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Numerical simulation of stochastic two-point reactor kinetics equations for reflected reactors

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Abstract

Deterministic numerical solutions of point reactor kinetic equations give us the mean values of the neutron population and delayed neutron precursor concentrations, whereas the actual dynamical process is stochastic. The neutron population and precursor concentrations fluctuate randomly with time. In the present study, a novel stochastic model for two-point reactor kinetics equations is developed and used to analyze the dynamical behavior of the source-free strongly reflected reactors with six groups of delayed neutron precursors. To derive the Itô stochastic differential equations system corresponding to this model, the two-point reactor kinetics equations are separated into three terms: prompt neutrons, delayed neutrons, and reflected neutrons. In the case of different perturbation scenarios, both with and without the Newtonian temperature reactivity feedback effects, this system of stochastic differential equations is solved using the Euler-Murayama numerical method. It is observed that the mean response of the system is comparable with the results of other deterministic numerical methods.

Keywords: Reflected reactor, Euler-Murayama Method, Stochastic models, Feedback effect

Yansıtıcılı reaktörlerin stokastik iki-nokta reaktör kinetik denklemlerinin sayısal simülasyonu

Öz

Nokta reaktör kinetik denklemlerinin sayısal çözümleri bize nötron popülasyonu ve gecikmiş nötron üreteçleri yoğunluklarının ortalama değerlerini vermektedir. Gerçek dinamik süreç stokastik bir süreç olduğu için, nötron popülasyonu ve üreteç yoğunlukları zamanla rastgele dalgalanmaktadır. Bu çalışmada, harici nötron kaynağı olmayan ve altı grup gecikmiş nötron üreteci olan güçlü yansıtıcılı reaktörlerin dinamik davranışını analiz etmek amacıyla iki-nokta reaktör kinetik denklemleri için yeni bir stokastik model geliştirilmiştir. Bu modele karşılık gelen Itô stokastik diferansiyel denklemler sistemini türetmek için iki-nokta reaktör kinetik denklemleri üç terime ayrılır: ani nötronlar, gecikmiş nötronlar ve yansıyan nötronlar. Geri besleme etkilerinin dâhil edildiği ve edilmediği farklı pertürbasyon durumlarında, stokastik diferansiyel denklemler sistemi Euler-Murayama sayısal yöntemini kullanarak çözülür. Sistemin ortalama yanıtının diğer deterministik sayısal yöntemlerin sonuçlarıyla karşılaştırılabilir olduğu görülmektedir.

Anahtar Kelimeler: Yansıtıcılı reaktör, Euler-Murayama yöntemi, Stokastik modeller, Geri-besleme etkisi

1. Introduction

Deterministic point kinetics equations are the coupled differential equations for the neutron population and the precursor concentrations. Deterministic numerical solution methods have been used to solve the point kinetics equations to predict the dynamical behavior of the nuclear reactors. Solutions of the point kinetics equations give us the mean estimated values for the neutron population and delayed neutron precursor concentrations [1-4].

Due to the inability of the conventional one-point reactor kinetics model in the estimation of the dynamical behavior of the strongly reflected reactor, the two-point reactor kinetics model was developed by Cohn, and re-derived by Van Dam and Spriggs et al. [5-7]. In this model the reflected reactor coupling parameters which are denoted by f_{cr} and f_{rc} are used

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to describe the migration of neutrons between core and reflector. The source-free version of the two-point reactor kinetics equation with six groups of delayed neutron precursors are as follows [6,8]:

$$\begin{cases} \dfrac{dN_c(t)}{dt} = \dfrac{\rho(t) - \beta - f_{cr}\, f_{rc}}{\Lambda_c}\; N_c(t) + \dfrac{f_{rc}}{l_r} N_r(t) + \displaystyle\sum_{i=1}^{6} \lambda_i\, C_i\,(t) \\ \dfrac{dN_r(t)}{dt} = \dfrac{f_{cr}}{\Lambda_c}\; N_c(t) - \dfrac{N_r(t)}{l_r} \\ \dfrac{dC_i\,(t)}{dt} = \dfrac{\beta_i}{\Lambda_c}\; N_c(t) - \lambda_i\, C_i\,(t) \qquad ; \quad i = 1, \dots, 6 \end{cases} \tag{1}$$

