

# SEQUENTIAL DATA WEIGHTING PROCEDURES FOR COMBINED RATIO ESTIMATORS IN COMPLEX SAMPLE SURVEYS

Aylin Alkaya<sup>1</sup>, H. Öztaş Ayhan<sup>2</sup>, Alptekin Esin<sup>3</sup>

## ABSTRACT

In sample surveys weighting is applied to data to increase the quality of estimates. Data weighting can be used for several purposes. Sample design weights can be used to adjust the differences in selection probabilities for non-self weighting sample designs. Sample design weights, adjusted for nonresponse and non-coverage through the sequential data weighting process. The unequal selection probability designs represented the complex sampling designs. Among many reasons of weighting, the most important reasons are weighting for unequal probability of selection, compensation for nonresponse, and post-stratification. Many highly efficient estimation methods in survey sampling require strong information about auxiliary variables,  $x$ . The most common estimation methods using auxiliary information in estimation stage are regression and ratio estimator. This paper proposes a sequential data weighting procedure for the estimators of combined ratio mean in complex sample surveys and general variance estimation for the population ratio mean. To illustrate the utility of the proposed estimator, Turkish Demographic and Health Survey 2003 real life data is used. It is shown that the use of auxiliary information on weights can considerably improve the efficiency of the estimates.

**Key words:** combined ratio estimator, data weighting, design weight, nonresponse weighting, Post-stratification, weighting, sequential weighting.

## 1. Introduction

Applying weights to sample survey data is one of the important methods that are used to correct for sampling and nonsampling biases and to improve efficiency of estimations in sample surveys. The use of an insufficient sampling

---

<sup>1</sup> Department of Business Administration, Nevşehir Hacı Bektaş Veli University, 50300 Nevşehir, Turkey. E-mail: Aylin@nevsehir.edu.tr

<sup>2</sup> Department of Statistics, Middle East Technical University, 06800 Ankara, Turkey. E-mail: OAyhan@metu.edu.tr

<sup>3</sup> Department of Statistics, Gazi University, 06500 Ankara, Turkey. E-mail: Alpesin@gazi.edu.tr

framework, incorrect implementation of the sample selection process, inaccurate data collection and evaluation, nonresponses etc. can lead to biased estimates. The weights are applied to obtain unbiased estimates from the biased sample (Ayhan 1981). The rationale of weighting sample data is to make survey estimates to be representative of the whole population in the cases of selecting units with unequal probabilities; nonresponse; and coverage errors which creates bias and departures between sample and the reference population (Holt and Elliot 1991; Smith 1991). Weighting the data can be conducted sequentially for unequal selection probability, nonresponse, coverage errors, post-stratification as a process. At each step of sequential data weighting the calculated weights are multiplied the previous step weights.

In the first step of sequential data weighting, design weights  $W_i$  are assigned to the sampling units. Kish (1992) has stated that, design weights can be either the element's selection probability  $W_i = k(1/\pi_i)$  or proportional to that inverse  $W_i \propto 1/\pi_i$ . It is common to increase the sampling fraction  $f = n/N$  to  $kf$  ( $k > 1$ ) in order to reduce sampling errors in one or more domains, where the domain weights will be  $w_h \propto 1/f_h = N_h/n_h$  and sampling fractions will be  $f_h = n^*/N_h$ . Weighting data by  $W_i \propto 1/\pi_i$  is a simple process that should be "always" applied to samples with unequal  $\pi_i$ 's (*according to the design based theory*). The general and most useful form of weighting is to assign the weights  $W_i$  to the sample cases  $i$  with  $W_i = 1/\pi_i$ ,  $i = 1 \dots N$ . The selection probabilities  $\pi_i$  for all sampling units must be known for all probability samples by definition (Kish 1992).

For the sample,  $n$  units are selected from a finite population size  $N$  with known but unequal probabilities. Complex sample surveys such as stratification, clustering or multi stage sampling involve unequal selection probabilities. In these surveys to compensate for the differences in the probabilities of selection of samples weighting is introduced, the data is weighted with the inverse of the selection probabilities of units. The purpose is to weight each sampling unit to produce unbiased estimates of population parameters.

The second step of weighting is the adjustment for unit or total nonresponse. Nonresponse leads bias because usually nonrespondents differ from respondents. The lower the response rate, the higher the bias will be. Nonresponse weighting adjustments increase the weights of the sampled units for which data were collected. This means that every responding unit in the survey is assigned a weight, and estimates of population characteristics are obtained by processing weighted observations.

After nonresponse adjustments of the weights, further adjustments for noncoverage can be assigned to the weights as appropriate. Non-coverage refers to the failure of the sampling frame to cover the entire target population. In

practice, to reduce the effect of noncoverage and nonresponse the design weights are generally adjusted by a weighting method of calibration. The method depends on auxiliary variable(s) which uses auxiliary variable information to increase efficiency of the estimators. Calibration is called as a weighting method and in the literature many weighting methods such as raking, post-stratification, generalized regression estimator (GREG) and linear weighting are classified as a calibration weighting method. Efficient weighting for variable values observed in a survey is a topic with a long history. The earliest references to the use of weighting include the iterative proportional fitting technique as named raking by Deming and Stephan (1940). The reference of calibration starts with Deville (1988) and continues with Deville and Särndal (1992), Wu and Sitter (2001), Wu (2003), Estevao and Särndal (2006), Kott (2006), Särndal (2007). Some of the substantial references for GREG are Cassel, Särndal and Wretman (1976), Särndal (1980), Isaki and Fuller (1982), Wright (1983), Deville and Särndal (1992), Deville, Särndal and Sautory (1993), Kalton and Flores-Cervantes (2003), Ardilly and Tillé (2006) and Tikkiwal, Rai and Ghiya (2012) studies.

Post-stratification is a well-known and frequently used weighting method to reduce nonresponse and noncoverage bias. Post-stratification is stratification after selection of the sample in Cochran (1977: 135). Post-stratification studies continued by Guy (1979), Holt and Smith (1979), Bethlehem and Kersten (1985), Bethlehem and Keller (1987), Little (1993), Singh (2003), Lu and Gelman (2003), Cervantes and Brick (2009) and many other studies. The idea behind the post-stratification is to divide population into homogenous strata according to the information gathered from the sample population (Bethlehem and Kersten 1985). Additionally, in the last step of sequential weighting, extreme weights (high or low) can be adjusted using a methodology known as trimming, which is often done to reduce the variance of the weights.

Auxiliary information is used for improving the efficiency of the sample survey design. The most common estimation methods using auxiliary information are regression and ratio estimator. The ratio estimator uses auxiliary variable information to produce efficient estimates. Cochran (1940) was the first to show the contribution of known auxiliary information in improving the efficiency of the estimator of the population mean  $\bar{Y}$  in survey sampling (Singh 2003). The quantity that is to be estimated from a sample design is the ratio of two variables both of which vary from unit to unit. In this paper, the population parameter to be estimated is the two variable ratio,  $R$ . Under stratified random sampling designs, there are two ways to produce ratio estimates, one way is the separate ratio estimator and the second way is the combined ratio estimator. Many large scale complex sample surveys are based on combined ratio mean estimator. "Combined ratio mean" is more practical to compute than the "separate ratio mean".

