



INTERNATIONAL CONFERENCE ON MATHEMATICS AND MATHEMATICS EDUCATION (ICMME-2016)

FIRAT UNIVERSITY
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ABSTRACTS BOOK

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PREFACE

The International Conference on Mathematics and Mathematics Education “Journey of Mathematics in Harput” (ICMME-2016) will be held on May 12-14, 2016 in Elazığ, Turkey.

MATDER- Association of Mathematicians is an association founded in 1995 by mathematicians in Turkey. Up to now 14 national mathematics symposium was organized by MATDER.

These meetings have traditionally been the main general national conferences all areas of mathematics and mathematics education and have been well attended by mathematicians from academia and ministry of education. The last four conferences have been held in Niğde (2015), Karabük (2014), Ankara (2013) and Samsun (2012). This year ICMME-2016 will be held at Fırat University, Elazığ, Turkey on 12nd-14th May 2016 as an international conference. Elazığ is a city in Eastern Anatolia, Turkey which is near to the historical city Harput. It is located in the uppermost Euphrates valley.

As with the previous meetings, the objective of this conference is to provide a forum for workers in the field to meet and discuss current research.

This conference is supported by Fırat University and MATDER-Association of Mathematicians.

The topics of interest include,

Mathematics

- Algebra
- Algebraic Geometry
- Category Theory
- Complex Analysis
- Computer Sciences
- Control Theory and Optimization
- Differential Equations
- Differential Geometry
- Discrete Mathematics
- Dynamical Systems and Ergodic Theory

- Functional Analysis
- Geometry
- Mathematical Logic and Foundations
- Mathematical Physics
- Number Theory
- Numerical Analysis
- Operator Theory
- Probability Theory and Statistics
- Real Analysis
- Topology

Mathematics Education

- Research
- Problems
- Hypothesis
- Technology
- Policies
- Applications
- Innovations
- Sufficiencies

The main aim of this conference is to contribute to the development of mathematical sciences, mathematical education and their applications and to bring together the members of the mathematics community, interdisciplinary researchers, educators, mathematicians and statisticians from all over the world. The conference will present new results and future challenges, in series of invited and short talks, poster presentations, workshops and exhibitions. The presentations can be done in the languages of English and Turkish. All presented paper's abstracts will be published in the conference proceeding. Moreover, selected and peer review articles will be published in the following journals:

- TWMS Journal of Pure and Applied Mathematics (Indexed by Emerging Sources Citation and MathSciNet)

- TWMS Journal of Applied and Engineering Mathematics (Indexed by Emerging Sources Citation and MathSciNet)
- Gazi University Journal of Science (Indexed by Engineering Index (Compendex), Emerging Sources Citation Index (ESCI))
- Gazi University Journal of Science Part A: Engineering and Innovation
- Communications, Series A1:Mathematics and Statistic(Indexed by MathSciNet and ULAKBİM)
- MATDER Matematik Eğitim Dergisi
- Turkish Journal of Mathematics & Computer Science (TJMCS)

Also, the 4th Mathematics competitions between high schools and chess tournament will be organized.

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INVITED SPEAKERS

New direction of fractional calculus: Nonlocal and non-singular kernels



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Abstract

Based upon the Mittag-Leffler function, new derivatives with fractional order were constructed. With the same line of idea, improper derivatives based on the Weyl approach are constructed in this works. To further model some complex physical problems that cannot be modeled with existing derivatives with fractional order, we propose, a new derivative based on the more generalized Mittag-Leffler function known as Prabhakar function. Some new results are presented together with some applications.

Keywords: Atangana-Baleanu fractional derivatives; Weyl approach; Prabhakar Mittag-Leffler function; integral transform.

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Teaching models in maths education of gifted students



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Abstract

Teaching models that have been used widely in programs for gifted are investigated. In these models, especially Problem Based Learning (PBL) and Creative Problem Solving (CPS) are focused on. The importance of the use of these models of maths education in gifted students was studied. It is tried to evaluate the positive and the negative aspects of these models.

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Lower-Limit and Upper-Limit Conditions for Oscillation and Stability of First-Order Delay Differential and Difference Equations



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Abstract

In this presentation, we will talk about lower-limit and upper-limit conditions for oscillation and stability of first-order delay differential equations and (their discrete analog) difference equations.

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Advances in fractional difference equations and their applications



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Abstract

Fractional calculus is one of the emerging fields in mathematics and it has a lot of applications in many fields of science and engineering. In my talk I will focus on a new established field called fractional difference equations. Several applications to the real world problems will be presented.

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Lie Symmetry Methods: Applications to Nonlinear Ordinary and Fractional Differential Equations



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Abstract

Lie symmetry methods are provides a powerful approach for studying nonlinear ODEs. They use the notion of symmetry to generate solutions in a systematic manner. In this talk, we briefly introduction the Lie symmetry approach, then we give some examples. The Lie symmetry analysis of some time fractional partial differential equations are also in order.

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Sample Mnemonics for Teaching some Concepts in Mathematics



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Abstract

We present basic and complex models and ideas for the teaching of elementary and advanced concepts in various subjects in mathematics, based on thirty years of teaching experience.

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THE USE OF STOCHASTIC DIFFERENTIAL EQUATIONS IN DIFFERENT FIELDS



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Abstract

Stochastic differential equations occur where a system described by differential equation is influenced by random noise. In other words, they define differential equation in which one or more of the terms is a stochastic process, resulting in a solution which is also a stochastic process. SDEs are used to model various phenomena such as unstable stock prices or physical systems subject to thermal fluctuations. Typically, SDEs contain a variable which represents random white noise that is calculated as the derivative of Brownian motion or the Wiener process. In this paper, we particularly study the equation

$$dS(t) = a(t)dt + \sigma(t)dW(t)$$

and Ito formula which are very crucial for stochastic analysis. Moreover, it is emphasized that this type of equations are capital of importance of many areas. This claim is also proved by giving some applications.

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Critical Thinking



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Abstract

Egocentric thinking is the basis of our conflict. This approach is contrary to the scientific attitudes and behaviours. When we consider things from a 'critical thinking perspective', first we should consider our thoughts and value systems, then other people's manner of approaching events. We can then arrive at realistic standpoint. In this study, we will explore the definition of critical thinking, the characteristics of individuals with advanced critical thinking skills, factors which prevent critical thinking, how we learn to think critically, the intellectual development process and critical thinking and finally Atatürk's opinion about the subject.

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ON THE EDUCATION AND TEACHING OF MATHEMATICS



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Abstract

Mathematical thinking and reasoning are still the most trusted method and correct way in taking right decisions and solving problems. In this context, mathematical concepts should be understood correctly for drawing conclusions necessary and required practices. Also, the huge differences between memorize and understand should be realized.

That's why, we are in need of an effective education model with renovated curriculum in our faculties for raising well-trained teachers.

In this sense, the many years of experiences gained in the field of education and teaching mathematics in universities would be shared with participants.

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Historical development of teacher training models in Turkey and a new model at teacher training



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Abstract

Teacher training has different dimensions and is very important matter. The most important element of an educational system is the teacher. Teaching profession, the importance of the teacher education and the roles of the teachers are the concepts which become a current issue almost in every society. Historical development of teacher training models in Turkey is investigated. It is tried to evaluate the positive and the negative aspects of the these models. It is proposed a new model at teacher training in Turkey which to determine the qualification of teachers and, by this way, to train qualified teachers.

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Circuit design, control and numerical simulation for the fractional-order chaotic behavior in a new dynamical system



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Abstract

This work presents a novel 3D fractional-ordered chaotic system. The dynamical behavior of this system is investigated. An analog circuit diagram is designed for generating strange attractors. Simulation results demonstrate that the fractional-ordered nonlinear chaotic attractors exist in this new system. The obtained results agree very well with those obtained by numerical simulations using a Multistep Generalized Differential Transform Method. Moreover, an active control method is proposed to suppress chaos to unstable equilibria based on the Mittag-Leffler stability theory.

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MATHEMATICS

ALGEBRA AND NUMBER THEORY

A General System of Rational Difference Equations

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ABSTRACT

We investigate behavior of well-defined solutions of the system

$$x_n = \frac{a_1 y_{n-3k}}{\alpha_1 + \beta_1 y_{n-3k} x_{n-2k} y_{n-k}},$$
$$y_n = \frac{a_2 x_{n-3k}}{\alpha_2 + \beta_2 x_{n-3k} y_{n-2k} x_{n-k}},$$

where $n \in \mathbb{N}$, $k \in \mathbb{Z}^+$, the coefficients $a_1, a_2, \alpha_1, \alpha_2, \beta_1, \beta_2$, and the initial conditions are arbitrary real numbers.

Key Words: System of rational difference equation, closed-solution, Periodicity, Convergence.

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A Multiple Criteria Decision Making Method on Single Valued Bipolar Neutrosophic Set Based on Correlation Coefficient Similarity Measure

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ABSTRACT

Multiple criteria decision making problems are one of the most important problems in the decision making sciences including engineering technology and applied sciences. The aim of this study to developed a multiple criteria decision making method under single valued bipolar neutrosophic environment. To construct this method, we first introduce a similarity measure on single valued bipolar neutrosophic set based on correlation coefficient similarity measure of single valued neutrosophic set. Also, the method is applied to a real example in order to show the practicality and accuracy of the proposed method.

Keywords: Neutrosophic sets, Multiple criteria decision making, Single valued neutrosophic neutrosophic set, Similarity measure.

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A New Approach For Multi-Attribute Decision-Making Problems With Bipolar Neutrosophic Sets

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ABSTRACT

In this study, we give a new outranking approach for multi-attribute decision-making problems in bipolar neutrosophic environment. To do this, we firstly propose some outranking relations for bipolar neutrosophic number based on ELECTRE, and the properties in the outranking relations are further discussed in detail. Also, we developed a ranking method based on the outranking relations of for bipolar neutrosophic number. Finally, we give a real example to illustrate the practicality and effectiveness of the proposed method..

Key Words: Single valued neutrosophic sets, single valued bipolar neutrosophic sets, outranking relations, multi-attribute decision making.

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A New Mathematics Teaching Model: Versatile Developmental Mathematics Teaching

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ABSTRACT

In this study, the issues like the Versatile Developmental Mathematics Teaching Model (VDMT), which was developed by Yıldırım (2014), how an education may be conducted in accordance with this model, and how the contents developed for this model should be have been dealt with.

The different developmental areas of the individual sare the mental, social, emotional and physical development areas (Eylen Özyurt, 2011). According to VDMT, mathematics education must ensure that the students use their mental, social, emotional and physical capacities, which are different developmental areas of students. If students use their mental, social, emotional and physical capacities during the education process, they learn better and develop in these fields. In other words, VDMT ensures that students develop in these fields and this development increases the learning. This situation goes on in a cycle.

According to Yıldırım (2015: 18), in order for students to be able to use their mental, social, emotional and physical capacities which are the different developmental areas of students, during educational process;

1) Education must be applied with the combined approach. Cooperative learning (effective cooperative) and full learning method must be included in each combination, and one or more than one of the following methods must also be used; learning by discovering, making students find, question-answer, problem-based learning, activity-based learning, mathematical education based on the history, computer-assisted mathematics education, education with cartoons, education with games and etc.

2) The learning and teaching principles, and the development of mathematical skills must be cared for in VDMT, which is the case in every type of mathematical teaching.

3) A content that is in accordance with the VDMT must be prepared and this must be followed in teaching.

The Educational Content that is in Accordance with VDMT

In order to conduct education with the VDMT, the lesson must be planned in every detail before the actual teaching. The best way for this is the preparation of a content that will guide teachers and students along the process of the lessons. This situation will save the education from being random, and save time.

According to Yıldırım (2015:19), the content of the VDMT must be as follows;

- ❖ It must be proper to the combined approach that will be used in education.
- ❖ It must be proper to the learning and teaching principles.
- ❖ It must be proper to the development of mathematical skills.
- ❖ It must include the conceptual and operational knowledge on the subject to be taught.
- ❖ It must be constructed with questions and instructions.
- ❖ It must be in the style that will allow students write the answer of every instruction or question just below them.
- ❖ It must be proper to the content development principles.

Key Words: Versatile Developmental Mathematics Teaching (VDMT)

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A Note on Generalized π -extending Modules

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ABSTRACT

All rings are associative with unity and modules are unital right modules. We use R to denote such a ring and M to denote a right R -module. A module M is called *CS* or *extending* if every submodule of M is essential in a direct summand of M . It is well known that uniform, semisimple and injective modules satisfy extending property. On the other hand there have been many generalizations of the extending module notion including π -extending condition. Recall that a module M is said to be *π -extending* [1] if every projection invariant submodule of M (i.e., every submodule which is invariant under all idempotent endomorphisms of M) is essential in a direct summand of M . It was shown in [1] that the class of π -extending modules is closed under direct sum.

In this note we focus our attention on generalized π -extending modules. A module M is called *generalized π -extending* module if for any projection invariant submodule N of M , there exists a direct summand K of M such that $N \leq K$ and K/N is singular. It can be easily seen that every extending (π -extending or singular) modules satisfy the generalized π -extending condition. We investigate the class of these modules including direct sum and direct summand properties.

Key Words: extending module, projection invariant, π -extending module.

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A New Multi-Attribute Decision Making Model Based On Single Valued Trapezoidal Neutrosophic Numbers

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ABSTRACT

In this study, we develop a novel decision making method on single valued trapezoidal neutrosophic numbers (SVTN-numbers) is called superiormulti-attribute decision making method, where all the decision information takes the form of SVTN-numbers. To construct the method, we firstly develop some new similarity measure between two SVTN-numbers. Then we propose concept of impact values on SVTN-numbers by using the cut sets of SVTN-number. Finally, a numerical example is introduced to show the applicability and feasibility of the proposed multi-attribute decision making method.

Key Words: Single valued neutrosophic set, single valued trapezoidal neutrosophic numbers, similarity measure, impact value, superior multi-attribute decision making.

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An Outranking Approach Formulti-Criteria Decision-Making Problems with Neutrosophic Refined Sets

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ABSTRACT

In this paper, we introduced a new outranking approach for multi-criteria decision making problems to address uncertain situations in neutrosophic refined environment. Therefore, we give some outranking relations neutrosophic refined number based on ELECTRE. We also examined some desired properties of outranking relations in detail. Then, we developed a ranking method based on the outranking relationsin neutrosophic refined number. Finally, we give a numerical example to show the practicality and effectiveness of the proposed method..

Key Words: Single valued neutrosophic sets, neutrosophic refined sets, outranking approach, outranking relations, decision making.

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A Note on the Second-Order Difference Equations

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ABSTRACT

In this study, Sturm-Liouville difference equation with variable coefficient, potential function $q(n)$, is considered as follows,

$$\Delta^2 x(n-1) + q(n)x(n) + \lambda x(n) = 0, n = a, \dots, b, \quad (1)$$

$$x(a-1) + hx(a) = 0, \quad (2)$$

$$x(b-1) + kx(b) = 0, \quad (3)$$

where a, b are finite integers with $a \geq 0$; $a \leq b$, h is a real number, Δ is the forward difference operator, $\Delta x(n) = x(n+1) - x(n)$, λ is the spectral parameter, $q(n)$ is a real valued potential function for $n \in [a, b]$; n is a finite integer.

The representation of solutions is obtained by variation of constants method for the initial value problems (1)-(2) and (1)-(3). It is proved that these results hold for the equations by using the initial conditions (2)-(3). These results are discrete analogue of Levitan and Sargsjan [1] (p. 5).

In the most of the studies, this problem is considered with real coefficient, but we consider the problem with variable coefficient. Atkinson [2] considered discrete and continuous boundary value problems in his book and he took up boundary problems of Sturm-Liouville type recurrence formula with real coefficients. Kelley-Peterson [4], Agarwal [5], Bereketoğlu-Kutay [3] mentioned about the methods of solution for the second-order linear difference equations, Shi-Chen [6], Wang-Shi [7] and Shi-Wu [8] studied on this subject.

Key Words: Sturm-Liouville, difference equation, Casoratian, variation of constants.

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Commutator Subgroup of Modular Group And Fibonacci Numbers

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ABSTRACT

In this contribution, we are interested in the Fibonacci numbers corresponding to the commutator subgroup of the classical modular group. Correspondence with Fibonacci numbers and modular group was observed at references [1] and [2]. D. Singerman and J. Strudwick give some results about Fibonacci numbers correspondence with modular group in [4] .

Our aim is to investigate some relations about Fibonacci numbers via suborbital graphs obtained from action of commutator subgroup on extended rationals.

Key Words: modular group, commutator subgroup, Fibonacci numbers.

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Complex Tridiagonal Matrices Associated with Pell and Jacobsthal Numbers

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ABSTRACT

The famous integer sequences (e.g. Fibonacci, Pell) provide invaluable opportunities for exploration, and contribute handsomely to the beauty of mathematics, especially number theory. Among these sequences, Pell and Jacobsthal numbers have achieved celebrity status.

In this study, we consider two $n \times n$ complex tridiagonal matrices. Then, we show that the permanents of these matrices are respectively equal to the Pell and Jacobsthal numbers by using the contraction method.

Key Words: Pell sequence, Jacobsthal sequence, tridiagonal matrix, permanent.