In this system of equations, $N_c(t)$ represents the neutron population in the core region and is taken proportional to reactor power, $N_r(t)$ is the neutron density in the reflector region, Λ_c is the neutron generation time in the core region, l_r is the neutron lifetime in the reflector region, $\rho(t) = \rho_\infty(t) - f_{cr}(1 - f_{rc})$ is the system reactivity, $\rho_\infty(t)$ is the infinite core reactivity, f_{cr} is the fraction of fission neutrons leaking from the core to the reflector, f_{rc} represents the fraction of reflector neutrons returning back to the core, $C_i\,(t)$, λ_i and β_i are the concentration, decay constant and delayed neutron fraction for delayed neutron precursor group i, respectively, and β represents the total delayed neutron fraction ($= \sum_{i=1}^{6} \beta_i$).

By using the adiabatic model, the Newtonian temperature reactivity feedback due to the fuel temperature is expressed as follows [9,10]:

$$\frac{dT(t)}{dt} = K_c\, N_c(t) \tag{2}$$

and

$$\rho(t) = \rho_{ext}(t) - \alpha\,[T(t) - T_0] \tag{3}$$

Where K_c is the reciprocal of the reactor heat capacity, $\rho_{ext}(t)$ is the external reactivity, $T(t)$ and T_0 are the core-averaged fuel temperature at time t and zero, respectively, and α is the magnitude of the fuel temperature coefficient of the reactivity.

By integrating the expression given in equation (2) with respect to time and using the expression given in equation (3), the system reactivity in the presence of the Newtonian temperature feedback effect becomes in the form of:

$$\rho(t) = \rho_{ext}(t) - b \int_0^t N_c(t')dt' \tag{4}$$

where $b = \alpha K_c$is the nonlinear coefficient part of the reactivity which is also called as the shutdown coefficient of the reflected reactor.

Different type of numerical and analytical solution methods such as fundamental matrix method, analytical exponential method, analytical inversion method, and exact solution methods were used to solve either linear or non-linear two-point reactor kinetics equations [8, 9, 11-13].

The neutron interaction type is determined by using the cross sections, which are also referred to as the interaction probabilities. Therefore, the occurrence of any neutronic event is a stochastic or random process. The actual dynamical process is also stochastic and the neutron population and precursor concentrations fluctuate randomly with time. Although at high power levels the random fluctuations are negligible but at low power levels, such as at reactor start-up, random fluctuations in the population dynamics can be significant. In modern science, the fluctuations are treated as a fundamental property of the system which carry very often as much information as the mean value. Therefore, it is important to get informed about these fluctuations. Hence, nuclear phenomena should be described using the stochastic models [14-16].

The initial stochastic one-point reactor kinetics model was developed by Hayes and Allen [17]. They also introduced a special Monte Carlo technique as well as the stochastic piecewise constant approximation method to solve the stochastic one-point reactor kinetics equations [17,18]. The Euler–Maruyama and Taylor 1.5 strong order numerical methods are also used to predict the stochastic behavior of the neutron and precursor populations [19].

The efficient stochastic model for the one-point kinetics equations was derived by Nahla and Edress, and different solution methods are implemented to solve it [20-22]. In this model to transform the deterministic one-point reactor kinetics equations into a stochastic differential equations system, the deterministic point kinetics equations are separated into delayed and prompt neutrons terms.

In this manuscript, a system of stochastic two-point reactor kinetics equations is provided and solved by using the Euler-Maruyama solution method. To test the validity of the proposed model, the mean response for the neutron population in the core and reflector regions and the mean response for the precursor concentrations are compared with the results of the different deterministic numerical methods.

