ORIGINAL PAPER

ON A SOLVABLE SYSTEM OF NON-LINEAR DIFFERENCE EQUATIONS WITH VARIABLE COEFFICIENTS

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Abstract. *In this paper, we show that the system of difference equations*

$$x_n = \frac{x_{n-2}z_{n-3}}{z_{n-1}\left(a_n + b_n x_{n-2}z_{n-3}\right)}, y_n = \frac{y_{n-2}x_{n-3}}{x_{n-1}\left(\alpha_n + \beta_n y_{n-2}x_{n-3}\right)}, z_n = \frac{z_{n-2}y_{n-3}}{y_{n-1}\left(A_n + B_n z_{n-2}y_{n-3}\right)}, n \in \mathbb{N}_0,$$

where the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$, $(A_n)_{n\in\mathbb{N}_0}$, $(B_n)_{n\in\mathbb{N}_0}$, *and the initial values* x_{-j}, y_{-j}, z_{-j}, $j \in \{1,2,3\}$ *are non-zero real numbers, can be solved in the closed form. Also, we determine the forbidden set of the initial values by using the obtained formulas. Finally, we obtain periodic solutions of aforementioned system.*

Keywords: *periodicity; system of difference equations; forbidden set.*

1. INTRODUCTION

Solving non-linear difference equations and their systems is a very hot topics that continue to attract the attention of a wide range of researchers, we can consult the following papers [1-23]. One of important non-linear solvable difference equation is the following difference equation

$$x_{n+1} = \frac{x_{n-1}x_{n-2}}{x_n\left(\pm 1 \pm x_{n-1}x_{n-2}\right)}, \ n \in \mathbb{N}_0. \tag{1.1}$$

El-Metwally et al. obtained the solutions of the equation (1.1) and studied the behavior of the solutions long time ago in [24].

In an earlier paper, Ibrahim et al. in [25] studied the solutions of the rational difference equation

$$x_{n+1} = \frac{x_{n-1}x_{n-2}}{x_n\left(a_n + b_n x_{n-1}x_{n-2}\right)}, \ n \in \mathbb{N}_0, \tag{1.2}$$

where $(a_n)_{n\in\mathbb{N}_0}$ and $(b_n)_{n\in\mathbb{N}_0}$ are real two-periodic sequences and initial values x_{-2}, x_{-1}, x_0 are nonzero real numbers.

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A few years ago, in [26], Ahmed et al. investigated the periodic character and the form of the solutions of some rational difference equations systems of order-three

$$x_{n+1}=\frac{x_{n-1}y_{n-2}}{y_n\left(-1\pm x_{n-1}y_{n-2}\right)},\ y_{n+1}=\frac{y_{n-1}x_{n-2}}{x_n\left(\pm 1\pm y_{n-1}x_{n-2}\right)},\ n\in\mathbb{N}_0, \tag{1.3}$$

by induction with $x_{-2},x_{-1},x_0,y_{-2},y_{-1}$ and y_0 are nonzero real numbers. When the assumption of $x_n=y_n$ and $x_{-2}=y_{-2},\ x_{-1}=y_{-1},\ x_0=y_0$ in system (1.3), system (1.3) is reduced special case of the equation (1.2).

Recently, in [27] we showed that the following difference equations system

$$x_n=\frac{x_{n-2}y_{n-3}}{y_{n-1}\left(a_n+b_nx_{n-2}y_{n-3}\right)},\ y_n=\frac{y_{n-2}x_{n-3}}{x_{n-1}\left(\alpha_n+\beta_ny_{n-2}x_{n-3}\right)},\ n\in\mathbb{N}_0, \tag{1.4}$$

where the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$ and the initial values x_{-j},y_{-j}, $j\in\{1,2,3\}$ are non-zero real numbers can be solved in closed-form. In addition, we obtained the forbidden set of the initial values x_{-j},y_{-j}, $j\in\{1,2,3\}$ for system (1.4) and give a study of the long-term behavior of its solutions when for every $n\in\mathbb{N}_0$, all the sequences (a_n), (b_n), (α_n), (β_n) are constant.

Quite recently in [28], was found exact formulas for the solutions of the system

$$x_{n+1}=\frac{x_{n-k+1}y_{n-k}}{y_n\left(a_n+b_nx_{n-k+1}y_{n-k}\right)},\ y_{n+1}=\frac{y_{n-k+1}x_{n-k}}{x_n\left(c_n+d_ny_{n-k+1}x_{n-k}\right)},\ n\in\mathbb{N}_0, \tag{1.5}$$

where $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(c_n)_{n\in\mathbb{N}_0}$ and $(d_n)_{n\in\mathbb{N}_0}$ are non-zero real sequences. System (1.4) can obtain by taking $k=2$ in system (1.5).

Finally, we showed that the following higher-order system of nonlinear difference equations

$$x_n=\frac{x_{n-k}y_{n-k-l}}{y_{n-l}\left(a_n+b_nx_{n-k}y_{n-k-l}\right)},\ y_n=\frac{y_{n-k}x_{n-k-l}}{x_{n-l}\left(\alpha_n+\beta_ny_{n-k}x_{n-k-l}\right)},\ n\in\mathbb{N}_0, \tag{1.6}$$

where $k,l\in\mathbb{N}$, $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$ and the initial values x_{-j},y_{-j}, $j=\overline{1,k+l}$, are real numbers can be solved in [29]. Also, by using the solutions of system (1.6), we investigate the asymptotic behavior of well-defined solutions of the above difference equations system for the case $k=2,\ l=k$.

A natural question is to study three-dimensional form of equation (1.2) and system (1.4) solvable in closed-form. Here we study such a system. That is, we deal with the following system of difference equations

$$x_n=\frac{x_{n-2}z_{n-3}}{z_{n-1}\left(a_n+b_nx_{n-2}z_{n-3}\right)},\ y_n=\frac{y_{n-2}x_{n-3}}{x_{n-1}\left(\alpha_n+\beta_ny_{n-2}x_{n-3}\right)},\ z_n=\frac{z_{n-2}y_{n-3}}{y_{n-1}\left(A_n+B_nz_{n-2}y_{n-3}\right)},\ n\in\mathbb{N}_0, \tag{1.7}$$

where the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$, $(A_n)_{n\in\mathbb{N}_0}$, $(B_n)_{n\in\mathbb{N}_0}$ and the initial values x_{-j}, y_{-j}, z_{-j}, $j\in\{1,2,3\}$ are non-zero real numbers.

Definition 1.1. (Periodicity) Let $\{x_n, y_n, z_n\}_{n\geq -3}$ be solutions to difference equations system (1.7). The solutions $\{x_n, y_n, z_n\}_{n\geq -3}$ is said to be eventually periodic p if $x_{n+p}=x_n$, $y_{n+p}=y_n$, $z_{n+p}=z_n$ for all $n\geq n_0$. If $n_0=-3$ is said that the solutions are periodic with period p.