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Eigenvalues of Vertex-Weighted Graphs

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ABSTRACT

Let G be a connected and vertex-weighted graph. The vertex-weighted graph is a graph, in which its vertices are assigned with positive real number or positive definitive matrices and this weights are signed by $w(i)$ at i vertex. The eigenvalues of a vertex-weighted graph are defined as the eigenvalues of its adjacency matrix and the spectral radius of a vertex-weighted graph is also defined as the spectral radius of its adjacency matrix. The adjacency matrix of vertex-weighted graph is denoted by $A_V(G) = (a_{ij})_{n \times n}$, where

$$a_{ij} = \begin{cases} w(i) \cdot w(j) & ; \text{ if } i \sim j \\ 0 & ; \text{ otherwise} \end{cases}$$

This matrix eigenvalues' are denoted by $\rho_v^{(1)} \geq \rho_v^{(2)} \geq \dots \geq \rho_v^{(n)} = 0$

In this paper, the eigenvalues of $A_V(G)$ are studied and some bounds for this eigenvalues are founded.

Key Words: Vertex-weighted graph, adjacency matrix, spectral radius.

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Groups and Rings of Soft Sets

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ABSTRACT

Soft set theory, proposed by Molodtsov, has been regarded as an effective mathematical tool to deal with uncertainties.[1,2,3,4,5] In this work, we introduce two new operations on soft sets, called inverse production and characteristic production, by using Molodtsov's definition of soft sets. We prove that the set of all soft sets over a universe U is an abelian group under the each operations and called "the inverse groups of soft sets" and "characteristic group of soft sets"[6]. Then using the operations inverse product, soft intersection and characteristic product, soft union, we construct two isomorphic ring structures, whose elements are soft sets [6].

Key Words: Soft sets, group structure, ring structure, inverse product, characteristic product

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Gröbner-Shirshov Bases of Coxeter Groups

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ABSTRACT

Coxeter groups arise naturally in several areas of mathematics. They are studied in Lie theory, commutative algebra, representation theory, combinatorics, and geometric group theory. The Gröbner basis theory for commutative algebras was introduced by Buchberger [1]. A parallel theory of Gröbner bases was developed for Lie algebras by Shirshov [2]. The key ingredient of the theory is the so-called composition lemma which characterizes the leading terms of elements in the given ideal. Later, Bokut noticed that Shirshov's method works for associative algebras as well [3]. Thus this theory in noncommutative setting is called the Gröbner-Shirshov basis theory. Grobner-Shirshov bases are important tools for finding normal forms of elements of every semigroup or group presented by generators and defining relations such as Coxeter groups.

Gröbner-Shirshov bases of Coxeter groups of the type A_n, B_n and D_n is obtained by Bokut and Sharer in [4]. In this work we use different ordering to find corresponding Gröbner-Shirshov bases. Our normal forms will be different than they found because of the order we choose. We claim that our normal forms are more suitable for understanding of structures of these groups. Because we able to describe multiplication of the group elements in terms of normal forms. In other words we give an algorithm to find the corresponding normal form of the product of two normal forms which is not, in general, a normal form. Thus, we completely reveal the groups.

Gröbner-Shirshov bases of exceptional finite Coxeter groups is found in [5] and [6]. The first example of finding a Gröbner-Shirshov basis of an infinite Coxeter group is given in [7].

Key Words: Coxeter groups, Gröbner-Shirshov bases, normal forms.

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Homotopy of Lie Crossed Module Maps

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ABSTRACT

Group crossed modules were firstly introduced by J.H.C. Whitehead [5] as algebraic models for homotopy 2-types. Areas in which crossed modules have been applied include the theory of group presentations, algebraic K-theory and homological algebra.

Kassel and Loday [3] introduced crossed modules of Lie algebras as computational algebraic objects. A crossed module (M, P, ∂) of Lie algebras is given by a lie algebra morphism $\partial: M \rightarrow P$, together with an action of P on M such that the relations below, hold for each $m, m' \in M$ and $p \in P$,

$$\text{CM1) } \partial(p.m) = [p, \partial(m)]$$

$$\text{CM2) } \partial(m).m' = [m, m'].$$

Let (M, P, ∂) and (M', P', ∂') be two crossed modules. A crossed module morphism $f: (M, P, \partial) \rightarrow (M', P', \partial')$ is a pair $\theta: M \rightarrow M'$ and $\psi: P \rightarrow P'$ Lie algebra morphisms such that $\psi\partial = \partial'\theta$ and $\theta(p.m) = \psi(p).\theta(m)$ for all $m \in M$ and $p \in P$.

Homotopy relation between crossed module maps $C=(C, R, \partial) \rightarrow C'=(C', R', \partial')$ was introduced by Whitehead in [6]. In [1] homotopy was investigated in term of a monoidal closed complexes, and an interval object ([4]).

Let $C=(C, R, \partial)$ and $C'=(C', R', \partial')$ be group crossed modules and $f_1: P \rightarrow P'$ be a group morphism. An f_1 -derivation $s: R \rightarrow C'$ is a map satisfying for all $r, r' \in R$

$$s(rr') = (f_1(r')^{-1}.s(r)) s(r').$$

Let $f=(f_2, f_1)$ be a crossed module morphism C to C' . If s is an f_1 - derivation and $g=(g_2, g_1)$ defined by

$$g_1(r) = f_1(r)(\partial' \circ s)(r)$$

$$g_2(c) = f_2(c)(s \circ \partial)(c)$$

is also a crossed module morphism \mathbf{C} to \mathbf{C}' , then s is called a homotopy connecting \mathbf{f} to \mathbf{g} and denoted as $\mathbf{f} \simeq \mathbf{g}$.

In this work we give the definition of homotopy of Lie algebra crossed modules. Also we show that $\text{Hom}(\mathbf{C}, \mathbf{C}')$ is a groupoid whose objects are the Lie crossed module morphisms $\mathbf{C} \rightarrow \mathbf{C}'$, with morphisms being the homotopies between them.

Key Words: Lie Algebra, crossed module, groupoid.

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Laplacian Eigenvalues of Graphs Using 2-Adjacency

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ABSTRACT

Let G be a simple connected graph. If there is an edge between i and j , we called adjacency of this vertex and denoted by $i \sim j$.

This study has two chapter. The first of this, we redefine adjacency for any two vertices as defined if there is connected two edges between any two vertices, i and j . This vertices are called 2-adjacency and denoted by $i \sim_2 j$. Also let $L^{\sim_2}(G)$ be the 2-adjacency Laplacian matrix of G and $L^{\sim_2}(G) = (l_{ij}^{\sim_2})$ be defined as the $n \times n$ matrix $(l_{ij}^{\sim_2})$, where

$$l_{ij}^{\sim_2} = \begin{cases} d_i^{\sim_2} & ; & i = j \\ -1 & ; & i \sim_2 j \\ 0 & ; & \text{otherwise.} \end{cases}$$

We find a lower bound for 2-spectral radius which is denoted by $\mu^{\sim_2}(G)$.

The second chapter we defined 2-adjacency self-Laplacian matrix and denoted by $L^{\approx}(G)$. $L^{\approx}(G) = (l_{ij}^{\approx})$ be defined as the $n \times n$ matrix (l_{ij}^{\approx}) , where

$$l_{ij}^{\approx} = \begin{cases} d_i^{\approx} & ; & i = j \\ -2 & ; & i \sim_2 j \\ 0 & ; & \text{otherwise.} \end{cases}$$

We find a lower bound for 2-spectral radius which is denoted by $\mu^{\approx}(G)$.

The end of this study this two matrices eigenvalues of L^{\approx} and L^{\sim_2} are contrasted. And then, when we generalize for k -adjacency to found bound, we obtain some conclusion. We present an open problem $L(G)$.

Key Words: 2-adjacency, Laplacian matrix, lower bound.

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Matrix powers of golden ratio

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ABSTRACT

Let α and β be the solutions of the quadratic equation $x^2 - x - 1 = 0$. The numbers α and β are the only numbers such that the reciprocal of each is obtained by subtracting 1 from it, that is, $x - 1 = \frac{1}{x}$, where $x = \alpha$ or β . Thus α is the only positive number that has this property. Since $\alpha - 1 = \frac{1}{\alpha}$, we have $\alpha = \frac{1 + \sqrt{5}}{2}$.

Moreover, the number $\alpha = \frac{1 + \sqrt{5}}{2}$ is known golden ratio. Over the past five centuries, golden ratio has been very attractive for researchers because it occurs ubiquitous such as nature and art. The following relations between golden ratio and the Fibonacci and Lucas numbers are well-known

$$\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = \alpha = \frac{1 + \sqrt{5}}{2} \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{L_{n+1}}{L_n} = \alpha = \frac{1 + \sqrt{5}}{2}$$

where F_n and L_n denote the n th Fibonacci and Lucas number, respectively.

Exponential function and power of golden ratio have the following series expansions for $t \in \mathbb{R}$:

$$e^t = 1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \frac{1}{4!}t^4 + \frac{1}{5!}t^5 + \dots$$

$$e^{-t} = 1 - t + \frac{1}{2!}t^2 - \frac{1}{3!}t^3 + \frac{1}{4!}t^4 - \frac{1}{5!}t^5 + \dots$$

$$\alpha^t = 1 + \ln(\alpha)t + \frac{[\ln(\alpha)]^2}{2!}t^2 + \frac{[\ln(\alpha)]^3}{3!}t^3 + \frac{[\ln(\alpha)]^4}{4!}t^4 + \frac{[\ln(\alpha)]^5}{5!}t^5 + \dots$$

$$\alpha^{-t} = 1 - \ln(\alpha)t + \frac{[\ln(\alpha)]^2}{2!}t^2 - \frac{[\ln(\alpha)]^3}{3!}t^3 + \frac{[\ln(\alpha)]^4}{4!}t^4 - \frac{[\ln(\alpha)]^5}{5!}t^5 + \dots$$

Differential systems have many applications in applied sciences such as mathematics and engineering. Computing matrix exponentials plays a very important role in the solution of the differential systems. Several methods and explicit formulas for the matrix exponential have been obtained [1,2,3].

In this research, by using series expansion of golden ratio we give some properties of matrix powers of golden ratio and we give explicit formulas for some special matrices which was studied by F. Ding [2].

Key Words: Golden ratio, series expansion.

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Multy-Power Rsa

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ABSTRACT

We propose a RSA type cryptosystem. Our cryptosystem uses two different mod value. Namely modulo $n_2=pq$ in encryption and modulo $n_1=p^a q^a$ in decryption. Our suggestion is public too. Additionally our e is one digit smaller than n_1 and we use as message space Z_e not Z_{n_2} . Also there is one more main difference between our suggestion and traditional RSA. Our suggestion allows using powers of prime numbers as a mod value.

Also we know that factorizing $n_2=pq$ is harder than factorizing $n_1=p^a q^a$. So, if we use powers of primes in the mod value, then the security will decrease. However, using prime powers can be a useful idea for the future practical cryptosystems.

Key Words: Homomorphic Encryption, RSA.

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New Equalities on the Intuitionistic Fuzzy Modaloperators

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ABSTRACT

In this study, properties of the modal operators \square , \diamond introduced on intuitionistic fuzzy sets and some intuitionistic fuzzy operations (\rightarrow , $@$, \cup , \cap , $\$$, $\#$, $*$) were investigated. New equalities were obtained and proved. Some basic definitions that we build on our work are given as follow.

Definition 1 ([1]) Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$,

where $\mu_A(x): X \rightarrow [0, 1]$ is the membership function of the fuzzy set A .

The notion of fuzzy logic was defined by L.A.Zadeh in 1965 in [1].

Definition 2 ([2,3]) Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$,

where the functions

$$\mu_A(x), \nu_A(x): X \rightarrow [0, 1]$$

define respectively, the degree of membership and degree of nonmembership of the element $x \in X$, to the set A , which is a subset of X , and for every element $x \in X$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Intuitionistic fuzzy sets form a generalization of the notion of fuzzy sets. Intuitionistic fuzzy sets (shortly IFS) were defined by K. Atanassov in 1986 [2].

Definition 3 ([2,3]) Let X be a nonempty set. If A is an IFS drawn from X , then:

$$\square A = A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}$$

$$\diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle \mid x \in X \}.$$

Modal operators (\square , \diamond) defined over the set of all IFS's transform every IFS into a FS. They are similar to the operators 'necessity' and 'possibility' defined in some modal logics. The notion of modal operator (\square , \diamond) introduced on intuitionistic fuzzy sets were defined by K. Atanassov in 1986 [2]. Intuitionistic fuzzy operators are widely used in algebraic structures, control systems, agriculture areas, computer, irrigation, economy and many engineering fields.

Keywords: Intuitionistic fuzzy sets, modal operators, operations.

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New Ostrowski Inequalities with fuzzy Fractional Caputo Derivative

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ABSTRACT

The following result is known in the literature as Ostrowski's inequality. In 1938, the classical integral inequality is proved by A.M. Ostrowski as the following. The inequality of Ostrowski gives us an forecast for the deviation of the values of a smooth function from its mean value. Accurately, if $f = [a, b] \rightarrow \mathbb{R}$ is a differentiable function with bounded derivative, then

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(u) du \right| \leq M(b-a) \left[\frac{1}{4} + \frac{x - \frac{a+b}{2}}{(b-a)^2} \right] \quad (1,1)$$

for every $x \in [a, b]$. Moreover the constant $1/4$ is the best possible.

In this study, firstly fuzzy basic concept is studied and then we present the right and left fuzzy fractional Riemann-Liouville integrals, Hukuhara difference and the right and left fuzzy fractional Caputo derivatives. We introduce the very general fuzzy fractional Ostrowski type inequality and we obtain with the inclusion of three functions with the right Caputo derivative. We use the Hölder inequality to for this types.

Definition 1 ([1]) Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$, where $\mu_A(x): X \rightarrow [0, 1]$ is the membership function of the fuzzy set A .

The notion of fuzzy logic was defined by L.A. Zadeh in 1965 in [1].

Definition 2 ([2]) Let $f: [a, b] \rightarrow \mathbb{E}$, The Riemann-Liouville integral of fuzzy-valued function f is as follows:

$$(I_{\alpha^+}^{\beta} f)(x) = \frac{1}{\Gamma(\beta)} \int_a^x \frac{f(t) dt}{(x-t)^{1-\beta}}, \quad x > a, \quad 0 < \beta \leq 1, \quad (1,2)$$

Since, for all $0 \leq r \leq 1, [f(x)]^r = [\underline{f}(x; r), \bar{f}(x; r)]$.

Key Words:Fuzzy logic, Fuzzy fractional integral and derivative, Fuzzy fractional Ostrowski inequalities

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Novel Cryptosystems Based on Two Different Mod Values

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ABSTRACT

In this article, we construct two new cryptosystems which use two different mod values. Our main purpose is to create fast and secure schemes by using two different mod values. We do encryption according to mod N and decryption according to mod N_1 where N is a multiple of N_1 . We examine homomorphic properties and security of these encryption schemes. At the final part, we implement the schemes and measure their time complexities.

Key Words: Homomorphic Encryption, Large Integer Factorization, Chinese Remainder Theorem.

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On a Class of Arf Numerical Semigroups

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ABSTRACT

A subset S of \mathbf{N} is called a numerical semigroup if S is closed under addition and S has element 0 and $\mathbf{N}S$ is finite where \mathbf{N} denotes the set of nonnegative integers. In this study, we are interested two subclass of maximal embedding dimension numerical semigroups, which are those semigroups having the Arf property and saturated numerical semigroups. We introduce a new class of both Arf property and saturated numerical semigroups with multiplicity four. We consider numerical semigroups minimally generated by $\{4, k, k+1, k+2\}$. Where k is an integer greater than or equal to 5 and k is congruent to 1 (modulo 4). We prove that all these semigroups are both numerical semigroups with Arf property and saturated numerical semigroup.

There is not any formulas to calculate invariants as Frobenius number, gaps, $n(S)$ and genus of S even for numerical semigroup with multiplicity four. But this invariants have been calculated by imposing some conditions on elements of the numerical semigroup S . We calculate the Frobenius number, the genus and the set of gaps of each of these numerical semigroups.

Additionally, we give a relation between the set of pseudo- Frobenius numbers and the set of all fundamental gaps of these numerical semigroups.

Key Words: Numerical semigroups, saturated numerical semigroups, Arf numerical semigroups, Frobenius number, genus, Apéry set.

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On Incomplete q -Chebyshev Polynomials

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ABSTRACT

Cigler defined q -analogues of the Chebyshev polynomials and study properties of these polynomials in [3,4]. The q -Chebyshev polynomials of the second kind are defined by $U_n(x, s, q) = (1 + q^n)xU_{n-1}(x, s, q) + q^{n-1}sU_{n-2}(x, s, q)$; $n \geq 2$ with initial conditions $U_0(x, s, q) = 1$ and $U_1(x, s, q) = (1 + q)x$ and the q -Chebyshev polynomials of the first kind are defined by $T_n(x, s, q) = (1 + q^{n-1})xT_{n-1}(x, s, q) + q^{n-1}sT_{n-2}(x, s, q)$; $n \geq 2$ with initial conditions $T_0(x, s, q) = 1$ and $T_1(x, s, q) = x$ in [3]. It is clear that $U_n(x, -1, 1) = U_n(x)$ and $T_n(x, -1, 1) = T_n(x)$, where $U_n(x)$ and $T_n(x)$ are the Chebyshev polynomials of the second and first kind, respectively. In this paper, we derive the generating functions of q -Chebyshev polynomials of first and second kind. We get recurrence relations and several properties of some q -Chebyshev polynomials. We show that there are the relationships between q -Chebyshev polynomials and some polynomials. Then we give the tables of some special cases of q -Chebyshev polynomials, some polynomials and sequences. Also we display the graphs of the q -Chebyshev polynomials and incomplete q -Chebyshev polynomials.

