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Mistake Handling Activities in Mathematics Educatıon: Practice in Class

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Abstract

The purpose of the study is to examine students' ideas of mistake handling activities implemented in mathematics class. To this aim, mistake handling activities were carried out with 12 high school students. Focus group discussions related to mistake handling activities were carried out with students during four weeks. The data were gathered through students' written reflections, transcripts of focus group discussions and semi-structured interviews. All the data were analyzed through content analysis. It was revealed that mistake handling activities made a motivational impact on high school students on their mathematical learning. The themes emerged from the study are advantage, innovation, being interesting and critical perspective.

Keywords: Mistake handling activities, mathematics instruction, inequalities

1. Introduction

Mistake handling activities are based on constructivism. Mistake handling activities support knowing mistakes and errors are necessary to construct the knowledge fully with the right knowledge. MHA are based on negative knowledge theory. The negative knowledge theory based on constructivism and metacognition (Heinze, 2005; Gartmeier, Bauer, Gruber ve Heid, 2008; Akpınar ve Akdoğan, 2010). To Gedik (2014) mistake handling activities are the activities causing mental complexity by using mathematical definitions, theorems, proofs or solutions of problems and mathematical terms and notations out of the usual way. The education and instruction environments are ususally constructed upon only positive knowledge. Are the approaches true that trying to construct the education instruction environment upon positive knowledge, refusing the negative knowledge, or trying to take precautions preventing the construction of this knowledge? It is understood that such an approach is the truest approach when the current teaching and learning theories are taking into account.

The reason for this is the constructivist approach framed current teaching and learning environments do not accept the approach built only on the truthfulness (Konyalıoğlu and Gedik, 2015). Theoretically, avoiding from the mistakes, not taking into consideration the mistakes and working on only with the true ideas comes the behaviourist approach to the mind. However, in Turkey the education and teaching process and the teaching and learning environments are tried to framed based on the theory of constructivism. In constructivism, mistakes are thought as parts of negative knowledge and the using mistakes positively is promoted. Thus, it is conceivable supporting constructivism providing the construction of knowledge, enabling advance level learning, enabling reasoning and investigating; the use of mistakes with the true knowledge; drawing the line of truthfulness by using mistakes instead of behaviourist approach based on only true knowledge (Heinze, 2005). In behaviorist theory, mistakes are neglected and in cognitive theory, only misconceptions are taken into account. In constructivist theory, mistake is seen as an opportunity and an unavodiable part of the learning process and it is suggested that it should be dealt with before the formation in the individual’s mind. In behaviourist approach, it is stated that avoiding from making mistakes is necessary because the mistakes can cause making mistakes and learning in a wrong way. On the other hand, in constructivism, not avoiding from the mistakes are important mistakes are the necessary parts of learning and it should not be avoided from making mistakes. Thus, learning from mistakes is based upon constructivism, it provides the transformation of mistakes to the useful products and using the mistakes for useful purposes (Dalehefte, Seidel ve Prenzel, 2012).

Constructivism is a theory of knowledge beyond being a theory of learning and it aims the construction of true knowledge in the mind of individual. Mistakes are the auxiliaries of the true knowledge during the construction of knowledge at all points. Heinze (2005) stated that preventing mistakes is against to construction of knowledge which is the fundamental philosophy of constructivism. If learning is accepted as an active process requiring both perceptual and judgemental learning practice, class learning environment should encourage students investigating, reasoning their incorrect comprehension (Tulis, 2013). Some of the researchers and philosophers searching the answer of the question “What is the role of mistakes on the construction of new knowledge?” assert that the mistakes are not only unavoidable but also valuable for the improvement of a discipline (Borasi, 1986).Harteis, Bauer ve Gruber (2008) approaching mistakes as “the cases as learning stimulus” and characterizing them as “critical knowledge” define learning from mistakes as “constructing knowledge through reflection associated with an individual’s and others’ mistakes”(cited. Akpınar ve Akdoğan, 2010). Dalehefte and others (2012) stated that mistakes help to learn negative knowledge that is they have a crucial role on what is wrong and why it is wrong during knowledge acqusition process.