Lemma 1.2. [30] Let $(a_n)_{n\in\mathbb{N}_0}$ and $(b_n)_{n\in\mathbb{N}_0}$ be two sequences of real numbers and the sequences $y_{2m+i}, i\in\{0,1\}$ be solutions of the equations

$$y_{2m+i}=a_{2m+i}y_{2(m-1)+i}+b_{2m+i}, m\in\mathbb{N}_0. \tag{1.8}$$

Then, for each fixed $i\in\{0,1\}$ and $m\geq -1$, equation (1.8) has the general solutions

$$y_{2m+i}=y_{i-2}\prod_{j=0}^{m}a_{2j+i}+\sum_{l=0}^{m}b_{2l+i}\prod_{j=l+1}^{m}a_{2j+i}.$$

Further, if $(a_n)_{n\in\mathbb{N}_0}$ and $(b_n)_{n\in\mathbb{N}_0}$ are constant and $i\in\{0,1\}$, then

$$y_{2m+i}=\begin{cases} a^{m+1}y_{i-2}+b\left(\frac{1-a^{m+1}}{1-a}\right), & \text{if } a\neq 1,\\ y_{i-2}+b(m+1), & \text{if } a=1.\end{cases}$$

2. CLOSED-FORM SOLUTIONS OF SYSTEM (1.7)

Let $\{(x_n, y_n, z_n)\}_{n\geq -3}$ be solutions of system (1.7). If at least one of the initial values x_{-i}, y_{-i}, z_{-i}, $i=1,2,3,$ is equal to zero, then the solutions of system (1.7) is not defined. For example, if $x_{-3}=0,$ then $y_0=0$ and so z_1 is not defined. Similarly, if $y_{-3}=0$ (or $z_{-3}=0$), then $z_0=0$ (or $x_0=0$) and so x_1 (or y_1) is not defined. For $i=1,2,$ the other cases are similar.

On the other hand, if $x_{n_0}=0$ for some $n_0\in\mathbb{N}_0$, then according to the first equation in (1.7) we have that $x_{n_0-2}=0$ or $z_{n_0-3}=0.$ If $-3\leq n_0-2\leq -1$ or $-3\leq n_0-3\leq -1,$ then we have a $j_0\in\{1,2,3\}$, such that $x_{-j_0}=0$ or $z_{-j_0}=0.$ If $n_0\geq 3$ then by using the equations in (1.7) we have that $x_{n_0-4}=0$ or $z_{n_0-5}=0$ if $x_{n_0-2}=0,$ or $z_{n_0-5}=0$ or $y_{n_0-6}=0$ if $z_{n_0-3}=0.$ If $-3\leq n_0-4\leq -1$ or $-3\leq n_0-5\leq -1$ in the first case, or $-3\leq n_0-5\leq -1$ or $-3\leq n_0-6\leq -1$ in the second case, then we have a $j_1\in\{1,2,3\}$ such that $x_{-j_1}=0$ or $y_{-j_1}=0$ or $z_{-j_1}=0.$ Repeating this procedure we find a $p\in\{1,2,3\}$ such that $x_{-p}=0$ or $y_{-p}=0$ or $z_{-p}=0.$ As we have proved above, such solutions are not defined.

$$\frac{1}{x_n z_{n-1}}=\frac{a_n+b_n x_{n-2}z_{n-3}}{x_{n-2}z_{n-3}},\frac{1}{y_n x_{n-1}}=\frac{\alpha_n+\beta_n y_{n-2}x_{n-3}}{y_{n-2}x_{n-3}},\frac{1}{z_n y_{n-1}}=\frac{A_n+B_n z_{n-2}y_{n-3}}{z_{n-2}y_{n-3}},n\in\mathbb{N}_0. \quad (2.1)$$

Applying the substitution

$$u_n=\frac{1}{x_n z_{n-1}},v_n=\frac{1}{y_n x_{n-1}},w_n=\frac{1}{z_n y_{n-1}},n\geq -2, \quad (2.2)$$

then system (2.1) reduce to the following linear difference equations of order two

$$u_n=a_n u_{n-2}+b_n,v_n=\alpha_n v_{n-2}+\beta_n,w_n=A_n w_{n-2}+B_n,n\in\mathbb{N}_0. \quad (2.3)$$

In view of Lemma 1.2, for $i\in\{0,1\}$, the general solutions of equations in (2.3) are

$$u_{2m+i}=u_{i-2}\prod_{j=0}^{m}a_{2j+i}+\sum_{l=0}^{m}b_{2l+i}\prod_{j=l+1}^{m}a_{2j+i},$$

$$v_{2m+i}=v_{i-2}\prod_{j=0}^{m}\alpha_{2j+i}+\sum_{l=0}^{m}\beta_{2l+i}\prod_{j=l+1}^{m}\alpha_{2j+i}, \quad (2.4)$$

$$w_{2m+i}=w_{i-2}\prod_{j=0}^{m}A_{2j+i}+\sum_{l=0}^{m}B_{2l+i}\prod_{j=l+1}^{m}A_{2j+i},m\in\mathbb{N}_0.$$

From equations in (2.2) we have that

$$x_{2m+i}=\frac{w_{2m+i-1}}{u_{2m+i}}\frac{u_{2m+i-3}}{v_{2m+i-2}}\frac{v_{2m+i-5}}{w_{2m+i-4}}x_{2(m-3)+i},$$

$$y_{2m+i}=\frac{u_{2m+i-1}}{v_{2m+i}}\frac{v_{2m+i-3}}{w_{2m+i-2}}\frac{w_{2m+i-5}}{u_{2m+i-4}}y_{2(m-3)+i}, \quad (2.5)$$

$$z_{2m+i}=\frac{v_{2m+i-1}}{w_{2m+i}}\frac{w_{2m+i-3}}{u_{2m+i-2}}\frac{u_{2m+i-5}}{v_{2m+i-4}}z_{2(m-3)+i},m\in\mathbb{N},$$

where $i\in\{1,2\}$, and consequently

$$x_{6m+j}=\frac{w_{6m+j-1}}{u_{6m+j}}\frac{u_{6m+j-3}}{v_{6m+j-2}}\frac{v_{6m+j-5}}{w_{6m+j-4}}x_{6(m-1)+j},m\in\mathbb{N}_0,$$

$$y_{6m+j}=\frac{u_{6m+j-1}}{v_{6m+j}}\frac{v_{6m+j-3}}{w_{6m+j-2}}\frac{w_{6m+j-5}}{u_{6m+j-4}}y_{6(m-1)+j},m\in\mathbb{N}_0, \tag{2.6}$$

$$z_{6m+j}=\frac{v_{6m+j-1}}{w_{6m+j}}\frac{w_{6m+j-3}}{u_{6m+j-2}}\frac{u_{6m+j-5}}{v_{6m+j-4}}z_{6(m-1)+j},m\in\mathbb{N}_0,$$

where $j\in\{3,4,5,6,7,8\}$. From (2.6), we get that

$$x_{6m+l}=x_{l-6}\prod_{j=0}^{m}\frac{w_{6j+l-1}}{u_{6j+l}}\frac{u_{6j+l-3}}{v_{6j+l-2}}\frac{v_{6j+l-5}}{w_{6j+l-4}},$$

$$y_{6m+l}=y_{l-6}\prod_{j=0}^{m}\frac{u_{6j+l-1}}{v_{6j+l}}\frac{v_{6j+l-3}}{w_{6j+l-2}}\frac{w_{6j+l-5}}{u_{6j+l-4}}, \tag{2.7}$$

$$z_{6m+l}=z_{l-6}\prod_{j=0}^{m}\frac{v_{6j+l-1}}{w_{6j+l}}\frac{w_{6j+l-3}}{u_{6j+l-2}}\frac{u_{6j+l-5}}{v_{6j+l-4}},$$

where $m\geq -1$ and $l\in\{3,4,5,6,7,8\}$. From (2.7)