Key Words: q -Chebyshev polynomials, incomplete q -Chebyshev polynomials, q -Fibonacci polynomials.

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On Lie Ideals with tri-additive Permuting maps in Rings

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ABSTRACT

Derivations in prime rings firstly initiated by [5]. It is considered a fundamental construction in the theory of centralizing maps on prime rings. A great deal of work in this context are available in the literature (see for example [1] and [3]). In this study R will be represent an associative ring, Z will denote the center of R and U will denote a square closed Lie ideal (see [2] for more detailed knowledge about Lie ideal of associative rings). A map $D: R \times R \times R \rightarrow R$ is called permuting tri-additive if $D(x, y, z) = D(x, z, y) = D(z, x, y) = D(z, y, x) = D(y, z, x) = D(y, x, z)$ holds for all $x, y, z, w \in R$ and a mapping $d: R \rightarrow R$ defined by $d(x) = D(x, x, x)$ is called the trace of D . The trace of D satisfies the relation $d(x + y) = d(x) + d(y) + 3D(x, x, y) + 3D(x, y, y)$ for all $x, y \in R$. Permuting tri-derivations in prime and semi-prime rings was firstly introduced by Ozturk [4]. In this talk after giving some important results on permuting tri-derivations in prime and semi-prime rings we will present commutativity of prime and semi-prime rings with $\text{char}R \neq 2$, $\text{char}R \neq 3$ satisfying various identities involving the trace d of permuting tri-derivation D and we will prove that either $U \subseteq Z$ or $d = 0$. The objective of this study to obtain more general results by considering various conditions on a subset of the ring R viz. Lie ideal of R .

Key Words: Derivations, permuting tri-derivations, traces, prime rings, semi-prime rings, Lie ideals.

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On The Generalized Tribonacci Like Polynomials

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ABSTRACT

In [2,4], the authors gave the definition of the Tribonacci polynomials as follows

$$T_n(x) = x^2T_{n-1}(x) + xT_{n-2}(x) + T_{n-3}(x); T_0(x) = 0, T_1(x) = 1, T_2(x) = x^2$$

for $n > 2$. Also, the authors have investigated the some properties tribonacci polynomials.

In this paper, We define the generalized tribonacci like polynomials. Let $n > 2$ be integer. The recurrence relations of the generalized tribonacci like polynomials are

$$T_n(x, y, z) = xT_{n-1}(x, y, z) + yT_{n-2}(x, y, z) + zT_{n-3}(x, y, z) \quad (1)$$

with the initial conditions

$$T_0(x, y, z) = 0, T_1(x, y, z) = 1, T_2(x, y, z) = x$$

and

$$L_n(x, y, z) = xL_{n-1}(x, y, z) + yL_{n-2}(x, y, z) + zL_{n-3}(x, y, z) \quad (2)$$

with the initial conditions

$$L_0(x, y, z) = 3, L_1(x, y, z) = x, L_2(x, y, z) = x^2 + 2y.$$

Taking $x = y = z = 1$ in (1), we obtain the classic tribonacci number in [1,3].

Using x^2 instead of x , x instead of y and $z = 1$ in (1), we have the classical tribonacci polynomials.

The characteristic equation of (1) and (2) is $\lambda^3 - x\lambda^2 - y\lambda - z = 0$. The Binet's formulas of the generalized tribonacci like polynomials

$$T_n(x, y, z) = \frac{\alpha^{n+1}}{(\alpha - \beta)(\alpha - \gamma)} + \frac{\beta^{n+1}}{(\beta - \alpha)(\beta - \gamma)} + \frac{\gamma^{n+1}}{(\gamma - \alpha)(\gamma - \beta)} \quad (3)$$

and

$$L_n(x, y, z) = \alpha^n + \beta^n + \gamma^n \quad (4)$$

where α, β and γ are roots of the characteristic equation.

We have achieved the sum of tribonacci like polynomials as follows

$$\sum_{s=0}^n T_s(x, y, z) = \frac{T_{n+1}(x, y, z) + (y+z)T_n(x, y, z) + zT_{n-1}(x, y, z) - 1}{x + y + z - 1} \quad (5)$$

and

$$\sum_{s=0}^n L_s(x, y, z) = \frac{L_{n+2}(x, y, z) + (x-1)L_{n+1}(x, y, z) + zL_n(x, y, z) - (3-2x-y)}{x + y + z - 1} \quad (6)$$

for $x + y + z \neq 1$.

We have obtained the relation between $T_n(x, y, z)$ and $L_n(x, y, z)$ as

$$L_n(x, y, z) = xT_n(x, y, z) + 2yT_{n-1}(x, y, z) + 3zT_{n-2}(x, y, z). \quad (7)$$

Also, we investigated other some properties of the tribonacci like polynomials. Tribonacci numbers, tribonacci polynomials are special cases of the generalized tribonacci like polynomials. Thus, all properties in this paper are valid for the tribonacci numbers and tribonacci polynomials.

Key Words: Tribonacci Numbers, Binet 's Formula, Tribonacci Polynomials

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On Soft Neutrosophic Classical Sets

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ABSTRACT

In this paper, we define concept of soft neutrosophic classical sets and set theoretical operations such as; union, intersection, AND-product, OR-product between two soft neutrosophic classical sets, and we investigate some properties of these operations. Based on soft neutrosophic classical sets, we define four basic types of sets of degenerate elements and propose an efficient approach for group decision making problems. Furthermore, we develop a group decision method problems and give an application of the proposed method.

Key Words: Soft set, neutrosophic set, set operations.

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On the Linear Recurrence Sequences

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ABSTRACT

A recurrence relation is a way of defining a sequence. A few of the first terms of the sequence are given explicitly. Then, the recurrence relation gives relationships between terms of the sequence that are sufficient to uniquely determine all the remaining terms [6]. Some of these sequence are Fibonacci, Lucas, Pell, Cordonnier, Padovan, Perrin sequences, Chebyshev polynomials of the first and second kind, triangular numbers, generalized Fibonacci, Lucas, Pell, Pell-Lucas and Perrin sequences and polynomials (available in many different generalization) etc. Many researchers have studied determinantal and permanental representations of these sequences. Minc [3] gave the permanental representation of generalized order-k Fibonacci numbers. Kılıç and Stakhov [2] gave a permanent representation of Fibonacci and Lucas p-numbers. Öcal et al. [4] studied some determinantal and permanental representations of k-generalized Fibonacci and Lucas numbers. Ramirez [5] we derived some relations between Fibonacci-Narayana numbers, and permanents and determinants of one type of upper Hessenberg matrix. Kaygısız and Şahin [1] gave determinantal representation of generalized Lucas polynomials. Şahin and Ramirez [7] gave some determinantal and permanental representations of convolved Lucas polynomials. More examples can be found in the literature. In this study, we compare the methods of proof that used in these studies. Then we show the relationship between them. Finally, we give new proof of some these theorems.

Key Words: Recurrence relation, Fibonacci-type sequences, determinant.

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On the Local Stability Analysis of a Nonlinear Discrete-Time Population Model involving Delay and Allee Effects

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ABSTRACT

There is a need to study mathematical modelling for understand the nature events. This models which allow to analyze the thereal world phenomeon are formed by using difference equation and differantial equation. Therefore, the model of the biological system is very importantto examine dynamics of populations. Local stability of an equilibrium implies that solutions approach the equilibrium only if they are initially close to it.

In this work, we present the local stability conditions of equilibrium point of a general nonlinear discrete-time delay population model with and without Allee effects which occur at low population density. The mathematical analysis and numerical simulations demonstrate that the Allee effect at different times decreases the local stability of equilibrium point of the population model.

Key Words: Stability Analysis, Allee effect, Equilibrium point.

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On the q -harmonic and q -hyperharmonic Numbers

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ABSTRACT

The classical harmonic numbers are defined by for $n \in \mathbb{N}$

$$H_n = \sum_{k=1}^n \frac{1}{k}$$

and $H_0 = 0$.

The harmonic numbers and their generalizations have been studied in several papers. For example in [2], Conway and Guy defined hyperharmonic number of order r , $H_n^{(r)}$, for $n, r \geq 1$ by the following recurrence relations:

$$H_n^{(r)} = \sum_{k=1}^n H_k^{(r-1)}$$

where $H_n^{(0)} = \frac{1}{n}$ and if $n \leq 0$ or $r < 0$, $H_n^{(r)} = 0$. In [3], Benjamin et al. gave some combinatorics properties of hyperharmonic numbers by taking repeated partial sums of the harmonic numbers.

Recently in [4], Tuğlu et al. defined harmonic and hyperharmonic Fibonacci numbers. Moreover, they gave some combinatorial identities for the harmonic Fibonacci numbers by using the difference operator. On the other hand in [5], q -analogues of H_n are given by Dilcher, for $n \geq 0$

$$H_n(q) = \sum_{j=1}^n \frac{1}{[j]_q}$$

and

$$\tilde{H}_n(q) = \sum_{j=1}^n \frac{q^j}{[j]_q}$$

where $H_0(q) = \tilde{H}_0(q) = 0$ and $[j]_q = \frac{1-q^j}{1-q}$. In [6], Mansour and Shattuck defined q -analogue of the hyperharmonic numbers as follows:

$$H_n^{(r)}(q) = \sum_{t=1}^n q^t H_t^{(r-1)}(q)$$

where $H_n^{(0)}(q) = \frac{1}{q[n]_q}$. They considered a q -analogue of the hyperharmonic numbers and gave some combinatorics properties of these numbers.

Motivated by the above papers, in this paper we present q -analogues of certain previously established combinatorial identities by using q -difference operator. Moreover, we give some new combinatorial identities involving q -hyperharmonic numbers.

Key Words: q -harmonic numbers, q -hyperharmonic numbers, q -difference operator.

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On The Norms of Another Form Of R-Circulant Matrices with the Hyper-Fibonacci and Lucas Numbers

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ABSTRACT

Circulant and r-circulant matrices play important role in some areas such as coding theory. In this talk, we give some inequalities related to norms of r-circulant matrices with the hyper-Fibonacci and hyper-Lucas numbers. In [2], we computed spectral norm of circulant matrices in the forms $Circ(F_0^{(k)}, F_1^{(k)}, \dots, F_{n-1}^{(k)})$ and $Circ(L_0^{(k)}, L_1^{(k)}, \dots, L_{n-1}^{(k)})$ and r-circulant matrices in the forms $Circr(F_0^{(k)}, F_1^{(k)}, \dots, F_{n-1}^{(k)})$ and $Circr(L_0^{(k)}, L_1^{(k)}, \dots, L_{n-1}^{(k)})$, where $F_n^{(k)}$ and $L_n^{(k)}$ denote the hyper-Fibonacci and hyper-Lucas numbers, respectively. In [1,4-7], the authors dealt with norms of r-circulant matrices with the some famous numbers entries.

The sequences of the Fibonacci numbers are one of the most well-known sequences, and it has many applications to different fields. The Fibonacci numbers are defined by the second order linear recurrence relation: $F_{n+1} = F_n + F_{n-1}$ with $F_0 = 0, F_1 = 1$. Similarly, the Lucas numbers are defined by $L_{n+1} = L_n + L_{n-1}$ with $L_0 = 2, L_1 = 1$. Fibonacci and Lucas numbers have generating functions and many generalizations. In [3], Dil and Mezö introduced new concepts as "hyper - Fibonacci numbers" and "hyper - Lucas numbers". These concepts are defined as

$$F_n^{(r)} = \sum_{k=0}^n F_k^{(r-1)} \text{ with } F_n^{(0)} = F_n, F_0^{(r)} = 0, F_1^{(r)} = 1$$

and

$$L_n^{(r)} = \sum_{k=0}^n L_k^{(r-1)} \text{ with } L_n^{(0)} = L_n, L_0^{(r)} = 2, L_1^{(r)} = 2r + 1$$

where r is a positive integer, F_n and L_n are ordinary Fibonacci and Lucas numbers, respectively.

In this research, we derive some bounds for the spectral norms of r -circulant matrices with the hyper-Fibonacci and hyper-Lucas numbers of the forms $Circr(F_k^{(0)}, F_k^{(1)}, \dots, F_k^{(n-1)})$ and $Circr(L_k^{(0)}, L_k^{(1)}, \dots, L_k^{(n-1)})$ and their Hadamard and Kronecker products. For this, we firstly compute the spectral and Euclidean norms of circulant matrices of the forms $Circ(F_k^{(0)}, F_k^{(1)}, \dots, F_k^{(n-1)})$ and $Circ(L_0^{(k)}, L_1^{(k)}, \dots, L_{n-1}^{(k)})$. We use some relations concerning the spectral norm, Euclidean norm, row norm, column norm. Moreover, we give some examples related to special cases of our results.

Key Words: r -circulant matrix, hyper- Fibonacci numbers and Lucas numbers, norm.

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On Uninorms on Bounded Lattices

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ABSTRACT

Uninorms were defined on unit interval $[0,1]$ by Yager and Rybalov [6] and studied by Fodor et al [3]. Since arbitrary bounded lattice cases more general, uninorms were defined and extensively studied over these structures ([2],[4]).

Uninorms on bounded lattice $(L,0,1)$ are functions which have property of associativity, commutativity, monotonicity and neutral element e on L . If $e=0$, uninorms coincides t-conorms, if $e=1$ uninorms coincides t-norms on bounded lattice L . Keeping this property in mind, uninorms have close relation with with t-norms and t-conorms. Moreover, uninorms are more general structures than t-norms and t-conorms on bounded lattice L .

In this study, some properties of uninorms, t-norms and t-conorms on bounded lattices are investigated and the relations between uninorms, t-norms and t-conorms are presented. Some illustrative examples are added for clarity.

Key Words: Uninorm, T-norm, T-conorm.

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Periodic Solutions of Two-Dimensions Systems of Difference Equations

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ABSTRACT

We investigate the periodic nature of well-defined solutions of the following systems of difference equations

$$\begin{aligned}x_{n+1} &= y_{n-8} / (\mp 1 \mp y_{n-8} x_{n-5} y_{n-2}), \\y_{n+1} &= x_{n-8} / (\mp 1 \mp x_{n-8} y_{n-5} x_{n-2}), n = 0, 1, 2, \dots,\end{aligned}$$

where initial conditions are real numbers. This study is motivated by ref. [3].

Key Words: System of rational difference equations, solution, periodic solution.

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Reduction Theory in Soft Sets and its Matrix Representation

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ABSTRACT

The concept of soft sets was introduced by Molodtsov [1] in 1999 as a general mathematical tool for dealing with uncertainties. Çağman and Enginoğlu [2] defined soft matrices which are representative of the soft sets. This style of representation is useful for storing a soft set in computer memory. Eraslan [3] defined expansion and reduction of the softsets that are based on the linguistic modifiers. By using these new notions he constructed a decision making method called softreduction method, which selects a set of optimum alternatives. In this study, we first give basic definitions and theorems related to soft sets and soft matrices. We then define matrix representation of reduced soft sets. We finally give a numerical example to show the reduced soft sets can be successfully applied to many problems containing uncertainties.

Key Words: Soft sets, soft matrices, linguistic modifiers, reductions of soft sets.

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Saturated Numerical Semigroups With Multiplicity Four

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ABSTRACT

A subset S of \mathbf{N} is called a numerical semigroup if S is closed under addition and S has element 0 and $\mathbf{N}S$ is finite where \mathbf{N} denotes the set of nonnegative integers. A numerical semigroup S is saturated if the following condition holds: s_1, s_2, \dots, s_k belongs to S are such that $s_1 < s$ or equal to s , for all $1 < i < k$ or $i=1$ and $i=k$, and c_1, c_2, \dots, c_k belongs to \mathbb{Z} are such that $c_1s_1 + c_2s_2 + \dots + c_ks_k > 0$ or equal to 0 , then $s + c_1s_1 + c_2s_2 + \dots + c_ks_k$ belongs to S . The Frobenius number of S is the maximum integer not belonging to S , which is denoted by $F(S)$. $H(S) = \mathbf{N}S$ is the set of the elements gaps of S , and the cardinality elements of $H(S)$ is called genus of S , and denoted by $g(S)$. It is said that an integer x is a Pseudo-Frobenius number if $x+s$ belongs to S for $s > 0$, s belongs to S and x belongs to $\mathbb{Z} \setminus S$. In this study, we will characterize the all families of Saturated numerical semigroups with multiplicity four. These numerical semigroups generated by $4, k, k+1, k+2$ for $k > 5$ or $k=5$, $k=1 \pmod{4}$, and $4, k, k+2, k+3$ for $k > 7$ or $k=7$, $k=3 \pmod{4}$, and $4, k, k+t, k+t+2$ for $k > 6$ or $k=6$, $k=2 \pmod{4}$, respectively. We will prove that Saturated numerical semigroups such that multiplicity four. Also, we will give formulas Frobenius number $F(S)$, Pseudo Frobenius number $PF(S)$, gaps $H(S)$ and genus $g(S)$ of these numerical semigroups.

Key Words: Saturated numerical semigroup, Gaps, Frobenius number.