Sequential data weighting methodology (Deming and Stephan 1940, Stephan 1942) for the combined ratio estimator is handled by Ayhan (1991) and Verma (1991) and was elaborated by Ayhan (2003). The purpose of this paper is to present a combined ratio estimator under sequential weighting procedure rely on

Ayhan (2003)'s combined ratio estimator. In accordance with this purpose, combined ratio estimator is merely to provide an estimator for illustration. Alternative illustrations can also be made for the separate ratio estimators, in another context.

In the proposed estimator, the weights are based on selection probabilities, the observed values of auxiliary variables. Compared to the known combined ratio estimator, this method uses more information about auxiliary variables in regard to determining the weights. It can be expected that Ayhan (2003)'s combined ratio estimator which involves more information in determining weights will give additional gain on the accuracy of the parameter estimation.

Simple variance formulae depend on one variable and for linear estimators are extensively given in the literature. However, in variance estimation of complex estimators which depend on more than one variable or nonlinear estimator (e.g., ratio, regression or calibration estimator) there complex structural variance estimation methods should have to be required. Lu and Gelman (2003) develop a method for estimating the sampling variance of survey estimates with weighting adjustments. This study revealed a general equation for variance estimation of the population ratio estimator under sequential weighting through Lu and Gelman (2003) variance estimation equation.

The paper is organized as follows. In Section 2, ratio estimation in simple random sampling and combined ratio estimation in stratified sampling ratio estimation is introduced. In Section 3, an alternative combined ratio estimator which was proposed depending on Ayhan (2003)'s combined ratio estimator under sequential weighting in complex sample surveys is considered. Section 4 contains a general equation for variance estimation of the population ratio estimator in weighted data depending on Taylor-series method determination. Section 5 covers variance inflation factor in the comparison of the weighting methods. The methodology using the 2003 Turkey Demographic and Health Survey (TDHS 2003) is given in Section 6. The conclusions are summarized in Section 7.

## **2. Estimation of a two variable ratio**

Frequently, the quantity that is to be estimated from a sample design is the ratio of two variables both of which vary from unit to unit. Let  $U$  be a finite population consisting of  $N$  elements ( $u_1, u_2, \dots, u_N$ ) on which the variables  $y$  and  $x$  are defined. The values of variables ( $y, x$ ) for  $U_i$  be  $y_i, x_i, i = 1, \dots, N$ . Denoted by  $(Y, X)$  the population totals of ( $y, x$ ), respectively. The population parameter to be estimated is the two variable ratio,

$$R = \frac{\sum_{i=1}^N y_i}{\sum_{i=1}^N x_i} = \frac{Y}{X} \quad (1)$$

and corresponding sample estimate is,

$$r = \frac{\sum_{i=1}^n (y_i / \pi_i)}{\sum_{i=1}^n (x_i / \pi_i)} . \quad (2)$$

The ratio estimator  $r$  determined by Horvitz-Thompson (1952) and be accepted as a Hájek (1971) type estimator, where  $\pi_i = P(i \in s)$  defined as sample inclusion probability for unit  $i$ ,  $i = 1 \dots N$ .

There are too many reasons to take into account of the ratio estimator,  $r = y/x$ . One of them is related to a random variable not a sample size  $n$ . In addition, in many cases, sampling units are different from the basic units.

The purpose of using auxiliary variables in the estimation stage is to get better estimates. High levels of efficient estimation strategies involve extensive auxiliary information (Särndal et. al. 1992). When  $y$  and  $x$  are highly correlated, the ratio estimator provides greater reduction in the standard error and increases the accuracy of estimates. The ratio estimator is consistent but a biased estimator, this bias can be neglected. In most of the practical surveys, being a biased estimator seems substantially trivial besides yielding significant reduction of sampling error. When sample size is large enough, the ratio estimator is nearly normally distributed and the formula for its variance is valid. The results may be used if the sample size exceeds 30 (Cochran 1977).

The ratio estimation in SRS, the combined ratio estimation in stratified sampling and the proposed combined ratio estimation in complex sampling designs are presented here.

## 2.1. Ratio estimation in Simple Random Sampling

Let sampling units based on two correlated measures are  $y_i$  and  $x_i$ , which are selected from a population by simple random sample of size  $n$ . Naturally, SRS is a self-weighted sampling design, thus under a SRS design, while obtaining the ratio estimation and its variance we need to assign weights to the data. In SRS without replacement, the design weights are  $\pi_i = n/N$  for all sampling units,

$i = 1, \dots, N$ . Hence, from Equation (2) the sample ratio  $r$  which is the estimate of  $R$  is

$$r = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n x_i} = \frac{y}{x}. \quad (3)$$

$y$  and  $x$  values are random variables and differ from sample to sample. Here,  $r$  is a ratio of two random variables and is obtained from SRS design.

## 2.2. Ratio estimation in Stratified Random Sampling

When using ratio estimation for  $R$  with stratified random sampling, there are two different ways to produce estimates. One is to make a separate ratio estimate of the total of each stratum and add these totals. The second one is the combined ratio estimate that is derived from a single combined ratio. The combined ratio estimation will be taken into account. The combined ratio estimator for  $R$  can be defined as the ratio of two totals as

$$r_c = \frac{\hat{Y}_{st}}{\hat{X}_{st}} = \frac{\sum_{h=1}^H W_h y_h}{\sum_{h=1}^H W_h x_h}, \quad W_h = \frac{N_h}{n_h} \quad (4)$$

where  $\hat{Y}_{st}$  and  $\hat{X}_{st}$  are the standard estimates of the population totals  $Y$  and  $X$ ;  $y_h$  and  $x_h$  are the sample totals of the stratum  $h$  for  $Y$  and  $X$ , respectively ( $st$  for *stratified*).  $N_h$  number of units in the stratum  $h$ ,  $n_h$  sample size corresponding to the stratum  $h$  and  $W_h$  is  $h$ th stratum sample weight.

## 3. Proposed combined ratio estimator

The combined ratio estimator for  $R$  in complex sampling designs suggested by Ayhan (2003) will be continued. The weighting procedures are based on different subclasses (domains) for each type of weighting which is illustrated on Table 1. Design weights and nonresponse weights are obtained at *segregated class* levels, while post-stratification weights are based on either *cross class* or *mixed class* levels (Ayhan 2003). The table is designed to reflect different types of weights for each stage of the weighting operation, which can be considered as a combined conditional approach.

**Table 1.** Weighting layout for sequential weighting process

Design Weights for Segregated Classes			⇒	Nonresponse Weights for Segregated Classes			⇒	Post-stratification Weights for Cross/Mixed Classes	
${}_A W_1$				${}_A W_1^*$				${}_A W_1^{**}$	
	${}_A W_h$			${}_A W_h^*$			${}_A W_k^{**}$		
		${}_A W_H$			${}_A W_H^*$		${}_A W_K^{**}$		

Source: Ayhan (2003)

Table 1 illustrates the general sequential weighting process. Here,  ${}_A W_h$  design weights,  ${}_A W_h^*$   $h$ th stratum nonresponse weights and  ${}_A W_k^{**}$   $k$ th post stratum weights,  $k=1, \dots, K$ .