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\frac{w_{6j+2i+k-1}}{u_{6j+2i+k}}\frac{u_{6j+2i+k-3}}{v_{6j+2i+k-2}}\frac{v_{6j+2i+k-5}}{w_{6j+2i+k-4}},$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\frac{u_{6j+2i+k-1}}{v_{6j+2i+k}}\frac{v_{6j+2i+k-3}}{w_{6j+2i+k-2}}\frac{w_{6j+2i+k-5}}{u_{6j+2i+k-4}}, \tag{2.8}$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\frac{v_{6j+2i+k-1}}{w_{6j+2i+k}}\frac{w_{6j+2i+k-3}}{u_{6j+2i+k-2}}\frac{u_{6j+2i+k-5}}{v_{6j+2i+k-4}},$$

for $i\in\{1,2,3\}$ and $k\in\{1,2\}$. Employing (2.4) in (2.8), we get

$$x_{6m+2i+1}=x_{2i-5}\prod_{j=0}^{m}\frac{w_{-2}\prod_{s=0}^{3j+i}A_{2s}+\sum_{l=0}^{3j+i}B_{2l}\prod_{s=l+1}^{3j+i}A_{2s}}{u_{-1}\prod_{s=0}^{3j+i}a_{2s+1}+\sum_{l=0}^{3j+i}b_{2l+1}\prod_{s=l+1}^{3j+i}a_{2s+1}}\frac{u_{-2}\prod_{s=0}^{3j+i-1}a_{2s}+\sum_{l=0}^{3j+i-1}b_{2l}\prod_{s=l+1}^{3j+i-1}a_{2s}}{v_{-1}\prod_{s=0}^{3j+i-1}\alpha_{2s+1}+\sum_{l=0}^{3j+i-1}\beta_{2l+1}\prod_{s=l+1}^{3j+i-1}\alpha_{2s+1}}$$
$$\times\frac{v_{-2}\prod_{s=0}^{3j+i-2}\alpha_{2s}+\sum_{l=0}^{3j+i-2}\beta_{2l}\prod_{s=l+1}^{3j+i-2}\alpha_{2s}}{w_{-1}\prod_{s=0}^{3j+i-2}A_{2s+1}+\sum_{l=0}^{3j+i-2}B_{2l+1}\prod_{s=l+1}^{3j+i-2}A_{2s+1}}$$

$$=x_{2i-5}\prod_{j=0}^{m}\left(\frac{x_{-1}y_{-1}z_{-1}}{x_{-3}y_{-3}z_{-3}}\right)\frac{\prod_{s=0}^{3j+i}A_{2s}+z_{-2}y_{-3}\sum_{l=0}^{3j+i}B_{2l}\prod_{s=l+1}^{3j+i}A_{2s}}{\prod_{s=0}^{3j+i}a_{2s+1}+x_{-1}z_{-2}\sum_{l=0}^{3j+i}b_{2l+1}\prod_{s=l+1}^{3j+i}a_{2s+1}}\frac{\prod_{s=0}^{3j+i-1}a_{2s}+x_{-2}z_{-3}\sum_{l=0}^{3j+i-1}b_{2l}\prod_{s=l+1}^{3j+i-1}a_{2s}}{\prod_{s=0}^{3j+i-1}\alpha_{2s+1}+y_{-1}x_{-2}\sum_{l=0}^{3j+i-1}\beta_{2l+1}\prod_{s=l+1}^{3j+i-1}\alpha_{2s+1}}$$
$$\times\frac{\prod_{s=0}^{3j+i-2}\alpha_{2s}+y_{-2}x_{-3}\sum_{l=0}^{3j+i-2}\beta_{2l}\prod_{s=l+1}^{3j+i-2}\alpha_{2s}}{\prod_{s=0}^{3j+i-2}A_{2s+1}+z_{-1}y_{-2}\sum_{l=0}^{3j+i-2}B_{2l+1}\prod_{s=l+1}^{3j+i-2}A_{2s+1}},\tag{2.9}$$

$$x_{6m+2i+2}=x_{2i-4}\prod_{j=0}^{m}\frac{w_{-1}\prod_{s=0}^{3j+i}A_{2s+1}+\sum_{l=0}^{3j+i}B_{2l+1}\prod_{s=l+1}^{3j+i}A_{2s+1}}{u_{-2}\prod_{s=0}^{3j+i+1}a_{2s}+\sum_{l=0}^{3j+i+1}b_{2l}\prod_{s=l+1}^{3j+i+1}a_{2s}}\frac{u_{-1}\prod_{s=0}^{3j+i-1}a_{2s+1}+\sum_{l=0}^{3j+i-1}b_{2l+1}\prod_{s=l+1}^{3j+i-1}a_{2s+1}}{v_{-2}\prod_{s=0}^{3j+i}\alpha_{2s}+\sum_{l=0}^{3j+i}\beta_{2l}\prod_{s=l+1}^{3j+i}\alpha_{2s}}$$
$$\times\frac{v_{-1}\prod_{s=0}^{3j+i-2}\alpha_{2s+1}+\sum_{l=0}^{3j+i-2}\beta_{2l+1}\prod_{s=l+1}^{3j+i-2}\alpha_{2s+1}}{w_{-2}\prod_{s=0}^{3j+i-1}A_{2s}+\sum_{l=0}^{3j+i-1}B_{2l}\prod_{s=l+1}^{3j+i-1}A_{2s}}$$

$$=x_{2i-4}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)\frac{\prod_{s=0}^{3j+i}A_{2s+1}+z_{-1}y_{-2}\sum_{l=0}^{3j+i}B_{2l+1}\prod_{s=l+1}^{3j+i}A_{2s+1}}{\prod_{s=0}^{3j+i+1}a_{2s}+x_{-2}z_{-3}\sum_{l=0}^{3j+i+1}b_{2l}\prod_{s=l+1}^{3j+i+1}a_{2s}}\frac{\prod_{s=0}^{3j+i-1}a_{2s+1}+x_{-1}z_{-2}\sum_{l=0}^{3j+i-1}b_{2l+1}\prod_{s=l+1}^{3j+i-1}a_{2s+1}}{\prod_{s=0}^{3j+i}\alpha_{2s}+y_{-2}x_{-3}\sum_{l=0}^{3j+i}\beta_{2l}\prod_{s=l+1}^{3j+i}\alpha_{2s}}$$
$$\times\frac{\prod_{s=0}^{3j+i-2}\alpha_{2s+1}+y_{-1}x_{-2}\sum_{l=0}^{3j+i-2}\beta_{2l+1}\prod_{s=l+1}^{3j+i-2}\alpha_{2s+1}}{\prod_{s=0}^{3j+i-1}A_{2s}+z_{-2}y_{-3}\sum_{l=0}^{3j+i-1}B_{2l}\prod_{s=l+1}^{3j+i-1}A_{2s}},\tag{2.10}$$