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Sensitivity Of Schur Stability Of Systems Of Higher Order Linear Difference Equations With Periodic Coefficients

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ABSTRACT

Let $A(n)$ is a T -periodic matrix with $N \times N$ dimension and $x(n)$ is an N dimensional vector. Consider the following difference equation system:

$$x(n+1) = A(n)x(n), \quad n \in \mathbb{Z}. \quad (1)$$

The matrix $X(T) = A(T-1) \times \dots \times A(1) \times A(0)$ is called as monodromy matrix of the system (1) [1,2]. In the plane \mathbb{C} , define the region $\mathbb{C}_s = \{z \in \mathbb{C} : |z| < 1\}$. If $\sigma(X(T)) \subset \mathbb{C}_s$, where $\sigma(X(T))$ is the spectrum of $X(T)$ then the matrix $X(T)$ is called to be Schurstable matrix, or to be an discrete-asymptotic stable matrix [3]. It is well-known in the literature that eigenvalue problem of non-self-adjoint matrices is an ill-posed problem [4]. For this reason, the parameters revealing the quality of the stability obtained by avoiding calculation of eigenvalues are preferred to investigate stability. Schur stability parameter for the systems with periodic coefficients is $\omega_1(A, T) = \|F\|$, where the matrix F is the solution of the discrete-Lyapunov matrix equation (D-LME) $X(T)^*FX(T) - F + I = 0$. The D-LME has a positive defined symmetric solution $F = F^* > 0$ then the system (1) is Schurstable ($\omega_1(A, T) < \infty$), otherwise the system is not Schur stable [5-7].

In this work, the relationship between the condition number of the system (1) and the condition number of k th order system given as

$$x(n+k) = A(n)x(n), \quad A(n+T) = A(n), \quad n \in \mathbb{Z} \quad (2)$$

has been examined. According to this relationship, the continuity theorems given for the system (1) in [6-8] have been extended to the system (2). Also, the results have been supported with numerical applications too.

Key Words: periodic coefficients, monodromy matrix, Schur stability, sensitivity, perturbation systems.

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Single Valued Interval Valued Trapezoidal Neutrosophic Numbers and SVIVTN-Multi-Attribute Decision-Making Method

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ABSTRACT

On the basis of the combination of interval valued neutrosophic sets and single valued trapezoidal neutrosophic number, this study introduces single valued interval valued trapezoidal neutrosophic number (SVIVTN-number) as a further generalization of the concepts of trapezoidal fuzzy number, trapezoidal intuitionistic fuzzy number, single-valued trapezoidal neutrosophic number and so on. Then the operation rules of SVIVTN-number are defined and some desired properties are examined. Also, a multi-attribute decision-making problem, based on some SVIVTN aggregation operators, is developed. To do this, we firstly present some operators including SVIVTN-arithmetic averaging operators and SVIVTN-geometric averaging operators. We secondly, give definition of score function and accuracy function of the SVIVTN-numbers. Thirdly, a ranking method on the SVIVTN-numbers is developed. Finally, a practical example is shown to verify the practicality and feasibility of the ranking method.

Key Words: Neutrosophic sets, interval valued trapezoidal neutrosophic number, aggregation operators, score function and accuracy function, multiple criteria decision making.

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Some Conjectures on the Smallest Eigenvalues of Certain Positive Definite Matrices*

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ABSTRACT

One of the hardest tasks in matrix theory is to find the eigenvalues of an $n \times n$ matrix. The harder one than this may be to determine the matrix whose smallest eigenvalue is smaller than the smallest eigenvalues of those in a certain class of $n \times n$ matrices. In this talk, we will present a result and some new conjectures on finding such a matrix and discuss our attempts to prove them.

Let I be a subset of \mathbb{R} such that $0, 1 \in I$ and $n \geq 2$. Denote by $K_n(I)$ the set of all $n \times n$ lower triangular matrices with each diagonal element equal to 1 and each under-diagonal element in I . Let $L_n(I) := \{YY^T : Y \in K_n(I)\}$ and

$$\beta_n(I) := \min_{Z \in L_n(I)} \left\{ \mu_n^{(1)}(Z) : \mu_n^{(1)}(Z) \text{ is the smallest eigenvalue of } Z \right\}.$$

In [4], it is conjectured that $\beta_n(\{0, 1\})$ is equal to the smallest eigenvalue of $Y_0 Y_0^T$, where $Y_0 = (y_{ij}^0) \in K_n(\{0, 1\})$ is defined by $y_{ij}^0 = \frac{1 - (-1)^{i+j}}{2}$ if $i > j$. The constants $\beta_n(\{0, 1\})$ were originally defined in [3]. Recently, after verifying the truth of the conjecture for $n = 8$ and 9 using a C code in [1], we prove the conjecture in a joint paper with Keskin, Yıldız and Demirbüken [2]. Let $a, c, d \in \mathbb{R}$ such that $a > 0$ and $c \leq 0 \leq |c| < d$. In another joint paper with Keskin and Yıldız [5], we show that $\beta_n([-a, a])$ is equal to the smallest eigenvalue of $Y_1 Y_1^T$, where $Y_1 = (y_{ij}^1) \in K_n([-a, a])$ is defined by $y_{ij}^1 = (-1)^{i+j+1} a$ for $i > j$. Furthermore, in the same paper, we raise a new generalization of the conjecture which states that $\beta_n([c, d])$ is equal to the smallest eigenvalue of $Y_2 Y_2^T$, where $Y_2 = (y_{ij}^2) \in K_n([c, d])$ is defined by

$$y_{ij}^2 = \begin{cases} c & \text{if } i + j \text{ is even,} \\ d & \text{if } i + j \text{ is odd,} \end{cases}$$

for. Finally, we conjecture that such a matrix Y_2 is unique, so are Y_0 and Y_1 .

Key Words: smallest eigenvalue, positive definite matrix, lower triangular matrix.

(* This study is dedicated to the memory of Professor Şuur Nizamoğlu, my MSc advisor and my fellow citizen, whose untimely death in February 2016 saddened all of us.)

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Tableaux-based Decision Procedures for Contact Logics

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ABSTRACT

The aim of this paper is to give sound and complete tableaux-based decision procedures for contact logics, i.e. modal logics suitable for reasoning about regions in discrete spaces.

Balbiani et al [3] and Vakarelov et al [6] define regions in a Kripke frame (W, R) to be arbitrary subsets of W . In their setting, the operations over regions are the Boolean operations of join and complement whereas the relations between regions are the and relations. The binary relation C of contact between two regions a, b of W is defined as follows: $C(a, b)$ iff a contains a point x and b contains a point y such that xRy . As for the binary relation of part-of between two regions a, b of W , it is identified with the ordinary inclusion relation. As shown [2,3,6], the language of contact logics can be seen as a first-order language without quantifiers. See also [4,5]. Nevertheless, we call it modal because most concepts, tools and techniques typical of ordinary modal languages can be applied to it: filtration method, canonical model construction, etc. For instance, with respect to modal definability too, a Sahlqvist-like Correspondence Theorem can be obtained for contact logics [1]. It happens that some elementary properties that are definable in the ordinary language of modal logic are not definable in contact logics and, on the contrary, some second-order properties that are definable in contact logics are not definable in the ordinary modal language. Concerning the satisfiability problem, an interesting result for the contact logics is the following [3], the satisfiability problem with respect to the class of all Kripke models or with respect to the class of all reflexive and symmetric Kripke models is NP-complete. These definability and computability results show that the language of contact logics can be sometimes more expressive than the corresponding language of modal logic whereas the satisfiability problem can be, in

some cases, easier to decide. Tableaux-based approaches have been introduced in the 1950s, first for classical logic and then for intuitionistic and modal logics. Tableaux-based approaches bring together the proof-theoretical and the semantical approaches to the presentation of a logical system. The method of semantic tableaux is an efficient decision procedure for satisfiability. If a formula is satisfiable, the procedure will constructively exhibit a model of the formula. In this paper we give sound and complete tableaux-based decision procedures for contact logics. Developing such tableaux-based decision procedures, we obtain new decidability/complexity results. The paper is organized as follows. Section 1 is introduction. Section 2 is devoted to the syntax and semantics for contact logics. Section 3 is devoted to the basic definitions of tableau method, tableau rules and example. Section 4 is devoted to soundness and termination for the minimal tableau system. Section 5 is devoted to truth lemma and completeness for the minimal tableauxystem.

Key Words: Contact logics, satisfiability problem, tableaux-based decision procedures, decidability and complexity.

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The Bourque-Ligh Conjecture of LCM matrices*

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ABSTRACT

Let $S = \{x_1, x_2, \dots, x_n\}$ be a set of distinct positive integers. The least common multiple (LCM) matrix $[S]$ on S is an $n \times n$ matrix whose ij -entry is the least common multiple of x_i and x_j . The GCD matrix (S) is defined similarly. S is called gcd-closed if S has (x_i, x_j) for all $1 \leq i, j \leq n$. In 1992, Bourque and Ligh [1] conjectured that $[S]$ is nonsingular if S is gcd-closed. In 1996, Hong [3] proved that the conjecture is true for $n \leq 7$ and it is not true for $n \geq 8$. In [4], Hong also showed that if S is a gcd-closed set satisfying $\max_{x \in S} \{\omega(x)\} \leq 2$ then $[S]$ is nonsingular. Here $\omega(x)$ denotes the number of distinct prime factors of x and $\omega(1) = 1$. In a recent paper [2], Haukkanen and et. al. utilize lattice-theoretic structures and the Möbius function to explain the singularity classical LCM matrices. They showed that if $[S]$ is singular such that $|S| = 8$ then the Hasse diagram of S is a cube.

In this talk, we will present recent results and conjectures on the problem of the nonsingularity of $[S]$. Furthermore, we will give the structure of S for which $[S]$ is nonsingular when the Hasse diagram of S is a cube.

Key Words: LCM matrix, gcd-closed, Bourque-Ligh conjecture.

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The Categorical Properties of Racks

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ABSTRACT

A rack is a set R equipped with a bijective, self-distributive binary operation. Racks have been studied by several people under various names. Conway and Wraith [3] used the word rack, describing a rack as the “rack and ruin” of a group, left when the group operation is discarded and only the concept of conjugation remains. Joyce studied a special kind of rack, a “quandle”, extensively in [1], defining the fundamental quandle of knot both algebraically and topologically. His work is largely duplicated independently by Matveev who called them “distributive groupoids” in [5]. Kauffman in [4] calling racks “crystals” and defining the fundamental rack for a knot in S^3 . Brieskorn [2] studies racks under the name of “automorphic sets”.

In this work, we studied some categorical properties of racks.

Key Words: Rack, pullback, equaliser.

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The Domination Parameters of Fan and k -pyramid Graphs

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ABSTRACT

Graph theory has become one of the most powerful mathematical tools in the analysis and study of the architecture of a network. The networks are important structures and appear in many different applications and settings. The study of networks has become an important area of multidisciplinary research involving computer science, mathematics, chemistry, social sciences, informatics and other theoretical and applied sciences. The vulnerability value of a communication network shows the resistance of the network after the disruption of some centers or connection lines until a communication breakdown.

A network is usually represented by an undirected simple graph where vertices represent processors and edges represent links between processors. Let $G = (V(G), E(G))$ be a undirected simple connected graph. A set $S \subseteq V(G)$ is a dominating set if every vertex in $V(G) - S$ is adjacent to at least one vertex in S . The minimum cardinality taken over all dominating sets of G is called the domination number of G and it is denoted by $\gamma(G)$. Moreover, a 2-dominating set of a graph G is a set $D \subseteq V(G)$ of vertices of graph G such that every vertex of $V(G) - D$ has at least two neighbors in D . The 2-domination number of a graph G , denoted by $\gamma_2(G)$, is the minimum cardinality of a 2-dominating set of the graph G .

In 2004, Henning introduced the concept of average domination. The average lower domination number of a graph G , denoted by $\gamma_{av}(G)$, is defined as:

$$\gamma_{av}(G) = \frac{1}{|V(G)|} \sum_{v \in V(G)} \gamma_v(G),$$

where the lower domination number, denoted by $\gamma_v(G)$, is the minimum cardinality of a dominating set of the graph G that contains the vertex v .

In 2015, the new graph theoretical parameter namely the average lower 2-domination number has been defined. The average lower 2-domination number of a graph G , denoted by $\gamma_{2av}(G)$, is defined as:

$$\gamma_{2av}(G) = \frac{1}{|V(G)|} \sum_{v \in V(G)} \gamma_{2v}(G),$$

where the lower 2-domination number, denoted by $\gamma_{2v}(G)$, is the minimum cardinality of a dominating set of the graph G that contains the vertex v .

In this paper, the vulnerability of popular interconnection networks including cycles namely fan graphs and k -pyramid graphs have been studied and their domination numbers, 2-domination numbers, average lower domination numbers and average lower domination 2-numbers have been computed.

Key Words: Graph vulnerability, Connectivity, Network design and communication; Domination number; Average lower domination number, Fan graphs, k -Pyramid graphs.

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The Vulnerability Measures of Some Chain Graphs

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ABSTRACT

Networks are important structures and appear in many different applications and settings. In a communication network, the measures of vulnerability are essential to guide the designers in choosing a suitable network topology. They have an impact on solving difficult optimization problems for networks. The network topology is significant since the communication between processors is derived via message exchange in distributed systems. For this reason, a communication network is modeled by a graph to measure the vulnerability as stations corresponding to the vertices and communication links corresponding to the edges. The vulnerability value of a communication network shows the resistance of the network after the disruption of some centers or connection lines until a communication breakdown. The domination number is the most useful vulnerability parameter. Let $G = (V(G), E(G))$ be an undirected simple connected graph. A set $S \subseteq V(G)$ is a dominating set if every vertex in $V(G) - S$ is adjacent to at least one vertex in S . The minimum cardinality taken over all dominating sets of G is called the domination number of G and denoted by $\gamma(G)$.

In 2004, Henning introduced the concept of average domination. The average lower domination number of a graph G , denoted by $\gamma_{av}(G)$, is defined as:

$$\gamma_{av}(G) = \frac{1}{|V(G)|} \sum_{v \in V(G)} \gamma_v(G),$$

where the lower domination number $\gamma_v(G)$ is the minimum cardinality of a dominating set of the graph G that contains the vertex v .

In this paper, the average lower domination number of some hexagonal cactus chain networks, namely para-chain L_n , ortho-chain O_n and meta-chain M_n have been determined.

Key Words: Graph vulnerability, Connectivity, Network design and communication; Domination number; Average lower domination number, Hexagonal cactus chains.

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ANALYSIS

A Computational Method for Circular Consecutive- k -out-of- n :F(G) Systems

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ABSTRACT

A consecutive k -out-of- n :F system consists of n components such that the system breakdowns if and only if at least k consecutive components fail. The first report for the consecutive k -out-of- n :F system was presented by Kontoleon [2], but the name consecutive k -out-of- n :F system originates from a paper by Chiang and Niu [1]. Then, to pay attention the reliability of this system, much work has been done. Another special type of system related to the consecutive k -out-of- n :F system is the consecutive k -out-of- n :G system. A consecutive k -out-of- n :G system is an ordered sequence of n components such that the system works if and only if at least k consecutive components work. Similarly, for the reliability of this system, extensive work has been done. A consecutive k -out-of- n :F and G systems divided into linear and circular systems corresponding to the components arranged along a line or a circle.

In many physical systems, reliability evaluation such as ones encountered in telecommunications, the design of integrated circuits, microwave relay stations, oil pipeline systems and vacuum systems in accelerators, computer ring networks and spacecraft relay stations have been applied consecutive k -out-of- n system models.

In this paper, we propose a new method to compute the reliability of consecutive k -out-of- n :F(G) systems, with n circularly arranged components. The proposed method provides a simple way for determining the system failure probability. Also we write R-Project codes based on our proposed method to compute the reliability of the circular systems.

Key Words: Reliability evaluation, k -out-of- n system, R programming.

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A new Companion of Ostrowski Type Inequalities for Mappings of Bounded Variation and Applications

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ABSTRACT

In this paper, we firstly establish an identity using a kernel which has five sections that further generalize various results. Then, we obtain a new companion of Ostrowski type integral inequalities for functions of bounded variation. At the end, we apply our results for new efficient quadrature rules. The results presented here would provide extensions of those given earlier.

Key Words: Bounded Variation, Ostrowski type inequalities, Riemann-Stieltjes integral.

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A New Semiparametric Method for Image Processing

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ABSTRACT

In this study, a method has been proposed to be used in image processing applications, a semiparametric regression model, when variables are measured with errors and the densities of these errors are unknown.

Semiparametric regression model with measurement error in the nonparametric part which is seen as an alternative to the kernel deconvolution techniques is introduced which has been developed by us. This estimators' asymptotic normality properties are analyzed to observe whether it fits a normal distribution around the parameters it converges when the sample size of estimator obtained by this method n goes to infinity.

Image processing is processing of images using mathematical operations by using any form of [signal processing](#) for which the input is an image. The ease of use and cost effectiveness have contributed to the growing popularity of digital imaging systems. We make contact with the field of semiparametric errors in variables and present a development and generalization of tools and results for use in image processing and reconstruction. In particular, we adapt and expand proposed models' ideas for use in image denoising, upscaling, interpolation, fusion, and more. The finite sample properties of estimators were investigated by Monte Carlo simulation approach. Furthermore, we compared the performances of proposed method with kernel deconvolution and no measurement error case.

Key Words: Errors in Variables, Partially Linear Model, Semiparametric Regression, Unknown Error Density, Image Processing

A Two-Parameter New Distribution Model

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ABSTRACT

In many practical cases, known probability distributions do not provide adequate fits to real data. For example, if the data are asymmetric, the normal distribution will not be a good choice. So, several methods for generating new probability distributions by adding one or more parameters has been studied in the statistical literature recently (Rodrigo and et. al.,2013). Adamidis and Loukas (1998) introduced the two-parameter exponential –geometric (EG) distribution with decreasing failure rate. Kus (2007) introduced the exponential-Poisson distribution (EP) with decreasing failure rate and discussed several of its properties. Marshall and Olkin (1997) presented a method for adding a parameter to a family of distributions with application to the exponential and Weibull families.