Design weights for non-self-weighting sample designs can be computed for each stratum  $h$  with the same probability of selection  $p_h$  for a combined ratio mean (Ayhan 1991; Verma 1991). Ayhan (2003) extend the combined ratio estimator and design weights and the design weight  ${}_A W_h$  for  $h$ th strata is,

$$\begin{aligned}
 {}_A W_h &= P_0 / P_h = \left[ \sum_{h=1}^H x_h / \sum_{h=1}^H (x_h / P_h) \right] / [(X/x) p_h] \quad (5) \\
 P_0 &= \sum_{h=1}^H x_h / \sum_{h=1}^H (x_h / P_h) \\
 P_h &= (X/x) p_h, \quad p_h = n_h / N_h
 \end{aligned}$$

Here  $P_0$  is an adjustment factor for the overall weighted and unweighted sample sizes,  $p_h$  is the  $h$ th stratum units selection probability depends on auxiliary variable  $x$ .

The combined ratio estimator depends on the design weights (5) can be written as

$${}_A r_c = \frac{\sum_{h=1}^H {}_A W_h y_h}{\sum_{h=1}^H {}_A W_h x_h} \quad (6)$$

In sequential data weighting, a weighting procedure for nonresponse is essential for self-weighting and nonself-weighting sample design outcomes.

If there are nonrespondents in the sample, the design weights have to be adjusted for nonresponse. The nonresponse weight,  ${}_A W_h^*$  for  $h$ th strata is

$${}_A W_h^* = R_0 / R_h \tag{7}$$

$$R_0 = \frac{\sum_{h=1}^H ({}_A W_h x_h)}{\sum_{h=1}^H ({}_A W_h x_h / R_h)}. \tag{8}$$

where  $R_h$  is the response rate in stratum  $h$  and  $R_0$  is the overall response rate which is used to adjust the sample sizes to be the same,  $\sum_{h=1}^H ({}_A W_h {}_A W_h^* x_h) = x$ .

The combined ratio mean estimator depends on the design weights from Equation (6) and nonresponse weights from Equation (7) will be,

$${}_A r_c = \frac{\sum_{h=1}^H {}_A W_h {}_A W_h^* y_h}{\sum_{h=1}^H {}_A W_h {}_A W_h^* x_h}. \tag{9}$$

Finally, a weighting procedure for post-stratification of a complex sampling scheme requires additional weighting procedures for independent subclasses. Post-stratification weights are given by

$${}_A W_k^{**} = X_k / X \tag{10}$$

where  $\sum_{k=1}^K {}_A W_k^{**} \sum_{h=1}^H ({}_A W_h {}_A W_h^* x_k) = x$  is the overall sample adjustment procedure (Ayhan 2003). At the last step of sequential data weighting, if design weights are adjusted for nonresponse and post-stratification in complex sampling surveys, the combined ratio estimator is computed as

$${}_A r_c = \frac{\sum_{k=1}^K {}_A W_k^{**} \sum_{h=1}^H {}_A W_h {}_A W_h^* y_{khR}}{\sum_{k=1}^K {}_A W_k^{**} \sum_{h=1}^H {}_A W_h {}_A W_h^* x_{khR}} \tag{11}$$

where  $y_{khR}$  is the  $k$ th post strata,  $h$ th stratum sample total from respondents.



#### 4. General variance estimation for the population ratio estimator

Although weighting data or sequential data weighting procedures are commonly used, it can be difficult to estimate sampling variances of associated weighted estimates. Lu and Gelman (2003) proposed a method for estimating the sampling variances of survey estimates with weighting adjustments derived from design-based analytic and Taylor-series variance estimators of population mean estimator in a general way. A natural simplifying assumption is to pretend that the weighting is all inverse-probability, with independent sampling where the probability that unit  $i$  is selected with proportional to  $\pi_i = 1/W_i$ . To compute the variance for inverse-probability weighting, a general variance estimator for  $\theta = \bar{Y}$  acknowledged as a ratio form of the weighted mean

$$\hat{\theta} = \frac{\sum_{i=1}^n W_i y_i}{\sum_{i=1}^n W_i} \quad (12)$$

where the denominator of this expression is 1, but only after the weights have normalized. The variance of  $\hat{\theta}$  is given by

$$\hat{V}_{HT}(\hat{\theta}) = \sum_{i=1}^n W_i^2 (y_i - \hat{\theta})^2 \quad (13)$$

and  $\sum^n W_i = 1$  (Lu and Gelman 2003).

Taylor-series method consists of deriving from a complex non-linear statistic, a linear statistic which has the same asymptotic variance

$$\hat{V}(\hat{\theta}) \approx \hat{V}\left(\sum_{i=1}^n W_i z_i\right) \quad (14)$$

where  $z_i$  new variable whose expression depends on  $\hat{\theta}$  and called a linearized variable for  $\hat{\theta}$ . When  $\hat{\theta} = y/x$  then  $z_i = y_i - \hat{\theta} x_i$ ,  $i = 1, \dots, n$  (Osier and Museux 2006). Mean of this variable is  $\bar{Z} = \bar{Y} - R\bar{X} = 0$ . Therefore, in a weighted sample the variance estimation of mean  $\bar{z}$  is

$$\hat{V}(\bar{z}) = \frac{1}{N^2} \sum_{i=1}^n W_i^2 [(y_i - \bar{y}) - r(x_i - \bar{x})]^2 \quad (15)$$

General variance estimation for the estimators of population ratio  $\theta = R$ , the linear relation can be expressed by  $y = bx$ . In weighted data for the variance estimation of population ratio estimator is given by Taylor-series method as using variable  $z_i = y_i - \hat{\theta}x_i$ , where  $\hat{\theta} = r = y/x$ , the variable sample mean is

$\bar{z} = \bar{y} - \hat{\theta}\bar{x}$  and population total estimate is  $N \bar{z}$ . Thereby, since  $\hat{\theta} = r$  then the variance estimation of  $r$  is

$$\hat{V}(r) = \frac{1}{X^2} \sum_{i=1}^n W_i^2 [(y_i - \bar{y}) - r(x_i - \bar{x})]^2 \quad (16)$$

A general equation for variance estimation of the population ratio estimator under stratified random sampling design depending on Taylor-series method can be introduced depending on Equation (16). A new variable defined as  $z_{hi} = y_{hi} - \hat{\theta}x_{hi}$ , where  $\hat{\theta} = r_c$ . Since this variable sample mean is  $\bar{z}_{st} = \bar{y}_{st} - \hat{\theta}\bar{x}_{st}$  and population total estimate is  $N \bar{z}_{st}$ . Here  $\bar{y}_{st} = N^{-1} \sum_{h=1}^H N_h \bar{y}_h$  and  $\bar{x}_{st} = N^{-1} \sum_{h=1}^H N_h \bar{x}_h$  are the standard estimates of the population means  $\bar{Y}$  and  $\bar{X}$ , respectively, made from a stratified sample. Mean of the new variable is  $\bar{Z} = \bar{Y} - R\bar{X} = 0$ . Thereby, under stratified random sampling design the variance estimation of  $\bar{z}_{st}$ ,

$$\hat{V}(\bar{z}_{st}) = \frac{1}{N^2} \sum_{h=1}^H W_h^2 s_{zh}^2, \quad W_h = \frac{N_h}{n_h} \quad (17)$$

$$\hat{V}(\bar{z}_{st}) = \frac{1}{N^2} \sum_{h=1}^H W_h^2 \sum_{i=1}^{n_h} [(y_{hi} - \bar{y}_h) - r_c(x_{hi} - \bar{x}_h)]^2 \quad (18)$$

Where  $n_h$  is the sample size of stratum  $h$ ,  $y_{hi}$  is the  $i$ th value of variable  $y$  in stratum  $h$ ,  $x_{hi}$  is the  $i$ th value of variable  $x$  in stratum  $h$  (here  $(1 - f_h)$  are neglected where  $f_h$  is the sampling fraction for  $h$ th stratum). Therefore, defining  $z_i = y_i - \hat{\theta}x_i$  and  $\hat{\theta} = r_c$ , the variance estimation can be obtained as

$$\hat{V}(r_{st}) = \frac{1}{X^2} \sum_{h=1}^H W_h^2 \sum_{i=1}^{n_h} [(y_{hi} - \bar{y}_h) - r_c(x_{hi} - \bar{x}_h)]^2 \quad (19)$$

This general variance estimation formulation for the stratified sampling design can also be extended to be the basis for the other complex sampling designs.