$$y_{6m+2i+1}=y_{2i-5}\prod_{j=0}^{m}\frac{u_{-2}\prod_{s=0}^{3j+i}a_{2s}+\sum_{l=0}^{3j+i}b_{2l}\prod_{s=l+1}^{3j+i}a_{2s}}{v_{-1}\prod_{s=0}^{3j+i}\alpha_{2s+1}+\sum_{l=0}^{3j+i}\beta_{2l+1}\prod_{s=l+1}^{3j+i}\alpha_{2s+1}}\frac{v_{-2}\prod_{s=0}^{3j+i-1}\alpha_{2s}+\sum_{l=0}^{3j+i-1}\beta_{2l}\prod_{s=l+1}^{3j+i-1}\alpha_{2s}}{w_{-1}\prod_{s=0}^{3j+i-1}A_{2s+1}+\sum_{l=0}^{3j+i-1}B_{2l+1}\prod_{s=l+1}^{3j+i-1}A_{2s+1}}$$
$$\times\frac{w_{-2}\prod_{s=0}^{3j+i-2}A_{2s}+\sum_{l=0}^{3j+i-2}B_{2l}\prod_{s=l+1}^{3j+i-2}A_{2s}}{u_{-1}\prod_{s=0}^{3j+i-2}a_{2s+1}+\sum_{l=0}^{3j+i-2}b_{2l+1}\prod_{s=l+1}^{3j+i-2}a_{2s+1}}$$

$$=y_{2i-5}\prod_{j=0}^{m}\left(\frac{x_{-1}y_{-1}z_{-1}}{x_{-3}y_{-3}z_{-3}}\right)\frac{\prod_{s=0}^{3j+i}a_{2s}+x_{-2}z_{-3}\sum_{l=0}^{3j+i}b_{2l}\prod_{s=l+1}^{3j+i}a_{2s}}{\prod_{s=0}^{3j+i}\alpha_{2s+1}+y_{-1}x_{-2}\sum_{l=0}^{3j+i}\beta_{2l+1}\prod_{s=l+1}^{3j+i}\alpha_{2s+1}}\frac{\prod_{s=0}^{3j+i-1}\alpha_{2s}+y_{-2}x_{-3}\sum_{l=0}^{3j+i-1}\beta_{2l}\prod_{s=l+1}^{3j+i-1}\alpha_{2s}}{\prod_{s=0}^{3j+i-1}A_{2s+1}+z_{-1}y_{-2}\sum_{l=0}^{3j+i-1}B_{2l+1}\prod_{s=l+1}^{3j+i-1}A_{2s+1}}$$
$$\times\frac{\prod_{s=0}^{3j+i-2}A_{2s}+z_{-2}y_{-3}\sum_{l=0}^{3j+i-2}B_{2l}\prod_{s=l+1}^{3j+i-2}A_{2s}}{\prod_{s=0}^{3j+i-2}a_{2s+1}+x_{-1}z_{-2}\sum_{l=0}^{3j+i-2}b_{2l+1}\prod_{s=l+1}^{3j+i-2}a_{2s+1}},\tag{2.11}$$

$$y_{6m+2i+2}=y_{2i-4}\prod_{j=0}^{m}\frac{u_{-1}\prod_{s=0}^{3j+i}a_{2s+1}+\sum_{l=0}^{3j+i}b_{2l+1}\prod_{s=l+1}^{3j+i}a_{2s+1}}{v_{-2}\prod_{s=0}^{3j+i+1}\alpha_{2s}+\sum_{l=0}^{3j+i+1}\beta_{2l}\prod_{s=l+1}^{3j+i+1}\alpha_{2s}}\frac{v_{-1}\prod_{s=0}^{3j+i-1}\alpha_{2s+1}+\sum_{l=0}^{3j+i-1}\beta_{2l+1}\prod_{s=l+1}^{3j+i-1}\alpha_{2s+1}}{w_{-2}\prod_{s=0}^{3j+i}A_{2s}+\sum_{l=0}^{3j+i}B_{2l}\prod_{s=l+1}^{3j+i}A_{2s}}$$
$$\times\frac{w_{-1}\prod_{s=0}^{3j+i-2}A_{2s+1}+\sum_{l=0}^{3j+i-2}B_{2l+1}\prod_{s=l+1}^{3j+i-2}A_{2s+1}}{u_{-2}\prod_{s=0}^{3j+i-1}a_{2s}+\sum_{l=0}^{3j+i-1}b_{2l}\prod_{s=l+1}^{3j+i-1}a_{2s}}$$

$$=y_{2i-4}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)\frac{\prod_{s=0}^{3j+i}a_{2s+1}+x_{-1}z_{-2}\sum_{l=0}^{3j+i}b_{2l+1}\prod_{s=l+1}^{3j+i}a_{2s+1}}{\prod_{s=0}^{3j+i+1}\alpha_{2s}+y_{-2}x_{-3}\sum_{l=0}^{3j+i+1}\beta_{2l}\prod_{s=l+1}^{3j+i+1}\alpha_{2s}}\frac{\prod_{s=0}^{3j+i-1}\alpha_{2s+1}+y_{-1}x_{-2}\sum_{l=0}^{3j+i-1}\beta_{2l+1}\prod_{s=l+1}^{3j+i-1}\alpha_{2s+1}}{\prod_{s=0}^{3j+i}A_{2s}+z_{-2}y_{-3}\sum_{l=0}^{3j+i}B_{2l}\prod_{s=l+1}^{3j+i}A_{2s}}$$
$$\times\frac{\prod_{s=0}^{3j+i-2}A_{2s+1}+z_{-1}y_{-2}\sum_{l=0}^{3j+i-2}B_{2l+1}\prod_{s=l+1}^{3j+i-2}A_{2s+1}}{\prod_{s=0}^{3j+i-1}a_{2s}+x_{-2}z_{-3}\sum_{l=0}^{3j+i-1}b_{2l}\prod_{s=l+1}^{3j+i-1}a_{2s}},\tag{2.12}$$

$$z_{6m+2i+1}=z_{2i-5}\prod_{j=0}^{m}\frac{v_{-2}\prod_{s=0}^{3j+i}\alpha_{2s}+\sum_{l=0}^{3j+i}\beta_{2l}\prod_{s=l+1}^{3j+i}\alpha_{2s}}{w_{-1}\prod_{s=0}^{3j+i}A_{2s+1}+\sum_{l=0}^{3j+i}B_{2l+1}\prod_{s=l+1}^{3j+i}A_{2s+1}}\frac{w_{-2}\prod_{s=0}^{3j+i-1}A_{2s}+\sum_{l=0}^{3j+i-1}B_{2l}\prod_{s=l+1}^{3j+i-1}A_{2s}}{u_{-1}\prod_{s=0}^{3j+i-1}a_{2s+1}+\sum_{l=0}^{3j+i-1}b_{2l+1}\prod_{s=l+1}^{3j+i-1}a_{2s+1}}$$
$$\times\frac{u_{-2}\prod_{s=0}^{3j+i-2}a_{2s}+\sum_{l=0}^{3j+i-2}b_{2l}\prod_{s=l+1}^{3j+i-2}a_{2s}}{v_{-1}\prod_{s=0}^{3j+i-2}\alpha_{2s+1}+\sum_{l=0}^{3j+i-2}\beta_{2l+1}\prod_{s=l+1}^{3j+i-2}\alpha_{2s+1}}$$