Knowledge acqusition requires using mistakes related to the knowledge with the true aspect of the knowledge. Only aforementioned way provides a complete, permanent and advance level learning. On the other hand, it is tried to give only true knowledge, th e mistakes probing the individual to think, reason, and investigate are rarely took attention at formal education institutes. Parviainen and Eriksson(2006) stated that focusing on only true knowledge causes an individual to think on the same way and prevents new thinking ways. Additionally, focusing on only true knowledge prevents an individual to see the new potential. Once again, the studies (Movshovitz-Hadar ve Hadas, 1990; Swan 1987, Borasi, 1986; 1989; 1994; 1996) in the literature stated that the effective use of mistakes in learning and teaching environments quite important. A well-conducted educative mistakeis an important and necessary part of learning and teaching process. A qualified guidance, course plan based on mistakes and techniques and methods like explaining, case study, discussion and drama, and a strategy like having journals out of class were suggested in using mistakes at learning and teaching process.

Mistake handling activities (MHA) are presented to the students after the teaching of true knowledge through the use of a method during the instruction of a subject. The MHA can be applied easily with the aforementioned true knowledge by probing inconsistent questions and solutions. MHA are the activities that are not time consuming and the place is not a problem. MHA is not a method. MHA can be used in known strategies, methods, and techniques. MHA comprises mistakes, errors, misconceptions and all kinds of inconsistent cases as paradox confounding with true knowledge. Here, it is significant to give true knowledge in advance. Teacher’s knowledge of MHA to apply them to the instruction is also an important aspect of the process. Finally, teachers place questions that they can provide answers to the questions both conceptually and procedurally when they apply the MHA example. On the other hand, MHA can be applied both inclass and out of class.

2. Method

Qualitative method was used in the study because this method focuses more on the process rather than the results (Denzin and Lincoln, 1998). Yıldırım and Şimşek (2011) define qualitative research method as a process examining deeply by using various instruments, revealing the perceptions and cases realistically and holistically at natural environments. In qualitative research it is supported that individual behaviours should be researched through flexible and holistic understanding. It is a research method forming in the research process, prioritizing investigation and understanding. In this study, perception related to the MHA will be tried to handle holistically.

Case study was used in this qualitative study. Merriam (1998) statedthat case studies are defined intensively and holistically as analizing an example, phenomena or social units. Yin (1881) stated that case studies are as researching emprically a current phenomena within the context of real life especially in the cases that the limits of general cases and the phenomenon are not clear. Stake also defines the case studies as can be used in the studies related to human relations because of its'realistic, applicable and remarkable properties (Brown, 2008).

The purpose of the case studies is to reveal and examining deeply the individuals' thoughts, interpretions related to the different case and phenomenon. Defining different cases can be seen impportant in terms of making generalizations. However, here the purpose is not to make generalization. The purpose is to examine cultural and cognitive process. The results related to the cases are evaluated with reaching a general construct through interpretions rather than quantitavive data. Case studies are conducted to reveal ideas deeply and make cause and effect relations withing the framework of ideas (Bassey, 1999).

The sample consists of 12 high school students selected based on voluntariness. Written reflection, video transcripts of focus group discussions and semi-structured interviews were used in order to gather the data. Focus group discussions were used to observe handling MHA applications in classroom environment, written reflections were used to reveal individuals' ideas, and face to face interviews were used to detail written reflections and obtain information deeply.

The data obtained from the qualitative study were analyzed through content analysis. Content analysis requires analyzing the data deeply and enables the themes and dimensions which are not clear in advance (Yıldırım ve Şimşek, 2011).In content analysis, the data is brought close together in the framework of determined concepts and themes. After this process, the data are organized and interpreted in a form that a reader can understand easily (Yıldırım ve Şimşek, 2011).