2. Stochastic model formulation

To derive the stochastic two-point reactor kinetics equations, the deterministic two-point kinetics equations are separated into three terms as follows:

i. Prompt neutrons: $\frac{\rho(t)}{\Lambda_c} N_c(t)$

ii. Delayed neutrons: $\frac{\beta_i}{\Lambda} N_c(t) - \lambda_i C_i(t) \quad ; \quad i = 1, \dots, 6$

iii. Reflected neutrons: $\frac{f_{cr}}{\Lambda_c} N_c(t) - \frac{N_r(t)}{l_r}$

To formulate the stochastic model, the time domain is divided into small time intervals of the length of $\Delta t = h$ second, such that the occurrence probability of more than one event during each time interval is small. It is also assumed that the changes in the neutron and precursor populations during each time interval are approximately normally distributed [17-19,22].

$$|\Delta\Psi\rangle = \begin{pmatrix} \Delta N_c \\ \Delta N_r \\ \Delta C_1 \\ \vdots \\ \Delta C_6 \end{pmatrix} = \begin{pmatrix} N_c(t_{m+1}) - N_c(t_m) \\ N_r(t_{m+1}) - N_r(t_m) \\ C_1(t_{m+1}) - C_1(t_m) \\ \vdots \\ C_6(t_{m+1}) - C_6(t_m) \end{pmatrix} \tag{5}$$

where $t_m = m \times h$ represents any time point within the time domain, where m changes from zero to M (number of time-bins).

The eight possibilities for the $|\Delta\Psi\rangle$ and their corresponding probabilities are listed as follows:

$$|\Delta\Psi_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} ; \quad P_1 = h\frac{\rho(t_m)}{\Lambda_c}\ N_c(t_m) \tag{6}$$

$$|\Delta\Psi_2\rangle = \begin{pmatrix} -f_{rc} \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} ; \quad P_2 = h\left(\frac{f_{cr}}{\Lambda_c}\ N_c(t_m) - \frac{N_r(t_m)}{l_r}\right) \tag{7}$$

$$|\Delta\Psi_3\rangle = \begin{pmatrix} -1 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} ; \quad P_3 = h\left(\frac{\beta_1}{\Lambda}\ N_c(t_m) - \lambda_1 C_1(t_m)\right) \tag{8}$$

$$\vdots$$

$$|\Delta\Psi_8\rangle = \begin{pmatrix} -1 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} ; \quad P_8 = h\left(\frac{\beta_6}{\Lambda}\ N_c(t_m) - \lambda_6 C_6(t_m)\right) \tag{9}$$

The first event represents a change in the core region neutron population due to prompt neutrons. The second event denotes the neutron transfer between the core and reflector regions. The rest events represent the changes in the neutron and precursor populations which are caused due to both the precursor born and transformation of a precursor to a delayed neutron.

The mean change in the small time interval, h, is obtained as follows:

$$E[|\Delta\Psi_m\rangle] = \sum_{k=1}^{8} P_k\,|\Delta\Psi_k\rangle = h\,\bar{\bar{A}}(t_m)\,|\Psi(t_m)\rangle \tag{10}$$

where

$$\bar{\bar{A}}(t_m) = \begin{pmatrix} \frac{\rho(t_m)-\beta-f_{cr}\,f_{rc}}{\Lambda_c} & \frac{f_{rc}}{l_r} & \lambda_1 & \lambda_2 & \dots & \lambda_6 \\ \frac{f_{cr}}{\Lambda_c} & -\frac{1}{l_r} & 0 & 0 & \dots & 0 \\ \frac{\beta_1}{\Lambda} & 0 & -\lambda_1 & 0 & \dots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & 0 & -\lambda_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \frac{\beta_6}{\Lambda} & 0 & 0 & 0 & \dots & -\lambda_6 \end{pmatrix}$$

$$|\Psi(t_m)\rangle = \begin{pmatrix} N_c(t_m) \\ N_r(t_m) \\ C_1(t_m) \\ \vdots \\ C_6(t_m) \end{pmatrix}$$