## 5. Variance inflation factor in the comparison of the weighting methods

The variability of weights increases, thereby the accuracy of estimates decreases. A useful measure of the accuracy of this loss is the variance inflation factor (*VIF*). *VIF* which is adopted to be the variability measure of weights can be used for comparing the weights and weighting methods. The measure *VIF*

represents the multiplying factor that is applied to the variance of a survey estimate due to the variability in the weights where equal weights are optimal (Kalton and Cervantes 2003; Kish 1992). Even though the use of the weights in the analysis of survey data tends to reduce the bias in the estimates, it could also inflate the variances of such estimates. The effect of using weights in the estimation of the population parameters can be defined by the *VIF*,

$$VIF = n \frac{\sum_{i=1}^n W_i^2}{\left(\sum_{i=1}^n W_i\right)^2} = 1 + CV^2(W_i) \quad (20)$$

$W_i$  is the  $i$  th sampling unit weight and  $CV^2(W_i)$  indicates the relative loss is defined as the coefficient of variation of weights (Kish 1992).

## 6. Application of the methodology

In this section we demonstrate the proposed methodology and study the efficiency of the combined ratio estimators by using data from 2003 Turkey Demographic and Health Survey (TDHS-2003). In the selection of the TDHS-2003 sample, weighted multi-stage stratified cluster sampling approach was used. Here, under stratified sampling design, the combined ratio estimator and proposed combined ratio estimator will be used for the estimation of population ratio.

### 6.1. Survey design

TDHS-2003 is the eighth Turkish national survey carried out by the Institute of Population Studies in Turkey. The major objective of the TDHS-2003 survey was to ensure that the survey would provide estimates with acceptable precision for the domains for most of the important demographic characteristics, such as fertility, infant and child mortality, and contraceptive prevalence, as well as for the health indicators. In TDHS-2003 to represent Turkey nationally and at the urban-rural and regional levels interviews were carried out with 8075 ever-married women in 10836 households. The sample design and sample size of the TDHS-2003 provides to perform analyses for Turkey as a whole, for urban and rural areas and for the five demographic regions of the country (West, South, Central, North and East). The sample of the research also allows for the analysis of 12 geographical regions (NUTS 1), which was established within the second half of the year 2002 within the context of Turkey's move to join the European Union. Among these 12 regions, İstanbul and the Southeastern Anatolian Project regions (due to their special situations) were oversampled. Thereby, settlements are divided into 40 strata,  $H=40$ .

From the 2000 Turkish General Population Census the population size for the ever-married women is  $N = 12630510$ . In the TDHS-2003 the eligible women

were identified as 8477 of whom 96 percent were interviewed and so interviews were carried out with 8075 ever-married women.

### 6.2. Two variable ratio estimation

One of the objectives of this paper is to measure a population ratio. Using data from the TDHS-2003, we have decided to examine the ratio of the number of live births to the number of living children of ever-married women. Therefore,  $y$ , indicates the number of living children and  $x$ , indicates the number of live births.  $R = Y / X = \text{Number of living children} / \text{Number of live births}$  will be estimated. In the estimation of  $R$  the known combined ratio estimator  $r_c$  and Ayhan (2003)'s proposed combined ratio estimator  ${}_A r_c$  are used and the comparison of the  $r_c$  and  ${}_A r_c$  estimates are illustrated in the following sections.

The initial information on all places of residences in Turkey was derived from the year 2000 Turkish General Population Census results which provided a computerized list of all settlements (provincial and district, sub-districts and villages), their populations and the households. From 2000 Turkish General Population Census results, the true population ratio for the ever-married women is  $R = Y / X = 30398682/32713021 = 0.929253$  and this means in Turkey nearly 93% of the live birth children are still living. Eligible women design weights,  $W_h$  strata design weights, response rates, respondent sample sizes,  $W'_h$   $h$ th strata adjusted design weights and final design weights by strata information are presented in Table 2.

**Table 2.** Eligible women design weights and response rates, respondent sample sizes, adjusted design weights and final design weights by strata, Turkey 2003

Strata	Inverse of sampling fraction $W_h$	Household level $1/r_h^{HH}$	Women level $1/r_h^{WOMEN}$	$n_{hr}$	Women adjusted design weights in entire sample $W'_h \cdot (N/N_r)$	Women standardized weight in entire sample $(W'_h)^s$	Women weight in entire sample $(W'_h)^s$ x1000000
1	1160555/960	891/779	672/630	630	1708.8	1.076474	1076474
2	1587651/60	870/682	478/449	449	2602.12	1.659981	1659981
3	24989/100	68/63	52/50	50	324.996	0.272980	272980
4	76858/60	46/46	35/34	34	1209.74	0.962433	962433
5	469931/500	410/391	285/269	269	2455.64	0.802196	802196
6	362247/240	220/218	119/115	115	5688.92	1.150401	1150401
7	685892/400	348/300	195/183	183	1680.23	1.546953	1546953
8	686133/150	144/137	96/94	94	1333.83	3.583791	3583791

**Table 2.** Eligible women design weights and response rates, respondent sample sizes, adjusted design weights and final design weights by strata, Turkey 2003 (cont.)