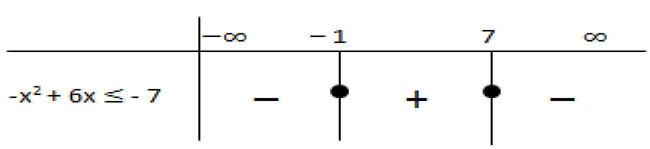
3. Findings

The findings of the study were analyzed as practice process findings and practice endingfindings.

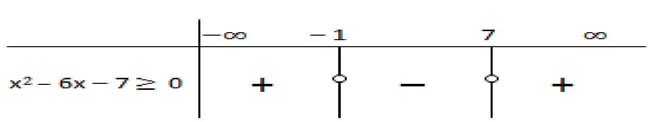
3.1. MHA Practice Process Findings

The practices related to the MHA are as follows which were implemented at the end of the course:
Practice 1: Please find the solution set of this inequality.
$-x^2+6x \leq -7$

Mistaken Solution 1 :$-x^2+6x \leq -7 \quad \Rightarrow \quad 0\leq x^2-6x-7 \quad \Rightarrow x=-1 \ \text{V} \ x=7$



Because $0\leq x^2-6x-7$ S.S. = [–1,7].
Mistaken Solution 2: $-x^2+ 6x \leq -7 \quad \Rightarrow \quad 0\leq x^2-6x-7 \quad \Rightarrow x=-1 \ \text{V}x=7$



S.S. = $(-\infty,-1) \cup (7,\infty)$.

Right Solution:$-x^2+6x \leq -7 \quad \Rightarrow \quad 0 \leq x^2-6x-7 \quad \Rightarrow x=-1 \ \lor x=7$

	$-\infty$	-1		7	∞
$x^2-6x-7 \geq 0$	+	•	−	•	+

S.S.= $(-\infty,-1] \cup [7,\infty)$.

The first behavior expected from students is to notice that they should find the solution set with the inequality which is the underlying inequality for the sign table. Again, it is expected from students that they should be able to understand the difference between "<" and "≤", if there is an equality they are expected to include the roots which do not make undefined the inequality. Despite that the first mistake in the solution of question is to form the sign table based on the inequality of $-x^2+6x \leq -7$, but find the solution set based on the inequality of $0 \leq x^2-6x-7$. The second mistake in the solution of question is doing the solution without taking into account the equality in the "≤" symbol.

Although the mistake in the first question is noticed by most of the students, the reasons of mistake were not noticed exactly by the students. One student showed the mistake by selecting a point from the table, other students showed the mistake by using their own solutions. However, the mistake done in the second solution was noticed by most of the students.

*Teacher:*Is there a mistake in these solutions? If so, in which?
*S*9: For the first one, I thought x is 8. It provided the inequality, but it is not in the solution set.
*S*3: To me the first one is right and the second one is wrong.
*S*12: Or, both of them are right and the solution set is ***R***.
*S*2: We put– x^2 other side of the equality. We made a mistake. The inequality should change its direction.
*S*4: When multiplying with minus it changes its direction.
*S*2: Right.
Teacher: A few minutes ago, your friend told us that the inequality was provided for x=8.
*S*9: In that case let's take 6 and try this.
*S*9: In the second, the circle should be full that is the points should be included.In second, the the circle should be full that is the points should be included.
*S*12: To me, both of them right.
*S*9: No, the first one is exactly wrong, the second one is right.
*S*4: We place mistakenly the minus and the plus sign and said $x$is big.
Teacher: Can both of them be right at the same time?
*S*2: We placex^2at the other side but we did not write.
*S*3: In the $0 \leq x^2-6x-7$ inequality, the sign of the term having the largest degree is +, but we put – in the first one.
*S*2: We should minuses in the first one but we took the reserve.
*S*9: the first one is wrong and the second one is right when we try it for –2 , 6 and 8. But,–1 and 7 should be included in the second one.
*S*2: At the bottom– $x^2+6x \leq -7$ is– $x^2+6x+7 \leq 0$ but we took the reverse.

