> Piri Reis University, Department of Computational Science and Engineering, Institute of Graduate Studies, İstanbul, Turkey

### Abstract

Chaos theory and chaotic circuits have attracted interest in applied sciences. Chaos theory studies the mathematical behavior of nonlinear dynamical systems that are sensitively dependent on initial conditions and where a slight change in one state of the system can cause significant differences in a later state. With the help of bifurcation theory, these systems have recently been classified as chaotic attractors. An attractor is said to be self-excited if the gravitational pressure overlaps a neighborhood of a critical point. Chaotic attractors that are not self-excited are known as latent attractors, such as Lorenz, Chua, and Rossler attractors. In many experimental studies of Chua on electrical circuits, a period-folding sequence that turns into chaos has been observed. Chua studied a three-dimensional system of autonomous differential equations representing a nonlinear electrical circuit [1]. In their studies, they demonstrated the chaos in two different quasi-periodic ways. First, they observed alternately that as the control parameter changes, quasi-periodic and phase-locked oscillations appear and disappear before chaos ensues through cyclic fold bifurcation of a phase-locked solution [2]. In another route, they showed alternately that quasi-periodic and phase-locked oscillations appear and disappear as the control parameter changes before chaos arise from a phase-locked oscillation via a period-doubling sequence [2]. We first investigate the chaotic regions for bifurcation analysis. In these regions, we study stability analysis and use Lyapunov exponents to show the chaotic regimes. We also investigate the Hopf bifurcation conditions and limit cycles. We will show the circuit simulation showing the results of these analysis in chaotic electrical circuits.

**Keywords:** Chaotic Circuit, Stability Analysis, Computer Algebra

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✉ Corresponding Author Email: [oybar@pirireis.edu.tr](mailto:oybar@pirireis.edu.tr)



## Solutions of Modified Schrödinger Equation by Using Analytical and Numerical Methods

Asıf Yokuş<sup>1</sup> ✉, Mehmet Yavuz<sup>2</sup>

<sup>1</sup> Department of Actuary, Faculty of Science, Firat University, Elazığ, 23100, Turkey

<sup>2</sup> Department of Mathematics and Computer Sciences, Necmettin Erbakan University, Konya, 42090, Turkey

### Abstract

Partial differential equations are mathematical models of physical events. The physical interpretation of the solution functions to these equations improves the perspective of many events that occur in nature. Many scientists who accepted the accuracy of this have directed their studies to this field. The solutions of these mathematical models become more valuable as they shed light on many events if they are physically meaningful. Many scientists develop solutions, methods and techniques of differential equations with their illustrative works. In this study, we discuss the modified Schrödinger equation (MNLS) which has a physically important place in mathematics and science, by using the  $(G'/G, 1/G)$ -expansion method. With this method, trigonometric, hyperbolic and rational solutions have been obtained. These solutions have an important place for scientists working with asymptotic behavior and shock wave structure. Many complex and long processes have been encountered while obtaining mentioned solutions. These difficulties are easily overcome with the help of computer technology, which is the technology of today. Moreover, we have used the Laplace decomposition method to obtain the numerical solution to the mentioned equation. By giving special values to the constants in the obtained solutions, the state of the wave at any moment  $t$  is presented with 3D, 2D and contour graphics. These methods used in the paper are useful and reliable methods for obtaining the solutions to nonlinear partial differential equations. In this context, these suggested methods can be recommended to obtain the solutions of nonlinear partial differential equations.

**Keywords:**  $(G'/G, 1/G)$ -expansion method, Laplace decomposition method, modified Schrödinger equation, exact solution, approximate solution, error analysis.

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✉ Corresponding Author Email: [asfyokus@yahoo.com](mailto:asfyokus@yahoo.com)

## A Computational Approach to Shallow Water Forced Korteweg–De Vries Equation on Critical Flow Over a Hole with Three Fractional Operators

Pundikala Veerasha<sup>1</sup> ✉, Mehmet Yavuz<sup>2</sup>

<sup>1</sup> Center for Mathematical Needs, Department of Mathematics, CHRIST (Deemed to be University), Bengaluru-560029, India

<sup>2</sup> Department of Mathematics and Computer Sciences, Necmettin Erbakan University, Konya, 42090, Turkey

### Abstract

The Korteweg–De Vries (KdV) equation is always provided a venue to study and generalize diverse physical phenomena. The pivotal aim of the study is to analyze the behaviors of forced KdV equation describing the free surface critical flow over a hole by finding the solution with the help of q-homotopy analysis transform technique (q-HATT). The projected method is elegant amalgamations of q-homotopy analysis scheme and Laplace transform. Three fractional operators are hired in the present study to show their essence in generalizing the models associated with power-law distribution, kernel singular, non-local and non-singular. The fixed-point theorem employed to present the existence and uniqueness for the hired arbitrary-order model and converges for the solution is derived with Banach space. The projected scheme springs the series solution rapidly convergent, and it can guarantee the convergence associated with the homotopy parameter. Moreover, for diverse fractional order the physical nature have been captured in plots. The achieved consequences illuminates, the hired solution procedure is reliable and highly methodical investigate the behaviors of the nonlinear models of both integer and fractional order.