$$=z_{2i-5}\prod_{j=0}^{m}\left(\frac{x_{-1}y_{-1}z_{-1}}{x_{-3}y_{-3}z_{-3}}\right)\frac{\prod_{s=0}^{3j+i}\alpha_{2s}+y_{-2}x_{-3}\sum_{l=0}^{3j+i}\beta_{2l}\prod_{s=l+1}^{3j+i}\alpha_{2s}}{\prod_{s=0}^{3j+i}A_{2s+1}+z_{-1}y_{-2}\sum_{l=0}^{3j+i}B_{2l+1}\prod_{s=l+1}^{3j+i}A_{2s+1}}\frac{\prod_{s=0}^{3j+i-1}A_{2s}+z_{-2}y_{-3}\sum_{l=0}^{3j+i-1}B_{2l}\prod_{s=l+1}^{3j+i-1}A_{2s}}{\prod_{s=0}^{3j+i-1}a_{2s+1}+x_{-1}z_{-2}\sum_{l=0}^{3j+i-1}b_{2l+1}\prod_{s=l+1}^{3j+i-1}a_{2s+1}}$$
$$\times\frac{\prod_{s=0}^{3j+i-2}a_{2s}+x_{-2}z_{-3}\sum_{l=0}^{3j+i-2}b_{2l}\prod_{s=l+1}^{3j+i-2}a_{2s}}{\prod_{s=0}^{3j+i-2}\alpha_{2s+1}+y_{-1}x_{-2}\sum_{l=0}^{3j+i-2}\beta_{2l+1}\prod_{s=l+1}^{3j+i-2}\alpha_{2s+1}},\tag{2.13}$$

$$z_{6m+2i+2}=z_{2i-4}\prod_{j=0}^{m}\frac{v_{-1}\prod_{s=0}^{3j+i}\alpha_{2s+1}+\sum_{l=0}^{3j+i}\beta_{2l+1}\prod_{s=l+1}^{3j+i}\alpha_{2s+1}}{w_{-2}\prod_{s=0}^{3j+i+1}A_{2s}+\sum_{l=0}^{3j+i+1}B_{2l}\prod_{s=l+1}^{3j+i+1}A_{2s}}\frac{w_{-1}\prod_{s=0}^{3j+i-1}A_{2s+1}+\sum_{l=0}^{3j+i-1}B_{2l+1}\prod_{s=l+1}^{3j+i-1}A_{2s+1}}{u_{-2}\prod_{s=0}^{3j+i}a_{2s}+\sum_{l=0}^{3j+i}b_{2l}\prod_{s=l+1}^{3j+i}a_{2s}}$$
$$\times\frac{u_{-1}\prod_{s=0}^{3j+i-2}a_{2s+1}+\sum_{l=0}^{3j+i-2}b_{2l+1}\prod_{s=l+1}^{3j+i-2}a_{2s+1}}{v_{-2}\prod_{s=0}^{3j+i-1}\alpha_{2s}+\sum_{l=0}^{3j+i-1}\beta_{2l}\prod_{s=l+1}^{3j+i-1}\alpha_{2s}}$$

$$=z_{2i-4}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)\frac{\prod_{s=0}^{3j+i}\alpha_{2s+1}+y_{-1}x_{-2}\sum_{l=0}^{3j+i}\beta_{2l+1}\prod_{s=l+1}^{3j+i}\alpha_{2s+1}}{\prod_{s=0}^{3j+i+1}A_{2s}+z_{-2}y_{-3}\sum_{l=0}^{3j+i+1}B_{2l}\prod_{s=l+1}^{3j+i+1}A_{2s}}\frac{\prod_{s=0}^{3j+i-1}A_{2s+1}+z_{-1}y_{-2}\sum_{l=0}^{3j+i-1}B_{2l+1}\prod_{s=l+1}^{3j+i-1}A_{2s+1}}{\prod_{s=0}^{3j+i}a_{2s}+x_{-2}z_{-3}\sum_{l=0}^{3j+i}b_{2l}\prod_{s=l+1}^{3j+i}a_{2s}}$$
$$\times\frac{\prod_{s=0}^{3j+i-2}a_{2s+1}+x_{-1}z_{-2}\sum_{l=0}^{3j+i-2}b_{2l+1}\prod_{s=l+1}^{3j+i-2}a_{2s+1}}{\prod_{s=0}^{3j+i-1}\alpha_{2s}+y_{-2}x_{-3}\sum_{l=0}^{3j+i-1}\beta_{2l}\prod_{s=l+1}^{3j+i-1}\alpha_{2s}},\tag{2.14}$$

for every $m\geq -1,\ i\in\{1,2,3\}$.

The forbidden set of the initial values for system (1.7) can be given in the following theorem.

Theorem 2.1. Assume that $a_n \neq 0, b_n \neq 0, \alpha_n \neq 0, \beta_n \neq 0, A_n \neq 0, B_n \neq 0, n \in \mathbb{N}_0$. Then the forbidden set of the initial values for system (1.7) is given by the set

$$F = \bigcup_{m\in\mathbb{N}_0} \bigcup_{i=0}^{1} \{(x_{-3}, x_{-2}, x_{-1}, y_{-3}, y_{-2}, y_{-1}, z_{-3}, z_{-2}, z_{-1}) \in \mathbb{R}^9 : x_{i-2}z_{i-3} = \frac{1}{c_m}, y_{i-2}x_{i-3} = \frac{1}{d_m}, z_{i-2}y_{i-3} = \frac{1}{e_m} \}$$

$$\bigcup \bigcup_{j=1}^{3} \{(x_{-3}, x_{-2}, x_{-1}, y_{-3}, y_{-2}, y_{-1}, z_{-3}, z_{-2}, z_{-1}) \in \mathbb{R}^9 : x_{-j} \neq 0, y_{-j} \neq 0, z_{-j} \neq 0\},$$

where $c_m := -\sum_{j=0}^{m} \frac{b_{2j+i}}{a_{2j+i}} \prod_{l=0}^{j-1} \frac{1}{a_{2l+i}} \neq 0, d_m := -\sum_{j=0}^{m} \frac{\beta_{2j+i}}{\alpha_{2j+i}} \prod_{l=0}^{j-1} \frac{1}{\alpha_{2l+i}} \neq 0, e_m := -\sum_{j=0}^{m} \frac{B_{2j+i}}{A_{2j+i}} \prod_{l=0}^{j-1} \frac{1}{A_{2l+i}} \neq 0.$