Strata	Inverse of sampling fraction $W_h$	Household level $1/r_h^{HH}$	Women level $1/r_h^{WOMEN}$	$n_{hr}$	Women adjusted design weights in entire sample $W'_h.(N/N_r)$	Women standardized weight in entire sample $(W'_h)^s$	Women weight in entire sample $(W'_h)^s$ x1000000
9	667273/240	211/204	139/135	135	1655.51	2.305124	2305124
10	202772/150	129/127	94/89	89	5173.43	1.058475	1058475
11	211704/60	48/47	50/48	48	2235.38	2.739621	2739621
12	352876/400	348/300	225/200	200	598.79	0.840259	840259
13	129118/100	83/75	46/46	46	2734.71	1.042909	1042909
14	109307/60	33/33	27/26	26	3442.6	1.841054	1841054
15	377921/100	90/86	70/62	62	846.61	3.259059	3259059
16	148605/60	56/56	39/38	38	1587.51	1.855263	1855263
17	182284/100	86/85	68/65	65	1305.67	1.408203	1408203
18	65446/60	45/45	21/21	21	867.53	0.796109	796109
19	47999/100	80/77	57/55	55	1349.47	0.377212	377212
20	83237/60	55/55	44/43	43	641.78	1.036076	1036076
21	915073/500	451/386	287/260	260	513.44	1.722755	1722755
22	431779/150	128/124	99/99	99	945.73	2.168697	2168697
23	298404/240	173/172	116/107	107	946.82	1.130884	1130884
24	276431/400	361/349	276/270	270	1527.76	0.533328	533328
25	1052242/900	808/734	593/557	557	1826.15	1.028638	1028638
26	681896/540	470/446	302/286	286	3430.47	1.085906	1085906
27	523267/500	457/438	354/343	343	4348.88	0.822517	822517
28	373756/240	210/205	162/159	159	2191.87	1.186317	1186317
29	336258/500	427/395	275/267	267	2945.04	0.546506	546506
30	318422/240	207/204	156/153	153	1263.75	1.001856	1001856
31	224473/200	180/176	138/136	136	1644.66	0.850111	850111
32	201222/90	82/82	60/59	59	1570.77	1.659488	1659488
33	310851/600	497/474	362/355	355	1628.01	0.404297	404297
34	349165/240	203/199	136/126	126	1883.16	1.169152	1169152
35	212359/500	462/452	392/384	384	1590.35	0.323444	323444
36	218260/240	200/199	158/151	151	2634.27	0.797725	797725
37	371366/500	478/449	383/371	371	1855.92	0.595771	595771
38	257644/240	227/220	208/195	195	1108.02	0.862345	862345
39	756933/1000	922/877	762/742	742	1368.89	0.596458	596458
40	356146 / 480	455 / 449	416 / 403	403	899.22	0.566475	566475

Source: TDHS 2003

The nonresponse adjustments for the sampling weights  $W_h$  are conducted at each strata,  $h = 1, \dots, H$ .

The adjusted nonresponse weights  $W_h(1/r_h^{HH})(1/r_h^{WOMEN})$  are defined by multiplying sampling weights by the inverse of household and women level response ratios. However, to provide equality of the adjusted sampling weights total to the population total, the adjusted sampling weights  $W_h(1/r_h^{HH})(1/r_h^{WOMEN})$  are multiplied with the value of,

$$N / \left\{ \sum_{h=1}^H \sum_{i=1}^{nhR} W_h(1/r_h^{HH})(1/r_h^{WOMEN}) \right\} = 12630510/10901679 = 1.158584.$$

Thus, the adjusted sampling weights are presented as  $W'_h(N/N_r)$  in Table 2. For example, the calculation for the adjusted value  $W'_h = 1474.9$  from Table 3 is as,

$$W'_h(N/N_h) = (1160555/960)(891/779)(672/630)1.158584 = 1474.9(1.158584) = 1708.799. \text{ Hence, } W'_h \text{ used for design weights } W_h.$$

**Table 3.** Unit variances of strata

Strata	$W'_h$	$S_{yh}^2$	$S_{xh}^2$	$S_{yhx}$	Strata	$W'_h$	$S_{yh}^2$	$S_{xh}^2$	$S_{yhx}$
1	280.51	3.157	5.763	1.88	21	1405.17	3.147	4.130	1.89
2	443.16	4.064	6.010	2.22	22	1419.55	1.510	1.867	1.32
3	516.83	1.758	2.628	1.38	23	1428.91	0.757	0.973	0.83
4	553.94	1.762	2.119	1.22	24	1450.24	1.181	1.316	1.33
5	730.72	3.243	4.145	1.9	25	1474.9	1.952	2.820	2.37
6	748.78	1.330	1.719	1.24	26	1576.19	1.051	1.384	1.20
7	776.14	9.318	12.35	1.21	27	1601.88	2.861	3.538	3.03
8	816.28	4.600	6.186	3.3	28	1625.4	3.151	4.250	1.76
9	817.22	5.255	7.000	2.2	29	1891.85	1.421	2.106	1.32
10	956.36	6.562	9.490	2.5	30	1929.41	1.028	1.835	1.23
11	1044.15	1.340	1.590	1.18	31	2119.52	1.080	1.467	1.25
12	1090.77	2.747	2.857	1.67	32	2245.95	1.713	2.475	1.44
13	1126.95	2.248	2.935	1.55	33	2273.7	1.874	3.713	1.66
14	1151.26	1.480	1.949	1.28	34	2360.39	1.282	1.653	1.19
15	1164.75	1.837	2.978	1.67	35	2541.93	2.691	3.078	1.7
16	1181.52	7.153	10.17	3.03	36	2960.91	1.595	2.120	1.33
17	1318.64	0.952	0.941	1.97	37	2971.38	1.869	2.272	1.44
18	1355.77	2.294	3.509	1.76	38	3753.62	2.056	3.400	1.63
19	1370.21	2.966	3.833	1.88	39	4465.31	3.188	4.027	1.89
20	1372.67	2.589	3.367	1.71	40	4910.24	1.341	1.737	1.21

### 6.3. Combined ratio estimator

Combined ratio estimator for  $R$  is

$$r_c = \frac{\sum_{h=1}^H W'_h y_h}{\sum_{h=1}^H W'_h x_h} = \frac{30466444}{33214885} = 0.917253.$$

This means that, the ever-married women, 91.7% of live born children are estimated to have lived. A general variance estimation proposed for the population ratio which is given by Equation (19) can be written as,

$$\hat{V}(r_c) = \frac{1}{X^2} \sum_{h=1}^H W_h^2 \frac{1}{n_h} (s_{yh}^2 - 2r_c s_{yhx} + r_c^2 s_{xh}^2). \quad (21)$$

The variance estimation of combined ratio estimator (conventional combined ratio estimator) depending on  $W'_h$  adjusted weights is

$$\hat{V}(r_c) = \frac{1}{X^2} \sum_{h=1}^H W_h'^2 \frac{1}{n_h} (s_{yh}^2 - 2r_c s_{yhx} + r_c^2 s_{xh}^2)$$

$$\hat{V}(r_c) = (3.22) 10^{-9}.$$

$s_{yh}^2$ ,  $s_{yhx}$ ,  $s_{xh}^2$  computed unit variance values of the strata are given in Table 3. There is an increase in the variance of the ratio estimate due to the use of design weights  $W'_h$ , and so that the  $VIF$  value is obtained as:

$$VIF(W'_h) = \left[ \frac{H \left( \sum_{h=1}^H (W'_h)^2 \right)}{\left( \sum_{h=1}^H W'_h \right)^2} \right] = 1.387289$$

For a  $VIF \approx 1.387$ , i.e., a reduction in the effective sample size of almost 38.7 percent.

### 6.4. Proposed combined ratio estimator

The estimator was proposed under sequential weighting process and so on the design weights are adjusted for nonresponse and post-stratification in TDHS-2003. First step is to obtain design weights  ${}_A W_h$ . Second step is to compute nonresponse weights  ${}_A W_h^*$ . The final step is weighting for post-stratification that is conducted by  ${}_A W_k^{**}$ . Here,  $h=1, \dots, H$ ,  $H=40$ . The  ${}_A W_h$  weight results are

presented in Table 4, calculation of  $R_h$  and  $R_0$  results are presented in Table 5.  ${}_A W_h^*$  weight results are presented in Table 6.  ${}_A W_k^{**}$  weight results are presented in Table 7.