The variance of change is also calculated as:

$$Var[|\Delta\Psi\rangle] = \sum_{k=1}^{8} P_k \, |\Delta\Psi_k\rangle\langle\Delta\Psi_k| = h \, \bar{\bar{B}}(t_m) \tag{11}$$

where

$$\bar{\bar{B}}(t_m) = \begin{pmatrix} \mu_c(t_m) & -f_{rc}\,\mu_{r1}(t_m) & -\mu_1(t_m) & -\mu_2(t_m) & \dots & -\mu_6(t_m) \\ -f_{rc}\,\mu_{r1}(t_m) & \mu_{r1}(t_m) & 0 & 0 & \dots & 0 \\ -\mu_1(t_m) & 0 & \mu_1(t_m) & 0 & \dots & 0 \\ -\mu_2(t_m) & 0 & 0 & \mu_2(t_m) & \dots & 0 \\ \vdots & \vdots & \vdots & & \ddots & \\ -\mu_6(t_m) & 0 & 0 & 0 & \dots & \mu_6(t_m) \end{pmatrix}$$

$$\mu_c(t_m) = \frac{\rho(t_m) + \beta + f_{rc}^2 f_{cr}}{\Lambda_c} \, N_c(t_m) - \frac{f_{rc}^2}{l_r} N_r(t_m) - \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\mu_r(t_m) = \frac{f_{cr}}{\Lambda_c} N_c(t_m) - \frac{N_r(t_m)}{l_r}$$

$$\mu_i(t_m) = \frac{\beta_i}{\Lambda_c} \, N_c(t_m) - \lambda_i C_i(t_m)$$

According to the central limit theorem, the random variate $\frac{|\Delta\Psi\rangle - E[|\Delta\Psi\rangle]}{\sqrt{Var[|\Delta\Psi\rangle]}}$ follows standard normal distribution [17-19]:

$$\frac{|\Delta\Psi\rangle - E[|\Delta\Psi\rangle]}{\sqrt{Var[|\Delta\Psi\rangle]}} = |\eta\rangle \tag{12}$$

where $|\eta\rangle = [\eta_1, \ \eta_2 \, , \dots , \eta_8]^T$, and η_i's are the random numbers which are chosen from standard normal distribution $N(0,1)$.

Thus we have

$$|\Delta\Psi\rangle = \mathrm{h}\,\bar{\bar{\mathrm{A}}}(\mathrm{t_m})\,|\Psi(\mathrm{t_m})\rangle + \sqrt{\mathrm{h}}\,\bar{\bar{\mathrm{B}}}^{\frac{1}{2}}(\mathrm{t_m})|\eta\rangle \tag{13}$$

The standard Wiener process $W = \{W(t) \; for \; t \geq 0\}$ is a stochastic process which satisfies the following properties [23,24]:

i. $W(t)$ is continuous for all t.
ii. $W(0) = 0$.
iii. For $0 < t_1 < t_2$, the random number given by $W(t_2) - W(t_1)$ is normally distributed with mean zero and variance $t_2 - t_1$, that is $W(t_2) - W(t_1) \sim \sqrt{t_2 - t_1}\, N(0,1)$
iv. For $0 < t_1 < t_2 < t_3 < t_4$, $W(t_2) - W(t_1)$ and $W(t_4) - W(t_3)$ are independent from each other

Relying on the Winer process properties, $\sqrt{\mathrm{h}}\,|\eta\rangle$ becomes equal to $|\Delta W\rangle$:

$$|\Delta W\rangle = \begin{pmatrix} \Delta W_1 \\ \Delta W_2 \\ \vdots \\ \Delta W_8 \end{pmatrix} \tag{14}$$

Where, $\Delta W_j = W_j(t_{m+1}) - W_j(t_m)$.