**Keywords:** Force KdV equation, fractional derivatives, q-homotopy analysis transform technique, fixed point theorem.

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✉ Corresponding Author Email: [pundikala.veerasha@christuniversity.in](mailto:pundikala.veerasha@christuniversity.in)

## A Risk-Averse Two-Stage Stochastic Programming Model for a Joint Multi-Item Capacitated Line Balancing and Lot-Sizing Problem

Yuchen Li<sup>1</sup>✉, Francisco Saldanha-da-Gama<sup>2</sup>

<sup>1</sup> Beijing University of Technology, School of Economics and Management, Beijing, China

<sup>2</sup> Departamento de Estatística e Investigação Operacional, Faculdade de Ciências da Universidade de Lisboa, Lisboa, Portugal

### Abstract

In late December 2019, the Chinese Health Commission reported the outbreak of an uncommon type of pneumonia (that would later be called "severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2)"---COVID-19) of unknown origin in Wuhan, Hubei Province. The spread of the virus has severely impacted the global economy and health systems. In the manufacturing sector, the pandemic has caused serious disruptions in many supply chains. In this paper, a comprehensive production planning problem under uncertain demand is investigated. The problem intertwines two NP-hard optimization problems: an assembly line balancing problem and a capacitated lot-sizing problem. The problem is modelled as a two-stage stochastic program assuming a risk-averse decision maker. Efficient solution procedures are proposed for tackling the problem. A case study related to mask production is presented. Several insights are provided stemming from the COVID-19 pandemic. Finally, the results of a series of computational tests are reported.

**Keywords:** Assembly line balancing, Lot-sizing, Uncertain demand

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✉ Corresponding Author Email: [liyuchen@bjut.edu.cn](mailto:liyuchen@bjut.edu.cn)

## Effects of Scanning Strategies on Thermal Behavior and Stress Fields During Selective Laser Melting Of 316L Stainless Steel

Zezhou Kuai<sup>1</sup>, Zhonghua Li<sup>2✉</sup>, Bin Liu<sup>1✉</sup>, Yang Shuai<sup>1</sup>

<sup>1</sup> North University of China, School of Materials Science and Engineering, Taiyuan, China

<sup>2</sup> North University of China, School of Mechanical Engineering, Taiyuan, China

### Abstract

Simulation of temperature fields and stress fields during selective laser melting (SLM) of 316L stainless steel powder were performed using the finite element method. The effects of scanning strategies on the SLM thermal behavior, stress evolution and residual stress were investigated. The commercial finite element analysis software ANSYS (APDL) was used to establish a single-layer multi-track three-dimensional transient numerical model of SLMed 316L stainless steel. The model considers the temperature-dependent material properties which consist of thermal conductivity, density, enthalpy, yield stress, thermal expansion coefficient and Young's modulus. Three partition scanning strategies were designed. In addition to the different sizes of the divided areas, the three scanning strategies have the same total scanning area and other settings. The results show that for partition scanning, the partition size largely determines the heat accumulation during SLM processing, which in turn affects the cooling rate and temperature gradient. Partition scanning reduces the residual stress of the part to a certain extent, and as the partition increases, the scanning length decreases and the residual stress decreases more significantly. The effect of heat accumulation under different scanning strategies is obviously different. As more regions are divided, the length of the scan track in each region becomes shorter, which will result in more significant heat accumulation effects in each region. The reduction of the scan length can effectively reduce the cooling rate and temperature gradient of the node. Strategy 2 reduces the maximum residual stress of Strategy 1 by 2%, and Strategy 3 reduces the maximum residual stress of Strategy 1 by 6%.

**Keywords:** Selective laser melting, Numerical simulation, Thermal behavior, 316L stainless steel

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✉ Corresponding Author Email: [lizhonghua6868@163.com](mailto:lizhonghua6868@163.com) ; [liubin3y@nuc.edu.cn](mailto:liubin3y@nuc.edu.cn)

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