Design weights:

We will start with obtaining  ${}_A W_h$  design weights. The adjustment factor  $P_0$  is

$$P_0 = \sum_{h=1}^H x_h / \sum_{h=1}^H (x_h / P_h) = 22443.5 / 0.013092223 = 1714262.026$$

for the overall weighted and unweighted sample sizes is to be the same, where  $p_h$ ,  $h$ th stratum units selection probability to the sample. The values  $X = 32713021$ ,  $x = 22443$ ,  $X/x = 1457.605$  and  $P_h$  are then computed as below given in Table 4.

**Table 4.**  $P_h$ ,  $x_h / P_h$  and combined design weights  ${}_A W_h$

Strata	Women adjusted design weights $W'_h.(N/N_r)$	$x_h$	$P_h = (X/x)p_h$	$x_h / P_h$	${}_A W_h = P_0 / P_h$
1	1708.8	1442.70	2490699.324	0.000579	0.688
2	2602.12	969.84	3792777.695	0.000255	0.452
3	324.996	123.00	473705.116	0.000259	3.618
4	1209.74	508.41	1763283.357	0.000288	0.972
5	2455.64	362.34	3579272.524	0.000101	0.478
6	5688.92	186.12	8292011.470	0.000022	0.206
7	1680.23	197.58	2449056.487	0.000080	0.699
8	1333.83	448.00	1944153.488	0.000234	0.881
9	1655.51	102.12	2413025.303	0.000042	0.710
10	5173.43	135.16	7540647.592	0.000017	0.227
11	2235.38	135.20	3258227.678	0.000041	0.526
12	598.79	125.95	872779.639	0.000144	1.964
13	2734.71	590.20	3986037.189	0.000148	0.430
14	3442.6	205.92	5017837.953	0.000041	0.341
15	846.61	650.70	1233995.175	0.005270	1.389
16	1587.51	1425.92	2313910.396	0.000616	0.740
17	1305.67	840.35	1903108.255	0.000441	0.900
18	867.53	787.65	1264487.585	0.000622	1.355
19	1349.47	371.28	1966949.916	0.000188	0.871
20	641.78	930.10	935440.667	0.000994	1.832



**Table 4.**  $P_h$ ,  $x_h / P_h$  and combined design weights  ${}_A W_h$  (cont.)

Strata	Women adjusted design weights $W'_h(N/N_r)$	$x_h$	$P_h = (X/x)p_h$	$x_h / P_h$	${}_A W_h = P_0 / P_h$
21	513.44	1025.28	748375.855	0.001370	2.290
22	945.73	953.47	1378469.728	0.000691	1.243
23	946.82	2144.38	1380058.482	0.001553	1.242
24	1527.76	78.88	2226820.459	0.000035	0.769
25	1826.15	354.20	2661745.418	0.000133	0.644
26	3430.47	449.55	5000157.602	0.000089	0.342
27	4348.88	171.84	6338806.459	0.000027	0.270
28	2191.87	85.02	3194808.713	0.000026	0.536
29	2945.04	152.00	4292608.344	0.000035	0.399
30	1263.75	68.04	1842006.830	0.000036	0.930
31	1644.66	107.07	2397210.645	0.000044	0.715
32	1570.77	254.66	2289510.638	0.000112	0.748
33	1628.01	1092.52	2372942.069	0.000460	0.722
34	1883.16	605.79	2744841.608	0.000220	0.624
35	1590.35	602.82	2318049.901	0.000260	0.739
36	2634.27	207.09	3839638.641	0.000053	0.446
37	1855.92	375.48	2705137.342	0.000138	0.633
38	1108.02	418.27	1615019.116	0.000258	1.061
39	1368.89	783.90	1995255.968	0.000392	0.859
40	899.22	1974.70	1310678.047	0.001506	1.307
Total		22443.50		0.013092	

Nonresponse weights:

In TDHS–2003, there are also non-respondent women in the survey. A weighting procedure for nonresponse is essential so we should adjust the design weights by assigning nonresponse weights to the data. Table 5 presents the calculation of the response rates  $R_h$ .

**Table 5.** Calculation of  $R_h$  and the Equation  $R_0$

Strata	$R_h$	$n_{hR}$	${}_A W_h x_h$	${}_A W_h x_h / R_h$	Strata	$R_h$	$n_{hR}$	${}_A W_h x_h$	${}_A W_h x_h / R_h$
1	0.82	630	2465286.0	3007711.90	21	0.775	260	526419.8	678937.77
2	0.73	449	2523640.0	3427234.70	22	0.969	99	901725.2	930813.09
3	0.89	50	39974.5	44872.97	23	0.917	107	2030342.0	2213915.50
4	0.97	34	615043.9	633133.44	24	0.946	270	120509.7	127423.38
5	0.90	269	889776.6	988509.07	25	0.853	557	646822.3	758053.41
6	0.95	115	1058822.0	1105702.20	26	0.899	286	1542168.0	1716072.10
7	0.80	183	331979.8	410348.86	27	0.929	343	747311.5	804735.05
8	0.93	94	597555.8	641451.46	28	0.958	159	186352.8	194499.83

**Table 5.** Calculation of  $R_h$  and the Equation  $R_0$  (cont.)

Strata	$R_h$	$n_{hR}$	${}_A W_h x_h$	${}_A W_h x_h / R_h$	Strata	$R_h$	$n_{hR}$	${}_A W_h x_h$	${}_A W_h x_h / R_h$
9	0.93	135	169060.7	180042.87	29	0.898	267	447646.1	498410.29
10	0.93	89	699240.8	750154.29	30	0.967	153	85985.5	88960.83
11	0.94	48	302223.4	321514.23	31	0.964	136	176093.7	182744.35
12	0.76	200	75417.6	98419.97	32	0.983	59	400012.3	406792.16
13	0.90	46	1614026.0	1786188.60	33	0.935	355	1778633.0	1901711.90
14	0.96	26	708900.2	736165.58	34	0.908	126	1140799.0	1256089.70
15	0.84	62	550889.1	650900.51	35	0.958	384	958694.8	1000319.50
16	0.97	38	2263662.0	2323232.30	36	0.951	151	545531.0	573688.93
17	0.94	65	1097220.0	1161364.90	37	0.910	371	696860.8	765865.46
18	1.00	21	683310.0	683310.00	38	0.909	195	463451.5	510077.56
19	0.92	55	501031.2	539481.08	39	0.926	742	1073073.0	1158541.50
20	0.97	43	596919.6	610801.43	40	0.956	403	1775690.0	1857464.10
Total							8075	34028101	37725657

From Equation (8)

$$R_0 = \frac{34028101}{37725657} = 0.901988.$$

Further, using  $R_h$  and  $R_0$  response rate values, the combined weights for nonresponse  ${}_A W_h^*$  from Equation (7) are obtained and given in Table 6.