Practice 2:

$$\frac{(2-x)^6.(x-1)}{x+1} \geq 0$$

Please find the solution set of this inequality.

Mistaken Solution:

	2–x=0	x–1=0	x+1=0
x=2	x=1	x= –1	

	$-\infty$ … -1	-1 … 1	1 … 2	2 … ∞
$(2-x)^6$	−	−	−	−
$x-1$	−	−	+	+
$x+1$	−	+	+	+
$\frac{(2-x)^6.(x-1)}{x+1} \geq 0$	−	+	−	−

S.S. = (–1,1].

Right Solution:

2–x=0 x–1=0 x+1=0

x=2 x=1 x= –1

	$-\infty$ … -1	-1 … 1	1 … 2	2 … ∞
$(2-x)^6$	+	+	+	+
$x-1$	−	−	+	+
$x+1$	−	+	+	+
$\frac{(2-x)^6.(x-1)}{x+1} \geq 0$	+	−	+	+

S.S. = ***R***–[–1,1)

Mistake made in the solution of question is taking the sign of $(2-x)^6$statement as minus when forming the sign table. It is required from students that they should notice this mistake that is the sign is not changed in double roots and for each $x\in$***R***, $(2-x)^6\geq 0$. The mistake in the solution was noticed by only one student.

*S*3: All the signs are negative when multiplying from bottom.
*S*2: It is not same when we look at the sign together
*S*9: The one in the middle should be empty (the student refers to (x–1)).
*S*3: We calculate all the signs wrong. All of them should be minus. Empty set.
*S*11: To me, there is an error at the upper (the student refers to $(2-x)^6$). It should be plus.
*S*6: Why is it so ?
*S*11: Because double root.
*S*6: But the sign is minus.
*S*11: What the teacher said-the result is positive when the root is double root. It's not negative.
*S*12: Also, we did not add {2} to the extent mistaken solution. But, at that point it is zero.

Practice 3: $|x+4|.(x^2-x)\geq 0$ Please find the solution set of this inequality.

Mistaken Solution:

x+4 = 0 x^2–x=0

x= –4 x=0 V x=1

	$-\infty$ … -4	-4 … 0	0 … 1	1 … ∞
$\|x+4\|$	−	+	+	+
x^2-x	+	+	−	+
$\|x+4\|.(x^2-x)\geq 0$	−	+	−	+

S.S = [–4,0] ∪ [1,∞).

Right Solution:

$x+4 = 0$ | $x^2-x=0$
$x= -4$ | $x=0 \vee x=1$

	$-\infty$	-4		0		1		∞
$\|x+4\|$	+	•	+		+		+	
x^2-x	+		+	•	−	•	+	
$\|x+4\|.(x^2-x) \geq 0$	+		+		−		+	

S.S. = $(-\infty, 0] \cup [1,\infty)$.

The behavior required from student is to notice absolute-value expression is equal to zero or bigger than zero. The mistake made in the solution of this question is to take $|x + 4|<0$ for $x \in (-\infty,-4)$. One of the students indicated that the solution is right by comparing the solution with her solution. On the other hand, another student indicated the mistake when she checked by selecting a point. Again, one student was able to identify the source of mistake.