This paper deals with a two-parameter new mix distribution called as RL (Rayleigh-Logarithmic) distribution with increasing failure rate. Provide the mathematical and statistical properties of the obtained distribution. Estimate the parameters by fisher information matrix and entropy are given for the new mix model. In the application part, a real data set which represents the marks of forty-eight slow space students in Mathematics in the final examination of the Indian Institute of Technology, Kanpur in year 2003 is receipted. This real set is analyzed for illustrative purposes.

Key Words: Logarithmic distribution, Maximum likelihood, Hazard function, Mix distribution, Bivariate Distributions.

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Abstract q -Harmonic Mappings For Which Analytic Part Is q -Convex Functions

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Abstract

A planar harmonic mapping in the open unit disc \mathbb{D} is a complex valued function f which maps \mathbb{D} onto the some planar domain $f(\mathbb{D})$. Since \mathbb{D} is simply connected domain the mapping f has a canonical representation $f = h(z) + \overline{g(z)}$, where $h(z)$ and $g(z)$ are analytic functions in \mathbb{D} and have the following power series.

$$h(z) = \sum_{n=0}^{\infty} a_n z^n, \quad g(z) = \sum_{n=0}^{\infty} b_n z^n$$

where $a_n, b_n \in \mathbb{C}$, $n = 0, 1, 2, \dots$ and usual we call $h(z)$ the analytic part of f and $g(z)$ is co-analytic part of f .

In the present article we will examine the subclass of planar harmonic mappings. Let $h(z)$ and $g(z)$ are analytic functions in the open unit disc $\mathbb{D} = \{z \mid |z| < 1\}$ and having the power series representation $h(z) = z + a_2 z^2 + \dots$ and $g(z) = b_1 z + b_2 z^2 + \dots$. If $f = h(z) + \overline{g(z)}$ be the solution of the non-linear partial differential equation $w_q(z) = \left(\frac{D_q g(z)}{D_q h(z)} \right) = \frac{\overline{f_z}}{f_z}$ with $|w_q(z)| < 1$, $h(z)$ q -convex function, then this class is called q -harmonic mappings for which analytic part is q -convex functions and the class of such functions is denoted by $SHS^*(q)$, where $D_q h(z) = \frac{h(z) - h(qz)}{(1-q)z} = f_z$, $D_q g(z) = \frac{g(z) - g(qz)}{(1-q)z} = \overline{f_z}$, $q \in (0, 1)$.

Key Words: q -harmonic mappings, growth theorem, distortion theorem.

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An Approximation Formula for The Critical Values of Hypothesis Tests about Two Population Means by Using Ranked Set Sampling

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ABSTRACT

McIntyre (1952) introduced a sampling design, called Ranked Set Sampling (RSS) which has a better design than the Simple Random Sampling (SRS) design for the estimation of the population mean. RSS is preferred for use in some fields such as the environment, ecology, agriculture and medicine in which measurement of the sampling units in terms of the variable of interest is quite difficult or expensive in terms of cost, time and other factors.

In this study, we investigate the hypothesis test for the difference of means of two populations under RSS when cycle size is 1. Since the theoretical distribution of the sample mean cannot be derived in RSS, the critical values of the test statistics cannot be obtained. For this reason, the critical values are obtained for the different sample sizes by using Monte Carlo method when the variances are known and unknown. In both cases, it is seen that the critical values for these test statistics obtained from Monte Carlo method converge to the critical values obtained from standard normal distribution. This convergence is valid even small sample sizes when the variances are known. Thus, we can use the critical values obtained from normal distribution when the variances are known. However, this convergence is relatively slow when the variances are unknown. For this reason, type I error is getting far from its nominal value when critical values from normal distribution are used for small sample sizes. Thus, functions for critical values depend on sample sizes for different nominal

alpha levels are obtained. By using this functions, critical values for common nominal alpha values are obtained for different sample size $n_1=n_2=n$ values. These critical values are used to obtain the power and type I error for RSS. Also, the obtained type I error and power of test based on RSS are compared to the obtained type I error and power of test based on SRS.

Key Words: Ranked Set Sampling, Hypothesis Test, Monte Carlo, Critical Value.

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An Extention of Weak Contractions in Partial Metrical Posets

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ABSTRACT

In 1992, Matthews [1] introduced the pair (X, p) called partial metric space to study denotational semantics of dataflow networks. Matthews also established an analogue of Banach fixed point theorem in the concept of partial metric space. Recently, the existence of fixed points in the partial metric spaces are considered by many researcher [2] – [6].

In this work, we extend some previous contractive contractions for the pair (X, p) that endowed with a partial order \preceq . Our results are more general than some existing results in the literature.

Key Words: Fixed point, weak contraction, partially ordered set, partial metric.

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Approximation by a Kantorovich Variant of the Generalization Szász Type Operators

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ABSTRACT

It is well-known that some kind of Kantorovich type operators are useful for reconstruction of the functions. For example, biomedical Images or other kind of images can be reconstructed and even enhanced by means of such operators, in order to solve various practical problems.

The aim of this paper is to present the rate of convergence at a point $x > 0$ to function locally of bounded variation. To prove our main result, we have used some methods and techniques of probability theory.

Key Words: Approximation, Bounded variation, Rate of convergence, Total variation.

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Approximation Properties of New Kantorovich Operators in Mobile Intervals

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ABSTRACT

The Kantorovich operators have been introduced by L.V. Kantorovich. In this paper, we introduce Kantorovich type operators in the mobile interval $x \in [0, 1 - \frac{1}{n+1}]$ and investigate some approximation properties of the operators. If n is sufficiently large then our operators coincide with the classical Kantorovich operators.

Key Words: Kantorovich operators, Mobile interval.

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ARIMA Versus Neural Network Model in Forecasting Time Series

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ABSTRACT

Being able to forecast the future value of time series variables is one of the main goal in statistical analysis. We can list the traditional path for forecasting economic events as following;

- At first, we must set a formula of economic event including the explanatory variables for this event,
- Then, we set a statistical model and collect data on consistent basis with sample structure,
- After these, we estimate the coefficients of statistical model by using the appropriate process,
- Finally, we can forecast the future value or the effect of the economic event by using these coefficients.

Instead of trying to identify the explanatory variables of economic event, the time lagged values of event can be used for forecasting. This process called time series analysis and it's very different from traditional statistical and econometric models.

In theory, ARIMA models are the most general class of models for forecasting a time series which can be made to be "stationary" by differencing (if necessary), perhaps in conjunction with nonlinear transformations such as logging or deflating (if necessary). A random variable being a time series is stationary, if its statistical properties are all constant over time. A stationary series has no trend, its variations around its mean have a constant amplitude, and it wiggles in a consistent fashion, i.e., its short-term random time patterns always look the same in a statistical sense. Also, a nonseasonal ARIMA model is classified as an "ARIMA(p,d,q)" model, where:

- p is the number of autoregressive terms,
- d is the number of nonseasonal differences needed for stationarity and
- q is the number of lagged forecast errors in the prediction equation.

Moreover, the generalized forecasting equation with respect to y is:

$$Y_t = \delta + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \dots + \varphi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (1)$$

This second way to forecast the future value of economic event is using Artificial Neural Network (NN). In machine learning and cognitive science, artificial neural networks (ANNs) are a family of

models inspired by biological neural networks and are used to estimate or approximate functions that can depend on a large number of inputs and are generally unknown.

In this study, we selected the (NAR) nonlinear autoregressive model. This model predicts the future values of a time series only by using its past values. In this respect, Y_t is as following,

$$Y_t = f(y_{t-1}, \dots, y_{t-d}) \quad (2)$$

At first, we tried to identify the relevant ARIMA and NAR model by using diagnostic tests and then, we compared the forecast performance of ARIMA and NAR by MATLAB.

Key Words: ARIMA, neural network, forecast.

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Banach Contraction Principle in Ultrametric Spaces Endowed with a Graph

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ABSTRACT

Fixed point theory with various applications has been a remarkable topic in the area of mathematics. For example, it has been used in computer science, engineering, game theory and image processing [2]. Banach contraction principle is considered as the most efficient method to solve some problems in mathematics. Since its structure is very simple and useful, researchers have used it in many existence problems in mathematical analysis.

The notion of ultrametric space (X, d) is a metric space which has the following strong triangle inequality $d(x, y) < \max\{d(x, z), d(z, y)\}$ for all $x, y, z \in X$. It is clear that the strong triangle inequality implies that the two larger of the distances $d(x, y)$, $d(x, z)$ and $d(y, z)$ are equal which is stated as every triangle in X is isosceles. It can be said that many properties in an ultrametric space are equivalent to the strong triangle inequality. But some properties, such as every open ball is closed, do not imply the strong triangle property. Up to now, several researchers gave various studies on ultrametric spaces. Ultrametric spaces were introduced by Van Rooij [8]. Gajic [3] proved a fixed point theorem for a class of generalized contractive mapping on ultrametric spaces. Rao et al. [7] presented two coincidence point theorems for three and four self maps in a spherically complete ultrametric space. Kirk and Shahzad [4] gave some fixed point results on ultrametric spaces. For other significant results on ultrametric spaces, see [5] and [6]. Recently, Alfuraidan [1] has introduced the contraction principle for mappings on a modular metric space with a graph.

In this work, we give a generalization of the Banach contraction principle on an ultrametric space endowed with a graph. Some examples are given to support the results. Our results generalize and extend some well-known results in the literature.

Key Words: Fixed point, ultrametric space, Banach contraction principle.

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Blood Transfusion Management Using Statistical Software

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ABSTRACT

Background. The development of cardiopulmonary bypass (CPB), thereby permitting open-heart surgery, is one of the most important advances in medicine in the 20th century [1]. Since the development of CPB, hemodilution is a frequently used to reduce blood viscosity [2]. This will result in a frequent decrease of patient's hematocrits during CPB with the risks of acute anemia [3]. Despite major advances in blood conservation strategies, transfusion rates remain high in some cardiac surgical patients [4]. The benefits of RBC transfusion include an increase in the oxygen-carrying capacity of blood, improved tissue oxygenation, and improved hemostasis [5]. However, blood transfusion is associated with an increased risk of morbidity and mortality [6]. This study was conducted to describe blood use of in patients undergoing CPB at University Hospital Hassan II of Fez – Department of Cardiac Surgery.

Methods. A retrospective study included 105 cardiac surgery patients aged between 17-72 years, undergoing CPB during the year 2015.

Data used in this study included: demographic Data, clinical Data, laboratory variables, Haemostatic management (hemofiltration, transfusion), in hospital outcomes, and CPB parameters, using SPSS Statistics, version 20.0.

Results. The gender distribution shows 65 % of women and 35 % of men. The average age of the patients was 45.21 years (range, 17-72). 12.4 % of patients received red blood cell (RBC) during CPB. 77 % of transfused group were female.

Valvular surgery was the most common type of surgical. (A) Rh positive blood type is surprisingly the most common among patients undergoing CPB.

Univariate analysis test showed a significant relationship between transfusion and ICU stay length, Nadir hematocrit during CPB, preoperative urea, preoperative creatinine, post-operative urea, post-operative creatinine, and Pump time, combined surgery, coronary surgery, and hemofiltration.

Binary logistic regression identified Nadir hematocrit during CPB as a protective factor, and coronary surgery as a risk factor.

Conclusion. Implementing a restrictive transfusion strategy is rational. Additional studies to investigate predictive factors for blood transfusion are needed. To ensure that the

right patient received the right amount of the right blood at the right time, implementing a virtual blood banking system is rational.

Key Words: Blood Transfusion, CPB, SPSS Statistics

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Boundedness of Hardy Inequality in the Spaces $\ell^{p(\cdot)}$

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ABSTRACT

As in generalized Lebesgue spaces $L_{\mathbb{R}}^{p(\cdot)}$, which is a class of measurable functions, boundedness of Hardy operator can be investigated in variable exponent convergent sequence spaces $\ell_{(\mathbb{N})}^{p(\cdot)}$ by the estimation of variable exponents with constants.

Let ℓ^p be convergent sequence space such that norm for every sequences in this space is defined by,

$$\|a_n\|_p = \left(\sum_{n=1}^{\infty} |a_n|^p \right)^{1/p} < \infty$$

and for a sequence (a_n) let Hardy operator, shown by H , be defined as $H(a_n) = \frac{1}{n} \sum_{k=1}^n a_k$ and

its conjugate as $\tilde{H}(a_n) = \sum_{k=1}^n \frac{a_k}{k}$.

Let $1 \leq p < q < \infty$ be real numbers and v_n, w_n be nonnegative weight sequences. So, the

necessary and sufficient condition for Hardy inequality $\left(\sum_{n=1}^{\infty} \left(\sum_{k=1}^n a_k \right) q \cdot v_n \right)^{1/q} \leq C \cdot \left(\sum_{n=1}^{\infty} a_n^p \cdot w_n \right)^{1/p}$

to hold with $a_n \geq 0$ is finding a constant defined as $C_{pq} = \sup_{n \in \mathbb{N}} \left(\sum_{k=n}^{\infty} v_k \right)^{1/q} \cdot \left(\sum_{k=1}^n w_k^{1-p'} \right)^{1/p'} < \infty$

[Leinder, 1970; Levin ve Steekin, 1960].

In this paper, we investigate necessary conditions for boundedness of Hardy inequality [8] in the cases of p being a function of the variable and variable exponents with following two theorems.

When variable exponent convergent sequence space given as $\ell_{\mathbb{N}}^{p(\cdot)} = \left\{ (a_n) : a_n > 0 \text{ ve } (a_n)^{p(n)} \text{ convergent} \right\}$ and the norm with $I_{p(\cdot)}((a_n)) = \sum_{n=1}^{\infty} |a_n|^{p(n)}$

modular given as $\|a_n\|_{\ell_{(\mathbb{N})}^{p(\cdot)}} = \inf \left\{ \lambda > 0 : I_p \left(\frac{a_n}{\lambda} \right) \leq 1 \right\}$ and $I_n(a_n) \leq 1 \Rightarrow \|a_n\|_{\ell_{(\mathbb{N})}^{p(\cdot)}} \leq 1$ let

$$\|a_n\|_{\ell_{(\mathbb{N})}^{p^+}} \leq I_p(a_n) \leq \|a_n\|_{\ell_{(\mathbb{N})}^{p^-}} : \|a_n\|_{p(\cdot)} \leq 1 \quad (1)$$

$$\Lambda = \left\{ p(\cdot) : \mathbb{N} \rightarrow \mathbb{R} : \left| p((2n) - p(n) \ln(n)) \leq c \right| \right\} \quad (2)$$

hold.

Theorem 1. Let $\beta \in \mathbb{R}, p : (0, l) \rightarrow [1, \infty)$ be an increasing function such that $\lim_{n \rightarrow 0} p(n) = p(0), \beta < 1 - \frac{1}{p(0)} = \frac{1}{p'(0)}, p^- > 1, \left(H(a_n) = \sum_{k=1}^n a_k \right)$. The necessary condition for

$$\|n^{\beta-1} H(a_n)\|_{\ell^{p(\cdot)}} \leq C \|n^\beta a_n\|_{\ell^{p(\cdot)}} \quad (3)$$

inequality to hold is condition (2).

Theorem 2. Let $p \in \mathbb{R}, \beta : (0, l) \rightarrow \mathbb{R}$ be a decreasing function such that $\beta(0) = \lim_{n \rightarrow 0} \beta(n)$ and $\beta(0) < 1 - \frac{1}{p} = \frac{1}{p'}, p^- > 1$. The necessary condition for

$$\|n^{\beta(n)-1} H(a_n)\|_{\ell^p} \leq C \|n^{\beta(n)} a_n\|_{\ell^p} \quad (4)$$

inequality to hold is condition (2) holding with $\beta(n)$.

Key Words: $\ell^p(\cdot)$ spaces, boundedness, Hardy operator.

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Comparison of Goodness of Fit Tests of Uniformity

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ABSTRACT

Goodness of fit (GOF) of a statistical model tests how well it fits a set of observations. There are many classical GOF tests to test uniformity in the literature. Kolmogorov-Smirnov (1933-1939), Anderson-Darling (1954), Cramer von Mises (1928-1930), Watson (1961) are the most important GOF tests. These tests can be applied to many difference distributions. In this study, classical GOF tests and Zamanzade (2014)'s GOF tests based on entropy are compared according to power and type one error rates. One of the main problems of these tests is that the exact distribution of the test statistic cannot be obtained easily. Thus, the critical values cannot be obtained exactly. Initially, we obtain the critical values of these test statistics by using Monte Carlo simulations in various sample sizes for nominal alpha (α) 0.05. According to these critical values, empirical type one errors and powers of test can be calculated. To compare the performance of the GOF tests, we use the alternative distribution families given by Stephens (1974). These distribution families are as follows; A_k, B_k, C_k : Single-mode with non-symmetrical, symmetrical single-mode and symmetrical bimodal distributions, respectively.

Kolmogorov-Smirnov, Anderson-Darling, Cramer von Mises, Watson and Zamanzade (2014)'s GOF tests (TB1, TB2) which are based on entropy for uniform distribution are compared in various situation of alternative distribution families.