**Table 6.**  ${}_A W_h^*$  combined weights for nonresponse

Strata	${}_A W_h^*$	${}_A W_h {}_A W_h^*$	Strata	${}_A W_h^* = R_0 / R_h$	${}_A W_h {}_A W_h^*$
1	1.1004	1880.447	21	1.1633	597.2943
2	1.2249	3187.459	22	0.9310	880.5547
3	1.0125	329.0642	23	0.9835	931.2369
4	0.9285	1123.265	24	0.9537	1457.079
5	1.0020	2460.737	25	1.0570	1930.422
6	0.9419	5358.535	26	1.0037	3443.17
7	1.1149	1873.316	27	0.9712	4224.055
8	0.9682	1291.477	28	0.9414	2063.474
9	0.9605	1590.252	29	1.0042	2957.633
10	0.9676	5006.144	30	0.9331	1179.33
11	0.9595	2144.986	31	0.9360	1539.491
12	1.1770	704.8326	32	0.9172	1440.83

**Table 6.**  ${}_A W_h^*$  combined weights for nonresponse (cont.)

Strata	${}_A W_h^*$	${}_A W_h {}_A W_h^*$	Strata	${}_A W_h^* = R_0 / R_h$	${}_A W_h {}_A W_h^*$
13	0.9982	2729.789	33	0.9644	1570.06
14	0.9366	3224.615	34	0.9931	1870.249
15	1.0657	902.2662	35	0.9411	1496.759
16	0.9257	1469.597	36	0.9485	2498.724
17	0.9547	1246.549	37	0.9913	1839.783
18	0.9019	782.5019	38	0.9927	1099.969
19	0.9712	1310.616	39	0.9738	1333.067
20	0.9229	592.3403	40	0.9435	848.4382

Weighting for post-stratification:

In TDHS-2003 survey the age group auxiliary variable  $x_2$  is used for post-stratification. The data separated into  $k=7$  age groups (post strata,  $k=1, \dots, 7$ ). Post-stratification weights  ${}_A W_k^{**}$  were defined by Equation (10) and the ratio estimator can be obtained by Equation (11). The components  $\sum_{h=1}^H {}_A W_h {}_A W_h^* y_{hkR}$ ,  $\sum_{h=1}^H {}_A W_h {}_A W_h^* x_{hkR}$  have been computed and presented in Table 7.

**Table 7.**  $n_{kR}$  and  $N_k$  distribution,  $\sum_{h=1}^H {}_A W_h {}_A W_h^* y_{hkR}$  and  $\sum_{h=1}^H {}_A W_h {}_A W_h^* x_{hkR}$  weights by age groups

Post strata	$x_2$ : Age group	$N_k$	$n_{kR}$	$\sum_{h=1}^H {}_A W_h {}_A W_h^* y_{hkR}$	$\sum_{h=1}^H {}_A W_h {}_A W_h^* x_{hkR}$
1	15–19	453511	240	180.5941	194.5517
2	20–24	1727365	1080	1749.078	1842.8710
3	25–29	2378665	1516	4214.355	4002.4680
4	30–34	2244391	1506	5734.090	5362.3310
5	35–39	2282957	1410	6394.855	5852.9540
6	40–44	1922351	1297	6846.528	6141.9070
7	45–49	1621270	1026	5581.468	4919.2090

In the estimation of  $R$ , the population total of the number of live births for the post-stratified sample by age groups must be known. From the 2000 General Census of Population  $X_k$ , the  $k$ th post-stratified population totals are obtained. The population totals and the post-stratification weights  ${}_A W_k^{**} = X_k / X$  are presented in Table 8.

**Table 8.**  $X_k$  population totals and  ${}_A W_k^{**}$  post-strata weights

$x_2$ :Age group	$X_k$	${}_A W_k^{**} = \frac{{}_A W_k^{**}}{X_k / X}$	$W_k^{**} \sum_{h=1}^H {}_A W_h {}_A W_h^* y_{khR}$	$W_k^{**} \sum_{h=1}^H {}_A W_h {}_A W_h^* x_{khR}$
15–19	294628	0.0359	7883.0720	8452.682
20–24	2078364	0.1368	263369.562	275931.954
25–29	4522719	0.1883	848192.363	886883.534
30–34	5700038	0.1777	1033509.385	1099511.889
35–39	7036619	0.1807	1173253.904	1279513.638
40–44	6707033	0.1522	1011205.470	1126042.687
45–49	6394157	0.1284	717786.487	820766.657
Total	32733558	-	5055200.244	5497103.043

$${}_A r_c = \frac{\sum_{k=1}^K {}_A W_k^{**} \sum_{h=1}^H {}_A W_h {}_A W_h^* y_{khR}}{\sum_{k=1}^K {}_A W_k^{**} \sum_{h=1}^H {}_A W_h {}_A W_h^* x_{khR}} = \frac{5055200.244}{5497103.43} = 0.919612$$

We can state that, 91.9% of live born children is estimated to have lived. The post-stratification weights and related unit variances are computed and presented on Table 9.

**Table 9.** Post-stratification weights and unit variances

$x_2$ : Age group	$W_k^{**} {}_A W_h {}_A W_h^*$	$s_{yk}^2$	$s_{xk}^2$	$s_{y x k}$
15–19	14040.19763	0.50	0.25	1.32
20–24	226920.2164	0.92	0.84	1.70
25–29	448066.179	1.63	2.65	2.11
30–34	415966.1629	1.44	2.09	1.33
35–39	399255.382	3.36	11.33	6.70
40–44	303439.722	1.50	2.25	1.80
45–49	212837.3363	1.39	1.95	2.70

The variance estimation given by Equation (19) can be defined as below for  ${}_A r_c$ :

$$\hat{V}({}_A r_c) = \frac{1}{X^2} \sum_{k=1}^K W_k^2 \frac{1}{n_h} (s_{yk}^2 - 2r_c s_{y x k} + r_c^2 s_{xk}^2) \quad (22)$$

The variance estimation value is

$$\hat{V}({}_A r_c) = (2.9) 10^{-9}$$

where  $W_k = W_k^{**} W_{hA} W_h^*$  and  $s_{yk}^2$   $s_{xk}^2$  unit variances of  $k$  th poststrata for  $y$  and  $x$ , respectively and  $s_{yjk}$  is covariance of  $k$  th poststrata for  $y$  and  $x$ . Inflation factor for  $W_k$  obtained as  $VIF(W_k) = 1.238408$ . There is nearly 10% reduction in the VIF value of proposed ratio estimator relative to conventional combined ratio estimator. VIF is reduced from 1.387 with conventional combined ratio estimator to 1.238 with proposed ratio estimator.

The comparison of the conventional combined ratio estimator and the proposed combined ratio estimator results of means, variance estimations and  $VIF$  are given on Table 10. In Table 10, we observe the values of mean, variance estimation and  $VIF$  of the combined ratio estimator and the proposed combined ratio estimator. From Table 10, it can be concluded that the proposed combined ratio estimator has the minimum variance estimation but it is seen that both have approximate variance estimation values. The variability level of weights according  $VIF$  values  ${}_A r_c$  seems as less variable than  $r_c$ .

**Table 10.** The comparisons of combined ratio estimator results

	Mean	$\hat{V}_{HT}(\hat{\theta})$	$VIF$
Conventional combined ratio estimator	$r_c = 0.917$	$(3.2) 10^{-9}$	$VIF(W_h) = 1.387289$
Ayhan (2003)'s combined ratio estimator	${}_A r_c = 0.919$	$(2.9) 10^{-9}$	$VIF(W_k^{**} W_{hA} W_h^*) = 1.238408$

## 7. Conclusions

Researchers believe that, the weights that provide excellent estimates for auxiliary variables will also provide good estimates for the interest variable. The new weights will continue to give unbiased estimates, but a realistic expectation is to remain near unbiasedness (Deville and Särndal 1992). Using the data weighted according to the auxiliary variable(s) which are known to be related to the interest variable lead to additional gains in the information. The weights in the combined ratio estimator  ${}_A r_c$  are defined on the basis of population and sample sizes and also information on the auxiliary variable. TDHS-2003 results have shown that, the combined ratio estimator which is defined by Ayhan (2003) provided a better

estimate of the parameter, by using auxiliary variable values in the calculation of weights. The proposed estimator has lower variance; it is not enough to prove that it is more efficient. The variance could be underestimated. We can say that, the estimator better reflects the effect of post-stratification.