Proof: At the beginning of Section 2, we have acquired that the set

$$\bigcup_{j=1}^{3} \{(x_{-3}, x_{-2}, x_{-1}, y_{-3}, y_{-2}, y_{-1}, z_{-3}, z_{-2}, z_{-1}) \in \mathbb{R}^9 : x_{-j} \neq 0, y_{-j} \neq 0, z_{-j} \neq 0\}$$

belongs to the forbidden set of the initial values for system (1.7). Now, we assume that $x_n \neq 0$, $y_n \neq 0$ and $z_n \neq 0$. Note that the system (1.7) is undefined, when the conditions $a_n + b_n x_{n-2} z_{n-3} = 0$, $\alpha_n + \beta_n y_{n-2} x_{n-3} = 0$ or $A_n + B_n z_{n-2} y_{n-3} = 0$, that is, $x_{n-2} z_{n-3} = -\frac{a_n}{b_n}$, $y_{n-2} x_{n-3} = -\frac{\alpha_n}{\beta_n}$ or $z_{n-2} y_{n-3} = -\frac{A_n}{B_n}$, for some $n \in \mathbb{N}_0$, are satisfied(Here we consider that $b_n \neq 0$, $\beta_n \neq 0$ and $A_n \neq 0$ for some $n \in \mathbb{N}_0$). From this and equations in (2.2), we get

$$u_{2(m-1)+i} = -\frac{b_{2m+i}}{a_{2m+i}}, v_{2(m-1)+i} = -\frac{\beta_{2m+i}}{\alpha_{2m+i}}, w_{2(m-1)+i} = -\frac{B_{2m+i}}{A_{2m+i}}, \tag{2.15}$$

for some $m \in \mathbb{N}_0$ and $i \in \{0,1\}$. Hence, we can determine the forbidden set of the initial values for system (1.7) by using the equations in (2.2). Now, we consider the functions

$$f_{2m+i}(t) := a_{2m+i} t + b_{2m+i},$$

$$g_{2m+i}(t) := \alpha_{2m+i} t + \beta_{2m+i}, \tag{2.16}$$

$$h_{2m+i}(t) := A_{2m+i} t + B_{2m+i},\ m \in \mathbb{N}_0, i \in \{0,1\},$$

which correspond to the equations of (2.3). From (2.3) and (2.16), we can write

$$u_{2m+i} = f_{2m+i} \circ f_{2(m-1)+i} \circ \cdots \circ f_i(u_{i-2}), \tag{2.17}$$

$$v_{2m+i}=g_{2m+i}\circ g_{2(m-1)+i}\circ\cdots\circ g_i\left(v_{i-2}\right), \tag{2.18}$$

$$w_{2m+i}=h_{2m+i}\circ h_{2(m-1)+i}\circ\cdots\circ h_i\left(w_{i-2}\right), \tag{2.19}$$

where $m\in\mathbb{N}_0$ and $i\in\{0,1\}$. By using (2.15) and implicit forms (2.17)-(2.19) and considering $f_{2m+i}^{-1}(0)=-\dfrac{b_{2m+i}}{a_{2m+i}}$, $g_{2m+i}^{-1}(0)=-\dfrac{\beta_{2m+i}}{\alpha_{2m+i}}$, $h_{2m+i}^{-1}(0)=-\dfrac{B_{2m+i}}{A_{2m+i}}$, for $m\in\mathbb{N}_0$ and $i\in\{0,1\}$, we have

$$u_{i-2}=f_i^{-1}\circ\cdots\circ f_{2m+i}^{-1}(0),\ v_{i-2}=g_i^{-1}\circ\cdots\circ g_{2m+i}^{-1}(0), w_{i-2}=h_i^{-1}\circ\cdots\circ h_{2m+i}^{-1}(0), \tag{2.20}$$

where $f_{2m+i}^{-1}(t)=\dfrac{t-b_{2m+i}}{a_{2m+i}}$, $g_{2m+i}^{-1}(t)=\dfrac{t-\beta_{2m+i}}{\alpha_{2m+i}}$, $h_{2m+i}^{-1}(t)=\dfrac{t-B_{2m+i}}{A_{2m+i}}$, for $m\in\mathbb{N}_0$, $i\in\{0,1\}$. From (2.20) we obtain $u_{i-2}=-\sum_{j=0}^{m}\dfrac{b_{2j+i}}{a_{2j+i}}\prod_{l=0}^{j-1}\dfrac{1}{a_{2l+i}}$, $v_{i-2}=-\sum_{j=0}^{m}\dfrac{\beta_{2j+i}}{\alpha_{2j+i}}\prod_{l=0}^{j-1}\dfrac{1}{\alpha_{2l+i}}$, $w_{i-2}=-\sum_{j=0}^{m}\dfrac{B_{2j+i}}{A_{2j+i}}\prod_{l=0}^{j-1}\dfrac{1}{A_{2l+i}}$, for some $m\in\mathbb{N}_0$ and $i\in\{0,1\}$. This means that if one of the conditions in (2.20) holds, then $m-$th iteration or $(m+1)-$th iteration in system (1.7) can not be calculated.

3. CASE OF CONSTANT COEFFICIENTS

In this section, we examine the forms of solutions of system (1.7) for the case when $a_n=a$, $b_n=b$, $\alpha_n=\alpha$, $\beta_n=\beta$, $A_n=A$, and $B_n=B$, for every $n\in\mathbb{N}_0$. Then, the system (1.7) becomes

$$x_n=\frac{x_{n-2}z_{n-3}}{z_{n-1}\left(a+bx_{n-2}z_{n-3}\right)}, y_n=\frac{y_{n-2}x_{n-3}}{x_{n-1}\left(\alpha+\beta y_{n-2}x_{n-3}\right)}, z_n=\frac{z_{n-2}y_{n-3}}{y_{n-1}\left(A+Bz_{n-2}y_{n-3}\right)}, n\in\mathbb{N}_0. \tag{3.1}$$

We start the following theorem describing the form of well-defined solutions of system (3.1).

Theorem 3.1. Let $\{(x_n,y_n,z_n)\}_{n\geq-3}$ be well-defined solutions of system (3.1). Then, for $m\geq-1$ and $i\in\{1,2,3\}$, $k\in\{1,2\}$ we get that