By dividing both sides of equation (13) by h and taking limit $h \to 0$, the corresponding system of Itô stochastic differential equations for the two-point reactor kinetics model is resulted as follows:

$$\frac{d}{dt}\,|\Psi(\mathrm{t})\rangle = \bar{\bar{A}}(t)|\Psi(\mathrm{t})\rangle + \bar{\bar{B}}^{\frac{1}{2}}(t)\,\frac{d}{dt}|W(t)\rangle \tag{15}$$

In this study, the Euler-Maruyama numerical method is used to solve the stochastic two-point reactor kinetics [25].

$$|\Psi(t_{m+1})\rangle = |\Psi(t_m)\rangle + h\,\bar{\bar{A}}(t_m)\,|\Psi(t_m)\rangle + \bar{\bar{B}}^{\frac{1}{2}}(t_m)|\Delta W\rangle \tag{16}$$

It is worth noting that, the result of each individual simulation is different from the other simulations results, therefore, the system mean response is obtained by calculating the average of the results of the several individual simulations. Similarly, the standard deviation corresponding to the mean neutron and precursor populations are easily calculated.

3. Computational results

To test the validity of the proposed stochastic model, the zero-power research reactor PROTEUS, which consists of a relatively small core surrounded by a thick graphite reflector, is taken into consideration [8,9]. For the different reactivity insertion scenarios such as step reactivity, ramp reactivity, and ramp reactivity insertion in presence of the Newtonian temperature feedback effect the mean response of the stochastic model is compared with the results of the deterministic numerical methods. The kinetics parameters of the reflected reactor PROTEUS in the critical condition are presented in Table (1).

Table 1. The kinetics parameters for the critical reflected reactor PROTEUS.

$\boldsymbol{\rho_\infty}$	$\boldsymbol{\Lambda_c(ms)}$	$\boldsymbol{l_r(ms)}$	$\boldsymbol{f_{rc}}$	$\boldsymbol{f_{cr}}$
0.4	0.4	4.0	0.5	0.8
Precursor group i		$\boldsymbol{\lambda_i(s^{-1})}$		$\boldsymbol{\beta_i}$
1		0.012444		2.371E- 4
2		0.030535		1.583E- 3
3		0.111438		1.417E- 3
4		0.301368		2.856E- 3
5		1.136307		8.314E -4

6	3.013683	3.037E- 4

The initial condition is expressed as follows:

$$|\Psi(t_0 = 0)\rangle = \begin{pmatrix} \mathrm{N_c}(t_0) \\ \mathrm{N}_r(t_0) \\ \mathrm{C}_1(t_0) \\ \vdots \\ \mathrm{C}_6(t_0) \end{pmatrix} = \mathrm{N_c}(t_0) \begin{pmatrix} 1 \\ \frac{l_r\, f_{cr}}{\Lambda_c} \\ \frac{\beta_1}{\Lambda_c\, \lambda_1} \\ \vdots \\ \frac{\beta_6}{\Lambda_c\, \lambda_6} \end{pmatrix} \tag{17}$$

3.1. Step reactivity insertion

In this test case, a positive step reactivity of 0.6 dollar is introduced into the system, at time $t = 0\ s$. The results obtained from the Stochastic Point Reactor Kinetics Model (SPRKM) for the neutron populations in the core and reflector regions are plotted in Figure (1). In each case, using the numerical solution of the SPRKM, two individual sample neutron populations and the mean neutron population of 2000 individual simulations are plotted. The time intervals length is also taken as $h = 0.001\ s$.