Teacher: All of you write and analyze the question and its solution at the board.
They were given as homework because of ending the course. The question was written to the board again.
Teacher: I'd like to learn your ideas.
*S*2: Right.
Teacher: Are we sure about there is not any mistake here? Did you check it?
*S*9: Yes.
*S*5: I solve the question again again and the result is the same.
Teacher: Have you ever tried it by taking any point?
*S*5: No. I haven't tried.
Teacher : So, let's try it.
*S*3: We found the roots of $x^2-x=0$ then put$x$to the other side ($x^2=x$) then$x$= -1 V x=1 comes.
Teacher: Shall we check it? Let's try it for x=1 then $1^2=1$ is right.
*S*6: For x=-1 -1=1. So, it's not.
*S*3: Right. I thought mistakenly.
*S*9: Can I ask something?Here is also included that $(-\infty, 4]$. I said it that day (in the previous lesson). I took x=-5. Here is also included.
Teacher: Let's try it. It is really verified.
*S*11: I Think we should delete the upper side (|x+4|). In that case $(-\infty, 0] \cup [1,\infty)$.
Teacher: Why should we delete it?
*S*11: It's positive when we select a point from $(-\infty, 4)$ for $|x+4|$.
Teacher: Are you agree with your friend?
*S*5: We didn't put the absolute-valued part to the solution.
*S*3: Yes. That part is always positive.

Practice4: $-3 < \frac{2}{x} \leq 5$ ise x hangi aralıkta olur?

Mistaken Solution 1:

$-3 = \frac{2}{x} \Rightarrow x = -\frac{2}{3}$

$5 = \frac{2}{x} \Rightarrow x = \frac{2}{5}$

S.S.= [-2/3,5/2]–{0}

MistakenSolution 2:

$\frac{1}{5} \leq \frac{x}{2} < -3 \quad \Rightarrow \quad -3 < \frac{x}{2} \leq \frac{1}{5} \quad \Rightarrow \quad -6 < x \leq \frac{2}{5}$

S.S.= (-6,2/5]

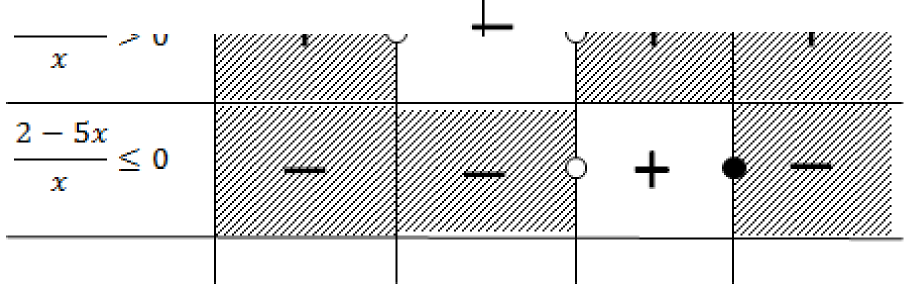
Mistaken Solution 3:

$-3 < \frac{2}{x} \leq 5 \quad \Rightarrow \quad -3x < 2 \leq 5x \quad \Rightarrow \quad \frac{2}{5} \leq x < -\frac{2}{3} \quad \Rightarrow \quad -\frac{2}{3} < x \leq \frac{2}{5}$

S.S.=(-2/3, 2/5]

Right Solution:

$-3 < \frac{2}{x}$	$\frac{2}{x} \leq 5$
$0 < 3 + \frac{2}{x}$	$\frac{2}{x} - 5 \leq 0$
$0 < \frac{3x+2}{x}$	$\frac{2-5x}{x} \leq 0$
$x = -\frac{2}{3} \quad \vee x = 0$	$x = \frac{2}{5} \quad \vee x = 0$



S.S. $= \left(-\infty, -\frac{2}{3}\right) \cup \left[\frac{2}{5}, \infty\right)$.

The first mistake in the solution of question is solving the question by accepting the inequality system as an equation system. The second mistake is the property of $x.y > 0$ and $x<y \Leftrightarrow \frac{1}{x} > \frac{1}{y}$ is right for $x.y<0$. The third mistake is to take in consideration the sign of the number when the inequality is expanded with a number. If the sign is minus the inequality changed its direction. Here, the sign of the *x*is not known. Students are expected to solve the question by considering this situation.