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{A^{3j+i+1}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{a^{3j+i+k}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b}$$
$$\times\frac{a^{3j+i}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{\alpha^{3j+i+k-1}\left(\left(1-\alpha\right)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta} \tag{3.2}$$
$$\times\frac{\alpha^{3j+i-1}\left(\left(1-\alpha\right)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{A^{3j+i+k-2}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B},$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{a^{3j+i+1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{\alpha^{3j+i+k}\left(\left(1-\alpha\right)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta}$$
$$\times\frac{\alpha^{3j+i}\left(\left(1-\alpha\right)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{A^{3j+i+k-1}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B} \tag{3.3}$$
$$\times\frac{A^{3j+i-1}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{a^{3j+i+k-2}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b},$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{\alpha^{3j+i+1}\left(\left(1-\alpha\right)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{A^{3j+i+k}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B}$$
$$\times\frac{A^{3j+i}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{a^{3j+i+k-1}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b} \tag{3.4}$$
$$\times\frac{a^{3j+i-1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{\alpha^{3j+i+k-2}\left(\left(1-\alpha\right)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta},$$

when $a\neq 1,\alpha\neq 1,A\neq 1,$

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+z_{k-3}y_{k-4}B\left(3j+i+1\right)}{a^{3j+i+k}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b}$$
$$\times\frac{a^{3j+i}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{\alpha^{3j+i+k-1}\left(\left(1-\alpha\right)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta} \tag{3.5}$$
$$\times\frac{\alpha^{3j+i-1}\left(\left(1-\alpha\right)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{1+z_{-k}y_{-k-1}B\left(3j+i+k-2\right)},$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{a^{3j+i+1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{\alpha^{3j+i+k}\left(\left(1-\alpha\right)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta}$$
$$\times\frac{\alpha^{3j+i}\left(\left(1-\alpha\right)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{1+z_{-k}y_{-k-1}B\left(3j+i+k-1\right)}\tag{3.6}$$
$$\times\frac{1+z_{k-3}y_{k-4}B\left(3j+i-1\right)}{a^{3j+i+k-2}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b},$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{\alpha^{3j+i+1}\left(\left(1-\alpha\right)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{1+z_{-k}y_{-k-1}B\left(3j+i+k\right)}$$
$$\times\frac{1+z_{k-3}y_{k-4}B\left(3j+i\right)}{a^{3j+i+k-1}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b}\tag{3.7}$$
$$\times\frac{a^{3j+i-1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{\alpha^{3j+i+k-2}\left(\left(1-\alpha\right)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta},$$

when $a\neq1,\alpha\neq1,A=1,$

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{A^{3j+i+1}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{a^{3j+i+k}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b}$$
$$\times\frac{a^{3j+i}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k-1\right)}\tag{3.8}$$
$$\times\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i-1\right)}{A^{3j+i+k-2}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B},$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{a^{3j+i+1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k\right)}$$
$$\times\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i\right)}{A^{3j+i+k-1}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B}\tag{3.9}$$
$$\times\frac{A^{3j+i-1}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{a^{3j+i+k-2}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b},$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i+1\right)}{A^{3j+i+k}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B}$$
$$\times\frac{A^{3j+i}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{a^{3j+i+k-1}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b}\tag{3.10}$$
$$\times\frac{a^{3j+i-1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k-2\right)},$$

when $a \neq 1, \alpha = 1, A \neq 1,$

$$x_{6m+2i+k} = x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{A^{3j+i+1}\left((1-A)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{1+x_{-k}z_{-k-1}b(3j+i+k)} \times\frac{1+x_{k-3}z_{k-4}b(3j+i)}{\alpha^{3j+i+k-1}\left((1-\alpha)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta} \times\frac{\alpha^{3j+i-1}\left((1-\alpha)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{A^{3j+i+k-2}\left((1-A)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B}, \tag{3.11}$$

$$y_{6m+2i+k} = y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+x_{k-3}z_{k-4}b(3j+i+1)}{\alpha^{3j+i+k}\left((1-\alpha)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta} \times\frac{\alpha^{3j+i}\left((1-\alpha)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{A^{3j+i+k-1}\left((1-A)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B} \times\frac{A^{3j+i-1}\left((1-A)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{1+x_{-k}z_{-k-1}b(3j+i+k-2)}, \tag{3.12}$$

$$z_{6m+2i+k} = z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{\alpha^{3j+i+1}\left((1-\alpha)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{A^{3j+i+k}\left((1-A)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B} \times\frac{A^{3j+i}\left((1-A)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{1+x_{-k}z_{-k-1}b(3j+i+k-1)} \times\frac{1+x_{k-3}z_{k-4}b(3j+i-1)}{\alpha^{3j+i+k-2}\left((1-\alpha)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta}, \tag{3.13}$$

when $a = 1, \alpha \neq 1, A \neq 1,$

$$x_{6m+2i+k} = x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+z_{k-3}y_{k-4}B(3j+i+1)}{a^{3j+i+k}\left((1-a)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b} \times\frac{a^{3j+i}\left((1-a)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{1+y_{-k}x_{-k-1}\beta(3j+i+k-1)} \times\frac{1+y_{k-3}x_{k-4}\beta(3j+i-1)}{1+z_{-k}y_{-k-1}B(3j+i+k-2)}, \tag{3.14}$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{a^{3j+i+1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k\right)}\times\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i\right)}{1+z_{-k}y_{-k-1}B\left(3j+i+k-1\right)}\times\frac{1+z_{k-3}y_{k-4}B\left(3j+i-1\right)}{a^{3j+i+k-2}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b},\tag{3.15}$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i+1\right)}{1+z_{-k}y_{-k-1}B\left(3j+i+k\right)}\times\frac{1+z_{k-3}y_{k-4}B\left(3j+i\right)}{a^{3j+i+k-1}\left(\left(1-a\right)-x_{-k}z_{-k-1}b\right)+x_{-k}z_{-k-1}b}\times\frac{a^{3j+i-1}\left(\left(1-a\right)-x_{k-3}z_{k-4}b\right)+x_{k-3}z_{k-4}b}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k-2\right)},\tag{3.16}$$

when $a\neq 1,\alpha=1,A=1,$

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{A^{3j+i+1}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{1+x_{-k}z_{-k-1}b\left(3j+i+k\right)}\times\frac{1+x_{k-3}z_{k-4}b\left(3j+i\right)}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k-1\right)}\times\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i-1\right)}{A^{3j+i+k-2}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B},\tag{3.17}$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+x_{k-3}z_{k-4}b\left(3j+i+1\right)}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k\right)}\times\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i\right)}{A^{3j+i+k-1}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B}\times\frac{A^{3j+i-1}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{1+x_{-k}z_{-k-1}b\left(3j+i+k-2\right)},\tag{3.18}$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+y_{k-3}x_{k-4}\beta\left(3j+i+1\right)}{A^{3j+i+k}\left(\left(1-A\right)-z_{-k}y_{-k-1}B\right)+z_{-k}y_{-k-1}B}\times\frac{A^{3j+i}\left(\left(1-A\right)-z_{k-3}y_{k-4}B\right)+z_{k-3}y_{k-4}B}{1+x_{-k}z_{-k-1}b\left(3j+i+k-1\right)}\times\frac{1+x_{k-3}z_{k-4}b\left(3j+i-1\right)}{1+y_{-k}x_{-k-1}\beta\left(3j+i+k-2\right)},\tag{3.19}$$

when $a=1, \alpha=1, A\neq 1,$

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+z_{k-3}y_{k-4}B(3j+i+1)}{1+x_{-k}z_{-k-1}b(3j+i+k)}\times\frac{1+x_{k-3}z_{k-4}b(3j+i)}{\alpha^{3j+i+k-1}\left((1-\alpha)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta}\times\frac{\alpha^{3j+i-1}\left((1-\alpha)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{1+z_{-k}y_{-k-1}B(3j+i+k-2)}, \tag{3.20}$$