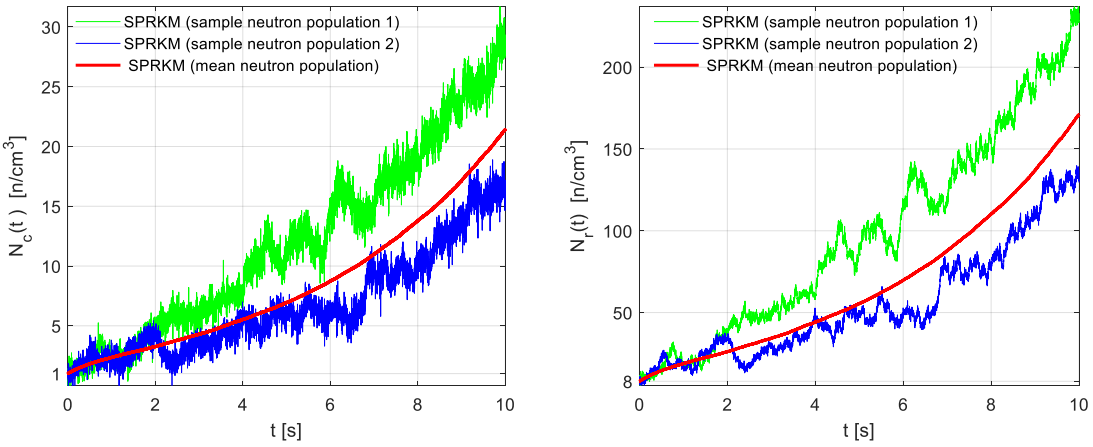

Figure 1. The core and reflector neutron populations for the positive step reactivity.

A comparison between the obtained results from the SPRKM and the results obtained from the fundamental matrix method (FMM) and analytical exponential method (AEM) are presented in Table (2). The standard deviations of the results obtained from the SPRKM are presented in parentheses next to the means. It is seen that the obtained results are in good agreement with the results of the deterministic numerical methods.

Table 2. Core and reflector neutron populations for the step reactivity insertion.

Time (s)	$N_c(t)$			$N_r(t)$		
	FMM [8]	AEM [11]	SPRKM (σ_{N_c})	FMM	AEM	SPRKM (σ_{N_r})
1.0	2.360463	2.360462	2.350399 (0.955340)	18.852197	18.852188	18.767001 (6.293199)
2.0	3.294818	3.294816	3.259099 (1.231999)	26.328428	26.328411	26.084001 (8.630404)
3.0	4.302021	4.302018	4.303596 (1.500201)	34.381319	34.381290	34.294006 (11.092999)

3.2. Ramp reactivity insertion

In this test problem, a $0.1\beta t$ positive linear ramp reactivity is inserted into the system, at $t = 0\ s$. The average response of the 2000 individual simulations for the core and reflector neutron populations are plotted in Figure (2). The results of the SPRKM are compared with the results of FMM and AIM Padé02 and presented in Table (3). It is observed that the results of SPRKM are accurate compared with the results of other deterministic methods.

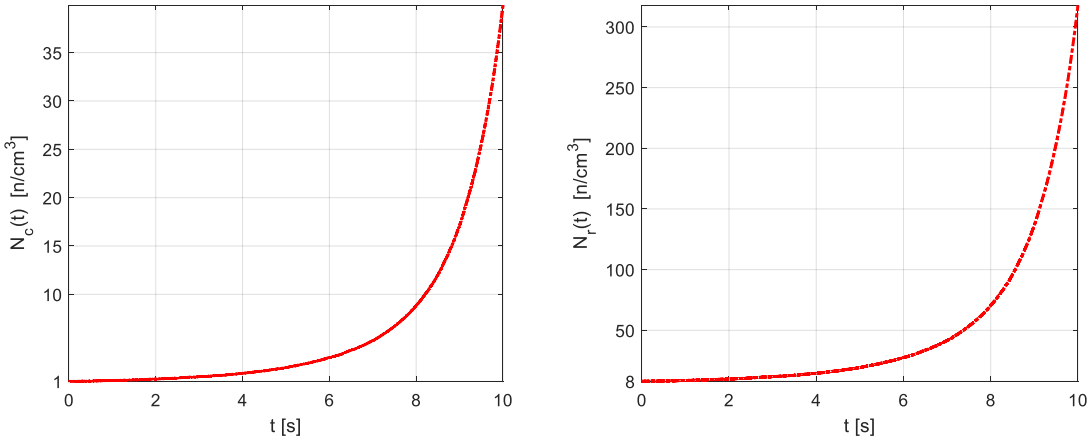

Figure 2. The core and reflector mean neutron populations for the positive ramp reactivity.