The empirical type one error rates of all considered tests are close to nominal level in all situations. It is observed that the investigated GOF tests for uniform distribution show different performances for different alternative families of distributions. Cramer von Mises Test, Watson Test and TB2 test yield more powerful results in certain situations. From simulation results, it can be said that Cramer vonMises, TB2 and Watson Tests reached their highest power value for each sample obtained from A_k, B_k and C_k distribution families, respectively.

Key Words: Uniformity, Entropy and Goodness of fit tests.

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Comparison of the Efficiencies of Different Shrinkage Parameters in Ridge Regression

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ABSTRACT

The most common method to estimate the regression coefficients is the Least Squares (LS) method. However, to have valid results, some assumptions need to be provided for the LS method. For example, it is desired that no multicollinearity between the independent (explanatory) variables exists. It is known that, in real world problems this assumption is very difficult to achieve. There are various methods have been developed to solve this problem; one of which is the 'biased estimation method'. One of the most widely used biased estimation methods is Ridge Regression. The purpose of Ridge Regression is to obtain smaller mean square errors (MSE) than the LS method. In this method, selection of the shrinkage parameter is very important. By using optimum shrinkage parameter, more accurate results can be obtained than LS method. Many different optimum k values are described in the literature. Hoerl and Kennard (1970) gave an optimum k value to minimize the MSE of Ridge estimator. Also, many different authors obtained the optimum k values according to different criteria. Theobald (1974), Hoerl et. al.(1975), Lawless and Wang (1976), Khalaf and Shukur (2005), Alkhamisi and Shukur (2008) and Muniz and Kibria (2009) are the other popular studies in this area. In this study, we compare these popular k values according to their MSE criteria and their accept rate of the testing procedure given in Liski (1982, 1983) via simulation. The simulation results show that different optimum k values give the best results for different parameter combinations.

Key Words: Ridge Estimator, Mean Square Error, Optimum Ridge Parameter, Monte Carlo.

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Coupled Fixed Point and Related Results in Multiplicative Metric Spaces

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ABSTRACT

Recently, Florack and Assen [4] showed that multiplicative calculus (non-Newtonian calculus) provided a suitable concept for biomedical image analysis especially in problems in which positive images or definite matrix fields and positivity preserving operators are of interest. There exists many fundamental problems and concepts that have not been considered in the framework of multiplicative calculus.

In this paper, we introduce and consider the notion of coupled fixed point and related results in the concept of multiplicative metric space. We therefore aim to contribute and develop this framework.

Key Words: Coupled fixed point, mixed monotone property, multiplicative metric space, multiplicative calculus.

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Estimate of the Norm of the Difference of Two Solutions for the Singular Sturm-Liouville Problem

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ABSTRACT

In this study, we obtain a bound for difference of the solutions for Singular Sturm-Liouville problems with Coulomb potential. Stability problems of differential operators is important issue but not common. The meaning of the stability problem is to calculate the difference of spectral functions, solutions and potentials when finite numbers of eigenvalues is coincided. These type problem for regular Sturm-Liouville problems was studied Ryabushko [5]. Marchenko and Maslov [3] dealt with similar issue in the case of the spectral function coincide on given interval.

Consider the problems

$$L_1 y = -y'' + \left(q(x) + \frac{A}{x} \right) y = \lambda y, \quad 0 < x \leq \pi \quad (1)$$

$$\begin{aligned} y(0) &= 0, \\ y'(\pi) - Hy(\pi) &= 0, \end{aligned} \quad (2)$$

and

$$\begin{aligned} y(0) &= 0, \\ y'(\pi) - Hy(\pi) &= 0, \end{aligned} \quad (3)$$

where the function $q(x) \in L^1[0, \pi]$ and A, H, H is a real constants, $\frac{y(x)}{x} \in C[0, \pi]$. Let $\{\lambda_{1,k}\}$ and $\{\mu_{1,k}\}$ be the spectrums of the problems (1)-(2) and (1)-(3), respectively.

We consider the new problems

$$L_2 y = -y'' + \left(q(x) + \frac{A}{x} \right) y = \lambda y, \quad 0 < x \leq \pi \quad (4)$$

with conditions (2) and (3), where the function $q(x) \in L^1[0, \pi]$. Let $\{\lambda_{2,k}\}$ and $\{\mu_{2,k}\}$ be the spectrums of the problems (4)-(2) and (4)-(3), respectively.

The solutions of problems (1)-(2) and (3)-(4) have following forms

$$\begin{aligned}\varphi(\lambda, x) &= \frac{\sin \sqrt{\lambda} x}{\sqrt{\lambda}} + \int_0^x \frac{\sin \sqrt{\lambda}(x-t)}{\sqrt{\lambda}} \left\{ \frac{A}{t} + q(t) \right\} \varphi(\lambda, t) dt, \\ \psi(\lambda, x) &= \frac{\sin \sqrt{\lambda} x}{\sqrt{\lambda}} + \int_0^x \frac{\sin \sqrt{\lambda}(x-t)}{\sqrt{\lambda}} \left\{ \frac{A}{t} + q(t) \right\} \psi(\lambda, t) dt,\end{aligned}$$

When the eigenvalues $\{\lambda_{j,k}\}$ and $\{\mu_{j,k}\}, (j=1,2)$ coincide numbers of $N+1$ for $k=1,2,\dots,N+1$, we must evaluate the difference of the solutions of (1)-(2), (3)-(4) and problems. The main theorem in this study is following:

Theorem 1. When the eigenvalues $\{\lambda_{j,k}\}$ and $\{\mu_{j,k}\}, (j=1,2)$ coincide numbers of $N+1$ such that $\lambda_{1,k} = \lambda_{2,k}$ and $\mu_{1,k} = \mu_{2,k}$ for $k=1,2,\dots,N+1$, then

$$|\varphi(\lambda, x) - \psi(\lambda, x)|^2 < \frac{2}{N} \exp \left\{ \frac{2\sigma(x)}{\sqrt{N}} + 2 \left\{ \sigma_1(x) - \sigma_1 \left(x - \frac{1}{\lambda} \right) \right\} \right\} \rho_{1,m} \left(\frac{N}{2} \right) \frac{8K}{N^2 \left(3 + \frac{6}{N} + \frac{3-4K}{N^2} \right)} e^{\frac{8K}{N^2 \left(3 + \frac{6}{N} + \frac{3-4K}{N^2} \right)}}$$

for $k > N+1$, $m < \frac{N}{2}$, where

$$K = \int_0^\pi \{q(t) - q(t)\} dt + O\left(\frac{\ln k}{k^2}\right), \sigma(x) = \int_0^x \left| \frac{A}{t} + q(t) \right| dt, \sigma_1(x) = \int_0^x \int_0^t \left| \frac{A}{\tau} + q(\tau) \right| d\tau dt.$$

Key Words: Stability problem, coulomb potential, eigenvalues.

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Existence of Nondecreasing Solutions of Some Nonlinear Integral Equations of Fractional Order

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ABSTRACT

Many nonlinear problems arising from areas of the real world, such as natural sciences, can be represented with operator equations. Especially, integral and differential equations of fractional order play a very important role in describing these problems. For example, some problems in physics, mechanics and other fields can be described with the help of integral and differential equations of fractional order. Some of these problems are theory of neutron transport, the theory of radioactive transfer, the kinetic theory of gases [1], the traffic theory and so on.

The purpose of this paper is to examine the class of functional integral equations of fractional order in the space of continuous functions on interval $[0, a]$. Using a Darbo type fixed point theorem associated with the measure of noncompactness, we present sufficient conditions for existence of nondecreasing solutions of some functional integral equations of fractional order. These existence results include several obtained from previous studies. Finally, we establish some examples to show that our results are applicable.

Key Words: Nonlinear integral equations, measure of noncompactness, nondecreasing solution, Darbo fixed point theorem, Riemann–Liouville fractional integral.

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Faber Polynomial Coefficient Estimates For Analytic Bi-Close-To-Convex Functions Defined By Fractional Calculus

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ABSTRACT

A function is said to be bi-univalent on the open unit disk if both the function and its inverse are univalent in unit disc. In this paper, using the Faber Polynomials, we obtain coefficient expansions for analytic bi-close-to-convex functions defined by fractional calculus and determine coefficients for such functions. We also demonstrate the unpredictable behavior of the early coefficients of subclasses of bi-univalent functions. For some special cases, also we show that our class is generalization class of them.

Let A denote the family of functions analytic in the open unit disk $D := \{z \in \mathbb{C} : |z| < 1\}$ and let S be the class of functions $f \in A$ that are univalent in D and normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n.$$

For α ; $0 \leq \alpha < 1$, we let $S^*(\alpha)$ denote the class of function $g \in S$ that are starlike of order α in D , that is, $\operatorname{Re}[zg'(z)/g(z)] > \alpha$ in D and $C(\alpha)$ denote the class of functions $f \in S$ that are close-to-convex of order α in D , that is, if there exists a function $S^*(0)$ so that $\operatorname{Re}[zf'(z)/g(z)] > \alpha$ in D (e.g. see [9] or [10]). We note that $S^*(\alpha) \subset C(\alpha) \subset S$ and that $|a_n| \leq n$ for $f \in S$ by de Branges' Theorem [11], also known as the Bieberbach Conjecture.

A fairly complete treatment, with applications of the fractional calculus, is given in the books [12] by Oldham and Spanier, and [13] by Miller and Ross. We refer to [14] for more insight into the concept of the fractional calculus.

Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Then $f(z)$ is said to be λ fractional close to convex in D if there

exists a function $g(z) \in S^*$ such that $\operatorname{Re} \left(\frac{D(D^\lambda f(z))}{g(z)} \right) > 0$ for all z in D . The class of these

functions is denoted by $K(\lambda)$

The bi-close to convex functions considered in this paper are largest subclass of bi-univalent functions and generalization of the results of the paper in [4].

Key Words: bi-univalent, close-to-convex, fractional operator, starlike..

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Functions of Non-iid Random Vektors Expressed as Functions of iid Random Vectors in The Multivariate Order Statistics

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ABSTRACT

In the study, the definition of multivariate order statistics is given Furthermore, the theorem related to distribution of order statistics of innid (independent not necessarily identically distributed) random variables to that of order statistics of iid (independent and identically distributed) random variables is generalized for multivariate case.

Key Words: Multivariate order statistics, independent not necessarily identically distributed, independent and identically distributed.

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f - Statistical Convergence of Order α for Double Sequences

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ABSTRACT

The idea of statistical convergence was proposed by Zygmund [11] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [9] and Fast [4] and later reintroduced by Schoenberg [8] independently. The idea of statistical convergence was later extended to double sequences by Tripathy [10], Mursaleen and Edely [7] and Moricz [6].

In 2014, Aizpuru et al. [1] defined new concepts of f -density and f -statistical convergence for double sequence of complex or real numbers, where f is an unbounded modulus.

In this study, we introduce concepts of $f_{\bar{\alpha}}$ -density and $f_{\bar{\alpha}}$ -statistical convergence for double sequence of complex or real numbers, where $0 \leq \bar{\alpha} \leq 1$. Also, some relations between $f_{\bar{\alpha}}$ -statistical convergence and strong $f_{\bar{\alpha}}$ -Cesaro summability of order $\bar{\alpha}$ are given.

Key Words : Modulus function, density, statistical convergence.

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Generalized Fractional Integral Operators on Vanishing Generalized Local Morrey Spaces

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ABSTRACT

In this talk, the Spanne-Guliyev type boundedness of the generalized fractional integral operators on the vanishing generalized local Morrey spaces $VLM_{p,\varphi}^{\{x_0\}}(R^n)$ and vanishing weak generalized local Morrey spaces $VWLM_{p,\varphi}^{\{x_0\}}(R^n)$ will be proved. Also the Adams-Guliyev type boundedness of the operators on the vanishing generalized Morrey spaces $VM_{p,\varphi}(R^n)$ and vanishing weak generalized Morrey spaces $VWM_{p,\varphi}(R^n)$ will be proved.

Key Words: vanishing generalized local Morrey spaces, generalized fractional integral operators.

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Generalized Lacunary Statistical Convergence of Sequences of Fuzzy Numbers

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ABSTRACT

Lacunary statistical convergence of order α in real number sequences was defined by Şengül and Et [8]. After then, Altınok *et al.* [2] introduced and examined the class of sequence $bv_\theta(\Delta, F)$ using a lacunary sequence θ and the difference operator Δ in sequences of fuzzy numbers, and study some of its properties like solidity and symmetricity.

The purpose of this paper is to generalize the study of Şengül and Et [8] so as to fill up the existing gaps in the theory of lacunary statistical convergence for sequences of fuzzy numbers. In this study, we define the sets of lacunary statistically convergent sequences of order β and strongly summable lacunary statistically convergent sequences of order β for sequences of fuzzy numbers using generalized difference operator Δ^m and give some inclusion relations between them. Furthermore, we obtain some results using a modulus function f .

Let $\theta = (k_r)$ be a lacunary sequence, $X = (X_k)$ be a sequence of fuzzy numbers and $\beta \in (0, 1]$ be given. The sequence $X = (X_k) \in w(F)$ is said to be $S_\theta^\beta(F, \Delta^m)$ -statistically convergent (or lacunary statistically convergent sequence of order β) if there is a fuzzy number X_0 such that

$$\lim_{r \rightarrow \infty} \frac{1}{h_r^\beta} \left| \left\{ k \in I_r : d(\Delta^m X_k, X_0) \geq \varepsilon \right\} \right| = 0,$$

where $I_r = (k_{r-1}, k_r]$ and h_r^β denote the β th power $(h_r)^\beta$ of h_r , that is $h^\beta = (h_r^\beta) = (h_1^\beta, h_2^\beta, \dots, h_r^\beta, \dots)$. In this case we write $S_\theta^\beta(F, \Delta^m)\text{-}\lim X_k = X_0$. The set of all $S_\theta^\beta(F, \Delta^m)$ -statistically convergent sequences will be denoted by $S_\theta^{\beta, \Delta^m}$. For $\theta = (2^r)$, we shall write $S^\beta(F, \Delta^m)$ instead of $S_\theta^\beta(F, \Delta^m)$ and we shall write $S(F, \Delta^m)$ instead of $S_\theta^\beta(F, \Delta^m)$ in the special case $\beta = 1$ and $\theta = (2^r)$.

Key Words: Fuzzy sequence, lacunary convergence, difference sequence.

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Generalizations of Ostrowski Type Inequalities on Time Scales

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ABSTRACT

In 1938, Ostrowski derived a formula to estimate the absolute deviation of a differentiable function from its integral mean, the so-called Ostrowski inequality holds and can be shown by using the Montgomery identity. These two properties was proved by Bohner and Matthews for general time scales, which unify discrete, continuous and many other cases.

In 1988, Hilger introduced the time scale theory in order to unify continuous and discrete analysis. Such theory has a tremendous potential for applications in some mathematical models of real processes and phenomena studied in population dynamics, economics, physics, space weather and so on. Recently, many authors studied the theory of certain integral inequalities on time scales.

The purpose of this work is to obtain generalizations of Ostrowski type inequalities, Ostrowski type inequalities for two functions and perturbed Ostrowski type inequalities on time scales. These results not only provide a generalization of the known results, but also give some other interesting inequalities on time scales as special cases.

Key Words: Ostrowski inequality, Perturbed Ostrowski inequality, Time scales.

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Hilbert Transform on Local Morrey-Lorentz Spaces

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ABSTRACT

In this talk we define a new class of functions called local Morrey-Lorentz spaces $M_{p,q;\lambda}^{loc}$, $0 < p, q \leq \infty$ and $0 \leq \lambda \leq 1$. These spaces generalize Lorentz spaces such that $M_{p,q;0}^{loc} = L_{p,q}$.

We show that in the case $\lambda < 0$ or $\lambda > 1$ the space $M_{p,q;\lambda}^{loc}$ is trivial, and in the limiting case $\lambda = 1$ the space $M_{p,q;\lambda}^{loc}$ is the classical Lorentz space $\Lambda_{\infty,t^{1/p-1/q}}$.

We prove that for $0 < q \leq p < \infty$ and $0 < \lambda \leq q/p$, the local Morrey-Lorentz spaces $M_{p,q;\lambda}^{loc}$ are equal to weak Lebesgue spaces $WL_{1/p-\lambda/q}$. Furthermore, we obtain the boundedness of the Hilbert transform in the local Morrey-Lorentz spaces.

Key Words: Morrey spaces, Lorentz spaces, local Morrey-Lorentz spaces, Hilbert transform, Hardy operator.

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Inverse Nodal Problem For P -Laplacian String Equation

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ABSTRACT

In this study, we solve inverse nodal problem for p -Laplacian string equation with boundary conditions. Asymptotic formulas of eigenvalues, nodal points and nodal lengths are obtained by using modified Prüfer substitution. The key step is to apply modified Prüfer substitution to derive a detailed asymptotic estimate for eigenvalues. Furthermore, a reconstruction formula for density function of p -Laplacian string equation is given by using nodal lengths. Obtained results are more general than the classical string problem.

Key Words: p -Laplacian, Inverse Nodal Problem, Prüfer Substitution, String Equation

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Inverse Nodal Problem For p -Laplacian Dirac System

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ABSTRACT

In this study, we solve inverse nodal problem for p -Laplacian Dirac system with boundary conditions depending polynomially on the spectral parameter. Asymptotic formulas of eigenvalues, nodal points and nodal lengths are obtained by using modified Prüfer substitution. The key step is to apply modified Prüfer substitution to derive a detailed asymptotic estimate for eigenvalues. Furthermore, we have shown that $r(x)$ and $q(x)$ functions in Dirac system can be established uniquely by using nodal parameter by the method used in [1]. Obtained results are more general than the classical Dirac system.