Two of the students were not able to identify the mistake although they indicated the solution as mistaken by comparing the solution with their own solution. Other students were not able to reach the right solution although they notice some of the mistakes in the solutions.

*S*1: In the first solution [-2/3,2/5] should be (-2/3,2/5]. There is no equality. Also, why do we remove zero?
*S*3: Because 0 makes it undefined. It is 0 when putting it its place.
*S*1: Right.
*S*11: In the second, it says $\frac{1}{5} \leq \frac{x}{2} < -3$, why it is not$-\frac{1}{3}$?.
*S*6: In the third one we expand *x*then we will do two inequalities seperately. (The student refers to -3*x*<2 ile 2≤5*x*).
*S*11: Yes, I agree with my friend. We should do so.
*S*12: I solved in another way and I found differently. (the solution is right).
*S*4: Me too. I also found differently. (the solution is right).

3.2. MHA Practice Ending Findings

The first question “Did you find it useful the MHA in mathematics?” was asked to the students. 10 of the students indicated that they found it useful, 1 of the students stated that it is not useful and one of the studnets did not make any comment. The student who thinks MHA is not useful said that he began to suspect from his knowledge.3 of the rest 10 students stated that they had different perspectives, 2 of them stated they increased their attetion and 2 of them stated that they learned new knowledge.

The second question "How did you find the MHA in mathematics?" was asked to the students. 7 of the students stated that MHA is interesting, 2 of the students stated that MHS is not interesting, 2 of the students stated that MHA is like a routin lesson process and 1 of the students did not make any comment. The students who see MHA as interesting stated that during MHA sense of wonder and making comments in the classroom encourage them to express themselves. Also, they stated that MHA made the lesson enjoyable and interesting as puzzles. 2 of the students stated MHA is a routin lesson process. 2 of the students stated that MHA prevented them to solve more questions.

The third question "What is the most distinct effect of the MHA for you?" was asked to the students. 4 of the students indicated that their necessity of having different perspectives increased. 3 of the students indicated that their criticality and sense of wonder increased. 2 of the students indicated that they began to be more careful. 1 one of the students indicated that there is not any difference during MHA. 1 one of the students did not make any comment.

The codes revealed by the instruments are "*usefulness, different perspective, sense of curiosity, being interesting, being enjoyable, increasing attention, sense of wonder, discussion, confusing, critical thinking.* The themes depended on these codes are; "*useful, new, being interestingand critical view*".

4. Conclusion

The congruity of the MHA to the constructivist approach accepted generally and the focus in formation of education systems was revealed with this study one more time. As Akıncıoğlu (2007) states that constructivist approach provides individuals to look at cases with different perspectives, improves critical thinking skills and arouse curiosity and probe problems causing mental imbalance states that constructivist approach provides individuals to look at cases with different perspectives, improves critical thinking skills and arouse curiosity and probe problems causing mental imbalance can be thought the remark of the congruity of MHA to the constructivist approach (Akıncıoğlu,2007) This result is also emphasized by Heinze (2005). The findings of the study provide tips that to what extent MHA is congruent with constructivist approach.

On the other hand, the discussion group environment, the classroom environment in which constructivist approach is used emphasized by Tulis (2013) allowed open communication directed to different solutions. Especially, the interest and curiosity about MHA can be thought as the first step of learning. Dalehefte, Seidel ve Prenzel (2012) stated that mistakes can be curiosity facilitator and interesting. Also, the findings of this study related to MHA is congruent with Borasi(1989) especially in terms of curiosity and providing attention. Again, different perspectives of students and the increase on attention are also congruent with the results of Ginat (2003) study. The idea of usefulness of the MHA is common in this study and this finding is congruent with the findings of Klymchuk ve Kachapova (2012) and Hesketh (1997) studies. Thus, the findings revealed that MHA has positive effects both cognitively and affectively in learning and teaching environment.

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