Table 3. Core and reflector neutron populations for the ramp reactivity insertion.

Time (s)	$N_c(t)$			$N_r(t)$		
	FMM	AIM Padé02 [12]	SPRKM (σ_{N_c})	FMM	AIM Padé02	SPRKM (σ_{N_r})
0.10	1.00171	1.00177	1.001393 (0.511915)	8.01271	8.01316	8.025715 (1.459975)
0.50	1.02817	1.02923	1.024317 (0.537758)	8.22249	8.23089	8.231267 (1.925292)
1.00	1.08316	1.05643	1.084554 (0.569949)	8.66127	8.68740	8.663623 (2.198768)
3.00	1.47994	1.50414	1.485416 (0.635719)	11.8302	12.0236	11.81655 (3.161711)

3.3. Reactivity insertion in presence of the temperature reactivity feedback

In this test case, the transient behavior of the reflected reactor with a linear ramp reactivity insertion and in presence of the Newtonian temperature feedback effect is simulated with SPRKM method. The system reactivity in each time point is approximately obtained as follows:

$$\rho(t_m) = a \times\ m \times\ h - \left(b \times\ h \times \sum_{j=1}^{m} N_c(t_j) \right) \quad ; \qquad m = 0, \dots, M \tag{18}$$

where a is the constant coefficient for the linear external reactivity.

For $a = 0.1\ s^{-1}$and two different b values of 10^{-11} (cm^3/s) and 10^{-13} (cm^3/s), the core and reflector mean neutron populations are plotted in Figure (3). It is observed that the neutron populations reach a peak value and finally due to the feedback mechanism the system reaches to a critical equilibrium condition with a different power level. By taking the time derivative of the reactivity equal to zero, the mean neutron population in the core region in the equilibrium state becomes equal to $N_{c_{eq}} = \frac{a}{b}$. Subsequently, in the reflector region, $N_{r_{eq}}$ is equal to $\frac{l_r\, f_{cr}}{\Lambda_c} \times \frac{a}{b}$. As seen in the figure, these equilibrium mean values are accurately estimated by the SPRKM.

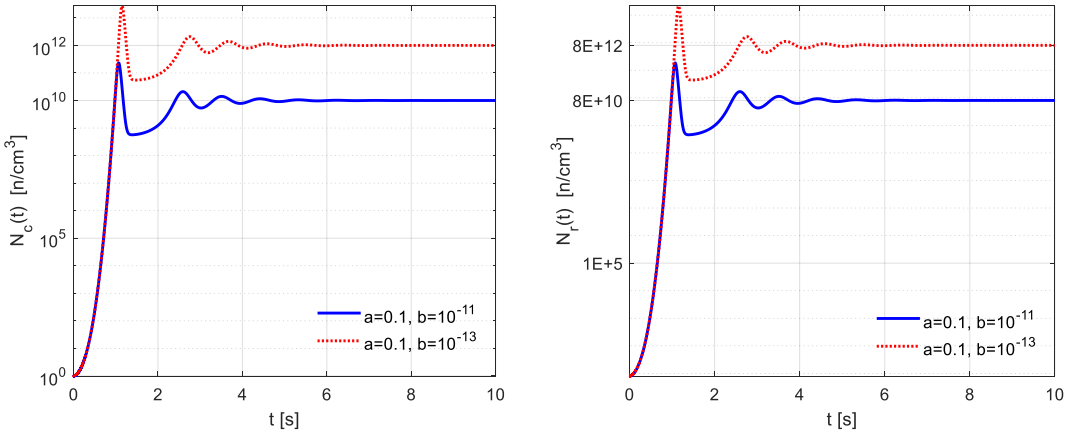

Figure 3. The core and reflector mean neutron populations for the ramp reactivity insertion in presence of feedback effect.