Keywords: p -Laplacian, Inverse Nodal Problem, Prüfer Substitution, Dirac System.

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Inverse Nodal Problem For p -Laplacian Sturm-Liouville Equation With Boundary Condition Polynomially Dependent On Spectral Parameter

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ABSTRACT

In this paper, solution of inverse nodal problem for one-dimensional p -Laplacian equation is extended to the case that boundary condition polynomially eigenparameter. To find the spectral datas as eigenvalues and nodal parameters, a Prüfer substitution is used. Then, reconstruction formula of the potential function is also given by nodal lengths. However, this method is similar to used in [1], our results are more general.

Keywords: Inverse Nodal Problem, Prüfer Substitution, Sturm-Liouville Equation.

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Korovkin Theory for Extraordinary Test Functions by A-Statistical Convergence

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ABSTRACT

Korovkin-type theorems have been extended with the aim of finding other subsets of functions satisfying the same property as $\{1, x, x^2\}$. In this paper we introduce A-statistical Korovkin subset for a positive linear operator T . We also characterize that a subset of the space of the closure of the functions which have compact support is an A-statistical Korovkin subset for T .

Key Words: A-statistical convergence, Korovkin subset.

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Limit Theorem for a Renewal – Reward Process with an Asymmetric Triangular Interference of Chance

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ABSTRACT

In this study, a semi-Markovian inventory model of type $(s; S)$ is considered and the model is expressed by means of renewal-reward process $(X(t))$ with an asymmetric triangular distributed interference of chance and delay. The ergodicity of the process $X(t)$ is proved and the exact expression for the ergodic distribution is obtained. Then, two-term asymptotic expansion for the ergodic distribution is gotten for standardized process $W(t)=(2X(t))/(S-s)$. Finally, using this asymptotic expansion, the limit theorem for the ergodic distribution of the process $W(t)$ is proved and the explicit form of the limit distribution is found.

Key Words: Inventory model of type (S,s) , Renewal – reward process, Asymmetric triangular distribution, Limit distribution.

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Matrix Maps Between Some Sequence Spaces

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ABSTRACT

The study of matrix maps between sequence spaces originates from a natural problem in classical summability theory, namely to characterize all infinite matrices that transform every convergent sequence into a convergent sequence. In this work we determine multipliers and duals of certain linear topological spaces. We also establish some estimates for the norms of bounded linear operators defined by those matrix mappings.

Key Words: Matrix operators, dual spaces.

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Modelling of Dengue Disease under Laplacian Random Effects

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ABSTRACT

Diseases are modelled mathematically mostly through a deterministic approach. In this study, we discuss a random approach to the mathematical modelling of epidemics using the example of dengue disease. Dengue fever is a mosquito-borne disease caused by the dengue virus. Deterministic mathematical models of dengue disease are based on the use of compartmental models such as SIR, SIS, SEIR and etc. The parameters of the compartmental models are added random effects to analyse the advantages of a random analysis for epidemics. The parameters of the deterministic model become random variables hence transforming the deterministic equation system into a system of nonlinear random differential equations. The analysis of the new random model offers more universal results for the disease. The interpretations based on the random dynamics of the disease offer a better foresight for the possible variations in the behaviour of the virus. A Random Model of the disease transmission and virus behaviour can be more successful in the guidance for treatment and medication. For this aim, numerical characteristics of the random model are investigated. Monte Carlo methods are used to simulate the characteristics of the model such as the expectation, variation and etc. Random behaviour of the virus dynamics are compared to the numerical results of the deterministic differential equation system to visualize the difference caused by the random effects. These differences are investigated to make comments on the disease dynamics.

Key Words: Random Effect, Random Differential Equation, Dengue Disease, Expected Value, Random Characteristics, Compartmental Models.

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Non-Isotropic Potential Theoretic Inequality

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ABSTRACT

In this study, it is re-established the new weighted inequalities were derived β –distance which is similar to the given inequality for the potential operator which is obtained by using Euclidean distance.

Here β –distance:

Let $\beta = (\beta_1, \beta_2, \dots, \beta_n)$, $\beta_k \geq \frac{1}{2}$, $k = 1, 2, \dots, n$ and $|\beta| = \beta_1 + \beta_2 + \dots + \beta_n$. For $x = (x_1, x_2, \dots, x_n), y = (y_1, y_2, \dots, y_n)$

$$|x - y|_\beta := \left(|x_1 - y_1|^{\frac{1}{\beta_1}} + |x_2 - y_2|^{\frac{1}{\beta_2}} + \dots + |x_n - y_n|^{\frac{1}{\beta_n}} \right)^{\frac{|\beta|}{n}}, \quad t > 0.$$

is non-isotropic distance or β –distance x and y , given in [2], ([8] – [11]), [15].

Definition: Let f and h be measurable functions such that $f \in L_1^{loc}(\mathbb{R}^n)$ and $h \geq 0$, we set the fractional integral generated by β –distance of order p as

$$I_p^\beta(f)(x) = \int_{\mathbb{R}^n} \frac{|f(y)|}{|x - y|_\beta^{(n-p)\frac{2|\beta|}{n}}} dy.$$

And we get generalized fractional integral generated by β –distance;

$$I_{p,h}^\beta(f)(x) = \int_{\mathbb{R}^n} \frac{|f(y)|}{|x - y|_\beta^{(n-p)\frac{2|\beta|}{n}} h(|x - y|_\beta)} dy.$$

Key Words: Adams trace inequality, Stummel class, Morrey spaces, non-isotropic distance.

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Numbers Producing Richard Distribution

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ABSTRACT

Today growth models and logistic distribution are some of the most emphasized topics for statisticians. Field of application's broadness also contributes to the importance of the subject. Recent developments in this topic are applied to many scientific fields, mainly in economics, medicine, industry, social sciences. In economics, researches are conducted on economic development, commercial companies' capital increase, production augmentation and topics alike. In medicine, observing a child's growth, the examined substance's increase on blood tests and similar analyses are some of the most common topics.

In data analysis for growth model, in order to convey an accurate analysis, it is crucial that the flexibility of the chosen link function is sufficient, in other words, the model contains plenty of parameters to provide this flexibility. This feature allows a more accurate statistical examination by increasing the compatibility of the model with the data. In the recognized link functions, the Richard function provides the most link flexibility.[3] Richard link function possesses two parameters as k and m , the k parameter being related to the curve's shape and the m parameter being related to the location of the curve.[2]

In this study, it's aimed to demonstrate that as m parameter changes, the average is not affected considerably, but the k parameter of the average is varied by producing numbers using Richard link function. Naturally, the fact that the average of the distribution of a Richard link function cannot be calculated creates an inability to interpret this result analytically. Regardless, it can be observed that visually created values depend actively on the k parameter in contrast with they are barely affected by the m parameter. This is a significant property of link function, provided by its analytical structure.[1]

Key Words: Growth Model, Logistic Distribution, Richard Distribution,

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On Asymptotically Deferred Statistical Equivalent of Sequences

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ABSTRACT

Marouf [3] introduced definitions for asymptotically equivalent sequences and asymptotic regular matrices. Patterson [4] extend these concepts by presenting an asymptotically statistically equivalent analogy of these definitions and natural regularity conditions for nonnegative Summability matrices. In this study we introduce the concepts of asymptotically deferred statistical equivalent and strong asymptotically deferred equivalent of sequences. Some relations between asymptotically deferred statistical equivalent and strong asymptotically deferred equivalent of order α of sequences are given.

Key Words: Statistical Convergence, Asymptotically Statistical Equivalent, Deferred Statistical Convergence ; Deferred Cesàro Summability.

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On Feng Qi-Type Integral Inequalities for Local Fractional Integrals

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ABSTRACT

In the last few decades, much significant development of integral inequalities had been established. Recall the famous

$$\int_a^b [f(t)]^{n+2} dt \geq \left[\int_a^b f(t) dt \right]^{n+1} \quad \text{integral inequality of Feng Qi type:}$$

(1)

where $f \in C^n(a,b)$, $f^{(i)} \geq 0$, $0 \leq i \leq n$, $f^{(n)} \geq n!$, $n \in \mathbb{N}$.

Let $f(x) \geq 0$ be a continuous function on $[0,1]$ satisfying

$$\int_x^1 f(t) dt \geq \int_x^1 t dt, \quad \forall x \in [0,1]. \quad (2)$$

Then the inequalities

$$\int_0^1 f^{\alpha+1}(t) dt \geq \int_0^1 t^\alpha f(t) dt, \quad (3)$$

and

$$\int_0^1 f^{\alpha+1}(t) dt \geq \int_0^1 t f^\alpha(t) dt, \quad (4)$$

hold for every positive real number $\alpha > 0$. (3) and (4) inequalities are known in the literature as Qi-type inequalities.

In this study, we establish the generalized Qi-type inequality involving local fractional integrals on fractal sets \mathbb{R}^α ($0 < \alpha < 1$) of real line numbers.

Some applications for special means of fractal sets \mathbb{R}^α are also given. The results presented here would provide extensions of those given in earlier works.

Firstly we give a important integral inequality which is generalized Qi inequality. Then, we establish weighted version of generalized Qi inequality for local fractional integrals. Finally, we obtain several inequalities related these inequalities using the local fractional integral.

Key Words: Qi inequalities, Integral inequalities, Local Fractional integral, Riemann-Liouville Fractional integrals.

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On Grüss Type Inequalities For Generalized Fractional Integral

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ABSTRACT

The aim of the present paper is to examine some new integral inequalities of Grüss type for h -Riemann-Liouville k -fractional integrals. From our consequence, new heavy or classical Grüss type inequalities have been established for some special cases. Moreover, special cases of the integral inequalities in this paper have been obtained by Dahmani and Tabharid, 2010 in [2].

Definition 1: The first is the Riemann-Liouville fractional integral of order $\alpha > 0$ for a continuous function f on $[a, b]$ which is defined by

$$I_a^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t - \tau)^{\alpha-1} f(\tau) dt; \quad \alpha \leq t \leq b. \quad (1.1)$$

Definition 2: Later, under the above definitions, in [9], Mubeen and Habibullah have introduced the k -fractional integral of the Riemann-Liouville type as follows:

$${}_k J_a^\alpha f(t) = \frac{1}{k\Gamma_k(\alpha)} \int_a^x (x - t)^{\frac{\alpha}{k}-1} f(t) dt; \quad \alpha > 0, \quad x > a. \quad (1.2)$$

Definition 3: Let $k > 0$, $f \in L[(a, b)]$ and $h(x)$ be an increasing and positive monotone function on $[a, \infty)$ and also derivative $h'(x)$ is continuous on $[a, \infty)$. The h -Riemann-Liouville k -fractional integral of a function f with respect to another function $h(x)$ is defined by

$${}_k I_{a^+; h}^\alpha f(x) = \frac{1}{k\Gamma_k(\alpha)} \int_a^x \frac{f(t)h'(t)dt}{(h(x) - h(t))^{1-\frac{\alpha}{k}}}, \quad (x > a; \quad \alpha > 0) \quad (1.3)$$

and

$${}_k I_{b^-; h}^\alpha f(x) = \frac{1}{k\Gamma_k(\alpha)} \int_x^b \frac{f(t)h'(t)dt}{(h(x) - h(t))^{1-\frac{\alpha}{k}}}, \quad (x < b; \quad \alpha > 0). \quad (1.4)$$

We observe that clearly ${}_k I_{a^+; h}^0 f(x) = {}_k I_{b^-; h}^0 f(x) = f(x)$.

Theorem: Let f and g be two integrable function on $[a, b]$ with, $m_1 < f(x) < m_2$, $m_3 < g(x) < m_4$ and let p be a positive function on $[a, b]$. Then for all $t > 0$, $k > 0$, $\alpha > 0$, there is following inequality

$$\begin{aligned} & |[({}_k I_{a^+; h}^\alpha pfg)(t)][({}_k I_{a^+; h}^\alpha p)(t)] - [({}_k I_{a^+; h}^\alpha pf)(t)][({}_k I_{a^+; h}^\alpha pg)(t)]| \\ & \leq (m_2 - m_1)(m_4 - m_3) \frac{[({}_k I_{a^+; h}^\alpha p)(t)]^2}{4}. \end{aligned}$$

Key Words: Fractional Integral, Riemann-Liouville Fractional Integrals, Grüss-Type Integrals, k - Fractional Integral.

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On Monotonic Solutions of Some Nonlinear Fractional Integral Equations

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ABSTRACT

It is well known that nonlinear integral and differential equations create an important branch of nonlinear analysis. A lot of nonlinear problems arising from areas of the real world are generally represented with integral and differential equations. Especially, integral and differential equations of fractional order play a very important role in modelling of some problems in physics, mechanics and other fields in natural sciences. For instance, these equations are used in describing of some problems in theory of neutron transport, the theory of radioactive transfer, the kinetic theory of gases [1], the traffic theory and so on.

In this study we examine the solvability of the following nonlinear integral equation of fractional order in $C[0,a]$ which is family of all real valued and continuous functions defined on the interval $[0,a]$

$$x(t) = f(t, x(t)) + \frac{(Tx)(t)}{\Gamma(\alpha)} \int_0^t \frac{u(t, s, (Gx)(s))}{(t-s)^{1-\alpha}} ds.$$

We present some sufficient conditions for existence of nondecreasing solutions of the above equation. Then using a Darbo type fixed point theorem associated with the measure of noncompactness we prove that this equation has at least one nondecreasing solution in $C[0,a]$. Finally we present some examples to show that our result is applicable.

Key Words: Nonlinear integral equations, measure of noncompactness, nondecreasing solution, Darbo fixed point theorem, Riemann–Liouville fractional integral.

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On Monotonic Solutions of a Nonlinear Integral Equation of Volterra Type

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ABSTRACT

Using a technique associated with measure of noncompactness we prove the existence of the nondecreasing solutions to an integral equations of Volterra type in the Banach space of real functions defined and continuous on a bounded and closed interval.

The main object of the present paper is to study the solvability of nonlinear Volterra integral equations. The theory of the equations of such a type is very developed. Nevertheless, there are a lot of problems concerning the solvability of those equations in some classes of functions which are not satisfactory and completely solved up to now.

The theory of integral equations has been well developed with the help of various tools from functional analysis, topology and fixed-point theory.

The aim of this paper is to investigate the existence of a nonlinear integral equation of Volterra type. Equations of such kind contain, among others, integral equations of convolution type. Our results will be established using a measure of noncompactness defined in [5].

The main tool used in our investigations is a special measure of noncompactness constructed in such a way that its use enables us to study the solvability of considered equations in the monotonic functions.

Let us mention that the theory of integral equations has many useful applications in describing numerous events and problems of real world.

Key Words: Nonlinear Volterra integral equations, Measure of noncompactness, Fixed point theorem.

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On New Fractional Hermite-Hadamard Type Inequalities for (α^*, m) – convex functions

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ABSTRACT

In recent years, theory of convex functions has received special attention by many researchers because of its importance in different fields of pure and applied sciences such as optimization and economics. Consequently the classical concepts of convex functions has been extended and generalized in different directions using novel and innovative ideas, see [3, 5, 6, 9, 10]. A function $f: I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ is said to be convex if the inequality $f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$

holds for all $x, y \in I$ and $t \in [0, 1]$. One of the most famous inequality for convex functions is so called Hermite-Hadamard inequality that stated as the follow;

Let I be an interval of real numbers and $a, b \in I$ with $a < b$. If $f: I \rightarrow \mathbb{R}$ is a convex function, then

$$f\left(\frac{a+b}{2}\right) \leq \int_a^b f(x) dx \leq \frac{f(a) + f(b)}{2}.$$

The Hermite-Hadamard inequality usually stated as a results valid for convex functions only, actually holds for some different type of convexity. For recent results concerning Hermite-Hadamard inequalities obtained via different type of convexity, see ([1] – [12]). In [6], V.G. Miheşan defined (α^*, m) –convexity as the following:

The function $f: [0, b] \rightarrow \mathbb{R}, b > 0$, is said to be (α^*, m) –convex, where $(\alpha^*, m) \in [0, 1]^2$, if we have

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y)$$

for all $x, y \in [0, b]$ and $t \in [0, 1]$.

Denote by $K_m^{\alpha^*}(b)$ the class of all (α^*, m) –convex functions on $[0, b]$ for which $f(0) < 0$. It can be easily seen that for $(\alpha^*, m) = (1, m)$, (α^*, m) –convexity reduces to m -convexity; $(\alpha^*, m) = (\alpha^*, 1)$, (α^*, m) –convexity reduces to α^* -convexity and for $(\alpha^*, m) = (1, 1)$, (α^*, m) convexity reduces to the concept of usual convexity defined on $[0, b], b > 0$. For recent results and generalizations concerning (α^*, m) convex functions, see [1].

The aim of the present paper is to investigate some new Hermite-Hadamard type integral inequalities for (α^*, m) –convex functions via Riemann-Liouville fractional integrals.

Key Words: Fractional Hermite-Hadamard inequalities, m –convex function, (α^*, m) -convex function

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On Nonlinear Singular Integrals Depending on Three Parameters

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ABSTRACT

In [1], Taberski studied both the pointwise convergence of functions in $L_1 < -\pi, \pi >$ and the approximation properties of their derivatives by a two parameter family of convolution type singular integral operators. In 1964, Taberski [2] stepped up his analysis to two-dimensional singular integral operators. Musielak [3] studied the problem of approximation by considering nonlinear integral operators and obtained significant results. After that, Swiderski and Wachnicki [4] studied the convergence of nonlinear singular integrals depending on two parameters. Nonlinear singular integral operators are used in many branches of mathematics such as medicine and engineering. Therefore, their popularity is gradually increasing.