$$y_{6m+2i+k}=y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+x_{k-3}z_{k-4}b(3j+i+1)}{\alpha^{3j+i+k}\left((1-\alpha)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta}\times\frac{\alpha^{3j+i}\left((1-\alpha)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{1+z_{-k}y_{-k-1}B(3j+i+k-1)}\times\frac{1+z_{k-3}y_{k-4}B(3j+i-1)}{1+x_{-k}z_{-k-1}b(3j+i+k-2)}, \tag{3.21}$$

$$z_{6m+2i+k}=z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{\alpha^{3j+i+1}\left((1-\alpha)-y_{k-3}x_{k-4}\beta\right)+y_{k-3}x_{k-4}\beta}{1+z_{-k}y_{-k-1}B(3j+i+k)}\times\frac{1+z_{k-3}y_{k-4}B(3j+i)}{1+x_{-k}z_{-k-1}b(3j+i+k-1)}\times\frac{1+x_{k-3}z_{k-4}b(3j+i-1)}{\alpha^{3j+i+k-2}\left((1-\alpha)-y_{-k}x_{-k-1}\beta\right)+y_{-k}x_{-k-1}\beta}, \tag{3.22}$$

when $a=1, \alpha\neq 1, A=1,$

$$x_{6m+2i+k}=x_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+z_{k-3}y_{k-4}B(3j+i+1)}{1+x_{-k}z_{-k-1}b(3j+i+k)}\times\frac{1+x_{k-3}z_{k-4}b(3j+i)}{1+y_{-k}x_{-k-1}\beta(3j+i+k-1)}\times\frac{1+y_{k-3}x_{k-4}\beta(3j+i-1)}{1+z_{-k}y_{-k-1}B(3j+i+k-2)}, \tag{3.23}$$

$$y_{6m+2i+k} = y_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+x_{k-3}z_{k-4}b(3j+i+1)}{1+y_{-k}x_{-k-1}\beta(3j+i+k)} \times\frac{1+y_{k-3}x_{k-4}\beta(3j+i)}{1+z_{-k}y_{-k-1}B(3j+i+k-1)} \times\frac{1+z_{k-3}y_{k-4}B(3j+i-1)}{1+x_{-k}z_{-k-1}b(3j+i+k-2)}, \tag{3.24}$$

$$z_{6m+2i+k} = z_{2i+k-6}\prod_{j=0}^{m}\left(\frac{x_{-3}y_{-3}z_{-3}}{x_{-1}y_{-1}z_{-1}}\right)^{2k-3}\frac{1+y_{k-3}x_{k-4}\beta(3j+i+1)}{1+z_{-k}y_{-k-1}B(3j+i+k)} \times\frac{1+z_{k-3}y_{k-4}B(3j+i)}{1+x_{-k}z_{-k-1}b(3j+i+k-1)} \times\frac{1+x_{k-3}z_{k-4}b(3j+i-1)}{1+y_{-k}x_{-k-1}\beta(3j+i+k-2)}, \tag{3.25}$$

when $a=1, \alpha=1, A=1.$

Proof: By using Lemma 1.2 and equations in (2.9)-(2.14), we can easily obtain the general solutions of system (3.1).

Lemma 3.2. Consider system (3.1). If $a\neq 1,\ \alpha\neq 1,\ A\neq 1,\ b\neq 0,\ \beta\neq 0$ and $B\neq 0$, then the system (3.1) has $6-$periodic solutions.

Proof: Let

$$p_n = x_{n-2}z_{n-3}, r_n = y_{n-2}x_{n-3}, s_n = z_{n-2}y_{n-3}, n\in\mathbb{N}_0. \tag{3.26}$$

Then from (3.1) we have that

$$p_{n+2} = \frac{p_n}{a+bp_n}, r_{n+2} = \frac{r_n}{\alpha+\beta r_n}, s_{n+2} = \frac{s_n}{A+Bs_n}, n\in\mathbb{N}_0. \tag{3.27}$$

If $b\neq 0,\ \beta\neq 0$ and $B\neq 0$, then system (3.27) has a unique equilibrium solution which $\left(\overline{p},\overline{r},\overline{s}\right)$ is different from $(0,0,0)$, that is,

$$p_n = \overline{p} = \frac{1-a}{b}\neq 0, r_n = \overline{r} = \frac{1-\alpha}{\beta}\neq 0, s_n = \overline{s} = \frac{1-A}{B}\neq 0, n\in\mathbb{N}_0. \tag{3.28}$$

If $\overline{p}=0$ or $\overline{r}=0$ or $\overline{s}=0$, then system (3.1) has not well-defined solutions. From (3.26) and (3.28), we get that

$$x_{n-2}=\frac{1-a}{bz_{n-3}}=\frac{(1-a)B}{b(1-A)}y_{n-4}=\frac{(1-a)B(1-\alpha)}{b(1-A)\beta x_{n-5}}=\frac{B(1-\alpha)}{(1-A)\beta}z_{n-6}=\frac{1-\alpha}{\beta y_{n-7}}=x_{n-8},\ n\geq 5,$$

$$y_{n-2}=\frac{1-\alpha}{\beta x_{n-3}}=\frac{(1-\alpha)b}{\beta(1-a)}z_{n-4}=\frac{(1-\alpha)b(1-A)}{\beta(1-a)By_{n-5}}=\frac{b(1-A)}{(1-a)B}x_{n-6}=\frac{1-A}{Bz_{n-7}}=y_{n-8},\ n\geq 5,\quad (3.29)$$

$$z_{n-2}=\frac{1-A}{By_{n-3}}=\frac{(1-A)\beta}{B(1-\alpha)}x_{n-4}=\frac{(1-A)\beta(1-a)}{B(1-\alpha)bz_{n-5}}=\frac{\beta(1-a)}{(1-\alpha)b}y_{n-6}=\frac{1-a}{bx_{n-7}}=z_{n-8},\ n\geq 5,$$

from which along with the assumptions in Lemma 3.2, the results can be easily seen.

CONCLUSION

In this paper, we have consider the following three-dimensional system of difference equations

$$x_n=\frac{x_{n-2}z_{n-3}}{z_{n-1}(a_n+b_nx_{n-2}z_{n-3})},y_n=\frac{y_{n-2}x_{n-3}}{x_{n-1}(\alpha_n+\beta_ny_{n-2}x_{n-3})},z_n=\frac{z_{n-2}y_{n-3}}{y_{n-1}(A_n+B_nz_{n-2}y_{n-3})},n\in\mathbb{N}_0,$$

which is a generalization of both equations in (1.1), (1.2) and systems in (1.3), (1.4), where the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$, $(A_n)_{n\in\mathbb{N}_0}$, $(B_n)_{n\in\mathbb{N}_0}$ and the initial values $x_{-j},y_{-j},z_{-j},\ j\in\{1,2,3\}$ are non-zero real numbers.

Firstly, we have obtained the closed form of well defined solutions of the aforementioned system using suitable transformation reducing the equations of our system to linear type. Also, we describe the forbidden set of the initial values using the obtained formulas. In addition, in the case where the coefficients are constant in the system, we have obtained the solutions for all possible cases of a,α,A. Finally, we have determined periodic solutions of this system.

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