The time evolution of the reactivity and normalized precursor concentrations are plotted in Figure (4). It is seen that the reactivity asymptotically tends to zero. For all precursor groups, the normalized concentrations in the equilibrium condition are the same and equal to:

$$\frac{C_{i_{eq}}}{C_{i(0)}} = \frac{\frac{\beta_i}{\Lambda_c \lambda_i} N_{c_{eq}}}{\frac{\beta_i}{\Lambda_c \lambda_i} N_c(0)} = \frac{a}{b} \tag{19}$$

It is known that, for a system in critical equilibrium condition, it takes a long time that the delayed neutron precursors reach the equilibrium condition. As it is seen in Figure (4-(b)), the precursor groups 5 and 6 reach the equilibrium in a shorter time. This is due to the large decay constant of these groups compared to other groups.

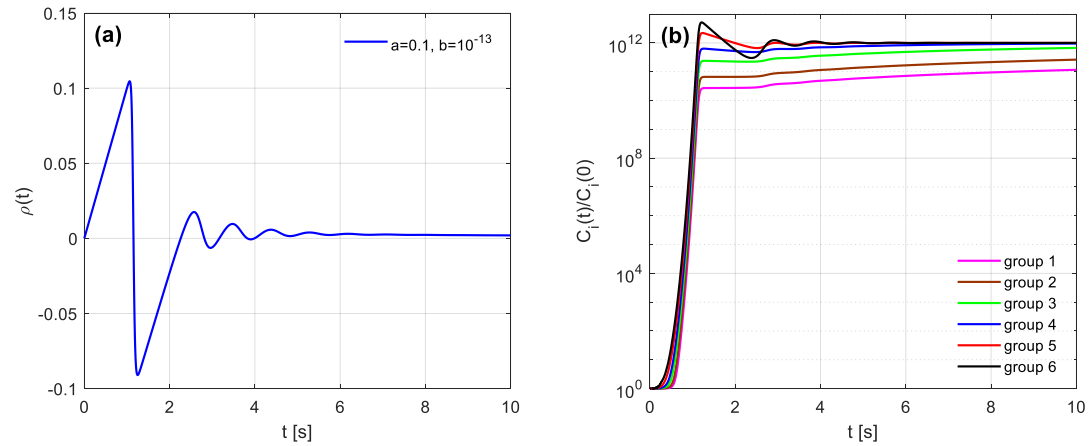

Figure 4. (a) Time evolution of the system reactivity for the a=0.1 and b=1E-13, (b) Normalized precursor concentrations for the a=0.1 and b=1E-13.

4. Summary and conclusions

The point reactor kinetics equations are deterministic and can only be used to estimate the mean values of the neutron population and delayed neutron precursor concentrations. The reactions in the nuclear reactor are not fully describable by deterministic laws. Therefore, nuclear phenomena should be described using the stochastic models. Both Monte Carlo techniques and stochastic point reactor kinetics models were used to model the random behavior of the neutron density and the precursor concentrations. It was observed that the stochastic point reactor models are computationally much faster than the Monte Carlo method [17-18]. In the present study, a new stochastic point reactor kinetics model is proposed to investigate the dynamical behavior of the reflected reactors. For different forms of reactivity insertions including

reactivity feedback, the developed system of stochastic differential equations is solved using the Euler-Murayama numerical method. The accuracy of the proposed methodology is confirmed by comparing the obtained results with the results of the other numerical methods presented in the literature. It should be noted that the developed stochastic method is only proper to estimate the mean particle populations and cannot be used to analyze the reactor noise. In order to investigate the reactor noise, all the birth and death events of particles must be taken separately into account. As a future work, we will try to derive an accurate stochastic point reactor kinetics model for noise analysis of the reflected systems.

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