## REFERENCES

- ARDILLY, P., TILLE, Y., (2006). *Sampling methods: exercises and solutions*. Translated from French by Leon Jang. Springer Science+Business Media, USA.
- AYHAN, H. Ö., (1981). Sources and bias of nonresponse in the Turkish Fertility Survey 1978, *Turkish Journal of Population Studies* 2–3, pp. 104–148.
- AYHAN, H. Ö., (1991). Post-stratification and weighting in sample surveys. Invited paper, Research Symposium '91, State Institute of Statistics, Ankara.
- AYHAN, H. Ö., (2003). Combined weighting procedures for post-survey adjustment in complex sample surveys. *Bulletin of the International Statistical Institute*, 60 (1), pp. 53–54.
- BETHLEHEM, J. G., KERSTEN, H. M. P., (1985). On the treatment of nonresponse in sample surveys. *Journal of Official Statistics*, 1 (3), pp. 287–300.
- BETHLEHEM, J. G., KELLER, W. J., (1987). Linear weighting of the sample survey data. *Journal of Official Statistics*, 3 (2), pp. 141–153.
- CASSEL, C. M., SÄRNDAL, C. E., WRETMAN J. H., (1976). Some results on generalized difference estimation and generalized regression estimation for finite populations. *Biometrika*, 63, pp. 615–620.
- CERVANTES, I. F., BRICK, J. F., (2009). Efficacy of Poststratification in Complex Sample Design. *Proceedings of the Survey Research Methods Section, American Statistical Association*, pp. 4642–4655.
- COCHRAN, W. G., (1977). *Sampling Techniques*. 3rd ed. Wiley, New York.
- DEMING, W. E., STEPHAN, F. F., (1940). On a least squares adjustment of a sample frequency table when the expected marginal totals are known. *Annals of Mathematical Statistics*, 11 (4), pp. 427–444.
- DEVILLE, J. C., SARNDAL, C. E., (1992). Calibration estimators in survey sampling. *Journal of the American Statistical Association*, 87, pp. 376–382.
- DEVILLE, J. C., SARNDAL, C. E., SAUTORY, O., (1993). Generalized raking procedure in survey sampling. *Journal of the American Statistical Association*, 88, pp. 1013–1020.
- ESTEVAO, V. M., SÄRNDAL, C. E., (2000). A functional form approach to calibration. *Journal of Official Statistics*, 16 (4), pp. 379–399.
- GUY, P. W., (1979). *Small sample theory for poststratification*, Doctor of Philosophy, Graduate College of Texas ASM University, Texas.

- HAJEK, J., (1971). Comment on a paper of D. Basu, In *Foundations of Statistical Inference*, Toronto, pp. 236–237.
- HOLT, D., ELLIOT, D., (1991). Methods of weighting for unit non-response. *The Statistician*, 40, pp. 333–342.
- HOLT, D., SMITH, T. M. F., (1979). Post stratification. *Journal of the Royal Statistical Society, Series A (General)*, 142 (1), pp. 33–46.
- HORVITZ, D. G., THOMPSON, D. J., (1952). A generalization of sampling without replacement from a finite universe. *Journal of the American Statistical Association*, 47, pp. 663–685.
- ISAKI, C. T., FULLER, W. A., (1982). Survey design under the regression superpopulation model. *Journal of the American Statistical Association*, 77, pp. 89–96.
- KALTON, G., CERVANTES, F. I., (2003). Weighting methods. *Journal of Official Statistics*, 19 (2), pp. 81–97.
- KISH, L., (1965). *Survey Sampling*. John Wiley & Sons, Inc., USA.
- KISH, L., (1992). Weighting for unequal Pi. *Journal of Official Statistics*, 8 (2), pp. 183–200.
- KOTT, P. S., (2006). Using calibration weighting to adjust for nonresponse and coverage errors. *Survey Methodology*, 32, pp. 133–142.
- LITTLE, R. J. A., (1993). Post-stratification: A modeler's perspective, *Journal of The American Statistical Association*, 88, pp. 1001–1012.
- LITTLE, R. J. A., VARTIVARIAN, S., (2005). Does weighting for nonresponse increase the variance of survey means? *Proceedings of American Statistical Association: Section on Survey Research Methods*, Minneapolis, pp. 3897–3904.
- LU H., GELMAN, A., (2003). A method for estimating design based sampling variances for surveys with weighting, post-stratification, and raking. *Journal of Official Statistics*, 19(2), pp. 133–151.
- OSIER, G., MUSEUX, J. M., (2006). Variance estimation for EU–SILC complex poverty indicators using linearization techniques. *European Conference on Quality in Survey Statistics*, Luxembourg, pp. 1–11.
- SÄRNDAL, C. E., (1980). On  $\pi$ -inverse weighting versus best linear unbiased weighting in probability sampling, *Biometrika*, 67, pp. 639–650.
- SÄRNDAL, C. E., SWENSON, B., WRETMAN, J., (1992). *Model assisted survey sampling*. Springer–Verlag, New York.
- SÄRNDAL, C. E., (2007). The calibration approach in survey theory and practice. *Survey Methodology, Statistics Canada*, 33 (2), pp. 99–119.
- SINGH, S., (2003). *Advanced sampling theory with applications*. Kluwer Academic Publishers, Dordrecht Boston, London.
- SMITH, T. M. F., (1991). Post–stratification. *The Statistician*, 40, pp. 315–321.

- STEPHAN, F. F., (1942). An iterative method of adjusting sample frequency tables when expected marginal totals are known. *Annals of Mathematical Statistics*, 13 (2), pp. 166–178.
- TDHS, (2004). Turkey Demographic and Health Survey 2003. Hacettepe University, Institute of Population Studies, Ankara, Turkey.
- TIKKIWAL, G. C., RAI, P. K., GHIYA, A., (2013). On the Performance of Generalized Regression Estimator for Small Domains, *Communication in Statistics: Simulation and Computation*, 42, pp. 891–909.
- VERMA, V., (1991). Sampling Methods. Manual for Statistical Trainers Number 2, Statistical Institute for Asia and the Pacific, Tokyo, Japan.
- VERMA, V., (2007). Recent advances in survey sampling. In: Ayhan, H.Ö. and Batmaz, I. (eds.), *Recent Advances in Statistics*. Turkish Statistical Institute Press, Ankara, Turkey, pp. 77–101.
- WRIGHT, R. L., (1983). Finite population sampling with multivariate auxiliary information. *Journal of the American Statistical Association*, 78, pp. 879–884.
- WU, C., SITTEK, R. R., (2001). A model-calibration approach to using complete auxiliary information from survey data. *Journal of the American Statistical Association*, 96, pp. 185–193.
- WU, C., (2003). Optimal calibration estimators in survey sampling. *Biometrika*, 90(4), pp. 937–951.