In this paper, we prove some theorems on pointwise convergence of nonlinear and radial type double singular integrals in the space of Lebesgue integrable functions.

Key Words: Radial kernel, nonlinear analysis, pointwise convergence.

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On Some Common Fixed Point Theorems For $(\mathcal{F}, \psi)_f$ - Contraction Mappings In Ordered Partial Metric Spaces

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ABSTRACT

In the present paper, some common fixed point theorems have been proved for two Banach pairs of mappings satisfying certain $(\mathcal{F}, \psi)_f$ -contraction conditions in the context of partially ordered partial metric spaces and some consequences have been obtained in these spaces.

Key Words: Partially ordered complete partial metric space, Fixed point, Common fixed point, $(\mathcal{F}, \psi)_f$ -contraction.

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On Some Developments Associated With The Quasilinear Operators

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ABSTRACT

In [1], Aseev introduced the concepts of “quasilinear spaces” and “quasilinear operators”, by weakening the requirement of linearity in the construction of linear functional analysis. This work enable us to consider both linear and nonlinear spaces of subsets and multivalued mappings from a single point of view. He presented some basic quasilinear counterparts of linear functional analysis by introducing the notions of norm and bounded quasilinear operators and functionals. His work has motivated us to improve “quasilinear space theory”. The works related to this theory can be found [2],[3] and [4]. Further, there are various ways introducing and handling quasilinear spaces. Another important treatment is those of Markow’s approach in [4] and [5]. However, we think that Aseev’s treatment provides the most suitable way to investigate a similar analysis on quasilinear spaces.

One of the fundamental deficiency in the theory of quasilinear spaces is the some basic definitions and theorems with regard to quasilinear operators. The purpose of this talk is to present the some developments associated with the quasilinear operators. We give some basic definitions and theorems which are quasilinear counterparts of some results in linear functional analysis and operator theory. We also observe some results unlike the concepts which are well-known in functional analysis.

Key Words: Quasilinear Spaces, Normed Quasilinear Spaces, Quasilinear Operators, Bounded Quasilinear Operators.

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On Some Hermite-Hadamard Type Inequalities for $\varphi - MT$ Convex Functions by Using Riemann-Liouville Fractional Integrals

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ABSTRACT

The convexity property of a given function plays an important role in obtaining integral inequalities. Proving inequalities for convex functions has a long and rich history in mathematics. Let $f: I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a convex mapping defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x) dx \leq \frac{f(a)+f(b)}{2} \quad (1.1)$$

Is known in the literature as Hermite-Hadamard inequality for convex mappings. Note that some of the classical inequalities for means can be derived from (1.1) for appropriate particular selections of the mapping f . Both inequalities hold in the reversed direction if f is concave.

The aim of this study is to establish Hermite-Hadamard type inequalities involving Riemann-Liouville fractional integrals for $\varphi - MT$ convex functions. Also we give some new inequalities of right-hand side of Hermite-Hadamard type are given for functions whose first derivatives absolute values $\varphi - MT$ functions via Riemann-Liouville fractional integrals.

Key Words: Hermite-Hadamard's inequalities, Riemann-Liouville fractional integral, integral inequalities, φ -convex functions, MT convex functions.

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On Some Properties Of New Sequence Spaces Defined By λ^2 -Bounded Sequences

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ABSTRACT

M. Mursaleen and A. K. Noman give the definition of new sequence spaces using the λ -boundedness and p -absolute convergence of type λ in [1]. They studied some inclusion relations and determined the Schauder basis. Then they computed α -, β - and γ -duals of these spaces and characterized some matrix classes in [2]. N. L. Braha and F. Başar introduce the new spaces of $A(\lambda)$ -null, $A(\lambda)$ -convergent and $A(\lambda)$ -bounded sequences and examined some topological properties of these spaces in [3]. Further, N. L. Braha defined the new paranormed spaces and give some properties of these spaces in [4]. In this paper, we introduce the notion of λ^2 -boundedness and p -absolute convergence type λ^2 . Then, we give the definition of space which was defined by using λ^2 -bounded sequences. We investigated some inclusion relations concerning with these spaces and construct Schauder basis.

Key Words: BK-Spaces, Schauder Basis, λ^2 -boundedness.

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On Some Topological Properties of Generalized Difference Sequence Spaces

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ABSTRACT

Let l_∞, c and c_0 respectively be the Banach spaces of bounded, convergent and null sequences $x = (x_k)_1^\infty$.

Kızılmaz [9] introduced the difference sequence spaces, $E(\Delta) = \{x = (x_k) : \Delta x \in E\}$ for $E = l_\infty, c$ and c_0 where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$. Later on the notion was generalized by Et and Çolak [2]. Recently, the following sequence spaces have been introduced by Karakaş et al. [1] $E(\Delta_p) = \{x = (x_k) : \Delta_p x \in E\}$ for $E = l_\infty, c$ and c_0 , where $p \in \mathbb{N}$ and $\Delta_p x = (\Delta_p x_k) = (px_k - x_{k+1})$.

We define the sequence spaces $E(\Delta_p^m) = \{x = (x_k) : \Delta_p^m x \in E\}$, for $E = l_\infty, c$ and c_0 , where $p, m \in \mathbb{N}$, $\Delta_p^m x_k = (\Delta_p^{m-1} x_k - \Delta_p^{m-1} x_{k+1})$ and so $\Delta_p^m x_k = \sum_{v=0}^m (-1)^v \binom{m}{v} p^{m-v} x_{k+v}$.

In this study we define the sequence spaces $l_\infty(\Delta_p^m), c(\Delta_p^m)$ and $c_0(\Delta_p^m)$, give some topological properties and inclusion relations of these sequence spaces.

Key Words: Sequence spaces, Difference sequence spaces

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On The Completeness of the Eigenfunctions for One Differential Operator with Jump Conditions

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ABSTRACT

It is well-known that many topics in mathematical physics require the investigation of eigenvalues and eigenfunctions of Sturm-Liouville type boundary value problems. In recent years, more and more researches are interested in the discontinuous Sturm-Liouville problems with eigenparameter-dependent boundary conditions, i.e., the eigenparameter appear not only in the differential equations but also in the boundary conditions. Moreover, some boundary value problems which may have discontinuities in the solution or its derivative at an interior point are also studied. Conditions are imposed on the left and right limits of solutions and their derivatives at one interior point c are often called “jump conditions” or “interface conditions”. Such problems often arise in varied assortment of physical transfer problems (see [1, 3, 6]). For instance, some problems with jump conditions arise in thermal conduction problems for a thin laminated plate (i.e., a plate composed by materials with different characteristics piled in the thickness). In this class of problems, jump conditions across the interfaces should be added since the plate is laminated. The study of the structure of the solution in the matching region of the layer leads to consideration of eigenvalue problem for a second order differential operator with piecewise continuous coefficients and jump conditions [6]. Basis properties of the system of eigenfunctions of the Sturm-Liouville problems with a spectral parameter in the boundary condition were studied in [7,8] and the existence of eigenvalues, estimates of eigenvalues and eigenfunctions, and expansions theorems were considered in [9]. In this study, we also deal with the class of Sturm-Liouville problems with jump conditions, by means of a combination of the methods [4],[5],[7] and [9]. Such properties as completeness, minimality and basis property of the eigenfunctions are established.

Key Words: Sturm-Liouville problem, basis, spectral parameter, eigenfunctions.

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On The Existence and Asymptotic Behaviour of The Solutions of Some Nonlinear Integral Equations On Unbounded Interval

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ABSTRACT

In this paper, we prove the existence of the solutions of a class of functional integral equations which contain a number of classical nonlinear integral equations as special cases. Those equations comprises a lot of particular cases of nonlinear Volterra integral equations that can be encountered in research papers and monographs concerning the theory of integral equations and their applications to the real world problems. Our investigations will be carried out in the space $BC(R^+, R)$, where $BC(R^+, R)$ denotes the space of bounded, continuous and real functions which are defined, on the real half-axis. The main tools here are the measure of noncompactness in conjunction with the classical Schauder fixed point theorem. This method enables us to overcome some difficulties appearing in the proof of existence results when we apply classical approach. Moreover, the choice of suitable measure of noncompactness allows us to characterize solutions of the considered nonlinear integral equation in terms of asymptotic behaviour .

Apart from this, the equations handled in this paper are more general than those investigated up to now. The applicability of our result is illustrated by some examples. Further we give some remarks showing the difference between our main result and some previous results.

Key Words: Nonlinear integral equation, Measure of noncompactness, Fixed point theorem.

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On The Existence and Uniform Attractivity of The Solutions of A Class of Nonlinear Integral Equations on Unbounded Interval

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ABSTRACT

In this paper, a class of functional integral equation is investigated in the space $BC(\mathbb{R}^+, \mathbb{R})$, where $BC(\mathbb{R}^+, \mathbb{R})$ denotes the space of bounded, continuous and real functions which are defined, on the real half-axis. That class comprises a lot of particular cases of nonlinear Volterra integral equations that can be encountered in research papers and monographs concerning the theory of integral equations and their applications to the real world problems. We will examine the problem of the existence of continuous solutions of the considered integral equations and the characterization of these solutions in the space $BC(\mathbb{R}^+, \mathbb{R})$. In our study we utilize the technique of measures of noncompactness and fixed point theorem of Darbo type to prove that the equation in question has solutions in the mentioned function space.

Apart from this, we will examine some important property of solutions of the equation which is called uniformly locally attractive or asymptotically stable.

It is worthwhile also mentioning that the integral equations handled in this paper are more general than those investigated up to now. In order to demonstrate the applicability of our result, we introduce some examples. Further we give some remarks showing the difference between our main result and some previous results.

Key Words: Nonlinear integral equation, Measure of noncompactness, Fixed point theorem.

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On the Fine Spectrum of Generalized Cesaro Operators

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ABSTRACT

There are many different ways to subdivide the spectrum of a bounded linear operator; some of them are motivated by applications to physics (in particular, quantum mechanics). The spectra of generalized Cesaro operators over the Hilbert space l_2 were computed by H.C.Rhaly. The main purpose of this paper is determine the fine spectra with the Goldberg classification of the generalized Cesaro operators over some sequence space. In most cases investigated, 0 belong to II_2 and $(1/m)$ belong to III_3 , where II_2 and III_3 are portions of the state space as described in [21]. In the present paper, also subdivision of the spectrum (approximation point spectrum, defect spectrum and compression spectrum) of the generalized Cesaro operators over some sequence space are investigated.

Key Words: Generalized Cesaro operators, Cesaro operator, Rhaly operator, spectrum.

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On the Negative Spectrum of Schrödinger's Operator Equation with a Singularity at Zero

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ABSTRACT

In this study, we investigate the negative spectrum for the operator L , acting in the Hilbert space $L_2(H, [0, \infty))$, which is given by the formula

$$Ly = -\frac{d^2y}{dx^2} + \frac{A(A+I)}{x^2}y - Q(x)y,$$

and which is considered under the boundary condition

$$y(0) = 0,$$

where A and $Q(x)$ are self-adjoint operators defined in the separable Hilbert space H ; and I is the identity operator.

In [1], for the operator L it is shown that

a) the negative spectrum is *discrete* when $\lim_{x \rightarrow \infty} \left\| \int_x^{x+1} Q(t) dt \right\|_H = 0$;

b) the negative spectrum is *finite* when $\lim_{x \rightarrow \infty} x \left\| \int_x^\infty Q(t) dt \right\|_H = 0$.

In the present study, we prove that, under some conditions, the following inequality holds for the negative eigenvalues λ_i of the operator L :

$$S_\gamma(L) = \sum_i |\lambda_i|^\gamma \leq \frac{1}{\gamma+1} \sum_{k=1}^\tau \frac{1}{2\gamma_k+1} \int_0^\infty x \|Q(x)\|^{\gamma+1} dx, \quad (\gamma \geq 0),$$

Here $\gamma_1 \leq \gamma_2 \leq \dots \leq \gamma_k \leq \dots$ are eigenvalues of the operator A , and τ is a number, satisfying

the condition $2\gamma_{\tau-1} + 1 \leq \int_0^\infty x \|Q(x)\| dx \leq 2\gamma_\tau + 1$.

Key Words: Operator equations, eigenvalues, negative spectrum, Hilbert space.

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On The Self-Adjoint Representation of Discontinuous Sturm-Liouville Differential Operator in the Direct Sum Space

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ABSTRACT

Self-adjoint spectral problems have more and more applications. For example, interesting non-classical wavelets can be obtained from eigenfunctions and associated functions for non-self-adjoint spectral problems. Therefore, such problems are increasing attention, especially the discreteness of the spectrum and the completeness of eigenfunctions. Self-adjoint spectral problems associated with differential operators having only a discrete spectrum and depending polynomially on the spectral parameter have been considered by Gohberg and Krein [4] and by Keldysh [5]. They studied the spectrum and root functions of such problems and showed the completeness of the root functions in the appropriate Hilbert function spaces. Non-self-adjoint differential operators whose spectrum may have a continuous part have been investigated by Glazman [3], Sims [10] and Race [9].

This work investigates the spectral properties of one discontinuous Sturm-Liouville problem with supplementary transmission conditions at the point of discontinuity. By suggesting our approach, we construct fundamental solutions and investigate some spectral properties of the eigenfunctions of the considered problem. We believe our construction will prove useful in the spectral analysis of these operators and in obtaining canonical forms of self-adjoint boundary-transmission conditions. Such forms are known for the continuous Sturm-Liouville problems. In the real world the phenomena of discontinuous jumps occur in the dynamics of processes. These processes are often seen in chemotherapy, population dynamics, optimal control, ecology, engineering, etc. The mathematical model that describes the phenomena is impulsive differential equations. Due to their significance, it is important to study the solvability of impulsive differential equations. For the background and applications of the theory of impulsive conditions to different areas, we refer the reader to the monographs and some recent contributions as [1,2,3,7,8,11].

Key Words: Sturm-Liouville problem, transmission conditions, eigenfunctions.

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On the Spectrum of Generalized Cesaro Operators

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ABSTRACT

The spectrum of Cesaro operator over the sequence spaces l_2 was studied by Brown, Halmos, Shields [13] in 1965, over the sequence spaces c , bv_0 and bv was worked by Okutoyi [27] in 1986, over the sequence space c_0 was studied by Reade [28] in 1985, over the sequence space l_p was worked [25] in 1972 and over the sequence space bv_p was studied by Akmedov and Başar [2] in 2008. The spectra of generalized Cesaro operators over the Hilbert space l_2 were computed by H.C.Rhaly. The main purpose of this paper is to show that generalized Cesaro operators is bounded and compact operator, and also to compute the spectrum of it on some sequence space.

Key Words: Generalized Cesaro operators, Cesaro operator, Rhaly operator, spectrum.

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On the Relation Between Ideal Limit Superior-Inferior and Ideal Convergence

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ABSTRACT

Let X be a nonempty set and \mathcal{J} be a family of the subsets of X . \mathcal{J} is an ideal if it has the properties

1. $A \cup B \in \mathcal{J}$ for all $A, B \in \mathcal{J}$,
2. $A \in \mathcal{J}$ and each $B \subset A$ imply $B \in \mathcal{J}$.

A nonempty family of sets $\mathcal{F} \subset 2^X$ is a filter on X if \mathcal{F} has the properties

1. $\emptyset \notin \mathcal{F}$,
2. $A \cap B \in \mathcal{F}$ for all $A, B \in \mathcal{F}$,
3. For each $A \in \mathcal{F}$ and each $B \supset A$ imply $B \in \mathcal{F}$

(see [6] and [7]).

An ideal \mathcal{J} is called non-trivial if $\mathcal{J} \neq \emptyset$ and $X \notin \mathcal{J}$.

A non-trivial ideal $\mathcal{J} \subset 2^X$ is called admissible if and only if $\mathcal{J} \supset \{\{x\} : x \in X\}$.

If $\mathcal{J} \subset 2^X$ is a non-trivial ideal then, $\mathcal{F} = \mathcal{F}(\mathcal{J}) = \{X \setminus A : A \in \mathcal{J}\}$ is a filter on X .

Let \mathcal{J}_δ be the class of all $A \subset \mathbb{N}$ with $\delta(A) = 0$. Then, \mathcal{J}_δ is a non-trivial admissible ideal.

A real valued sequence $x = (x_k)$ is said to be ideal convergent to L , if for every $\varepsilon > 0$, the set $A(\varepsilon) = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ belongs to \mathcal{J} (see [4], [5]). It is denoted by $\mathcal{J} - \lim_{k \rightarrow \infty} x_k = L$.

In this study, the notation of ideal supremum (\mathcal{J} -sup) and ideal infimum (\mathcal{J} -inf) were defined for real valued sequences.

Theorem: Let $x = (x_k)$ be a real valued sequence. Then
$$\mathcal{J} - \liminf_{k \rightarrow \infty} x_k = \mathcal{J} - \inf x_k = \sup L_{\mathcal{J}}(x)$$

and

$$\mathcal{J} - \limsup_{k \rightarrow \infty} x_k = \mathcal{J} - \sup x_k = \inf U_{\mathcal{J}}(x)$$

hold where $L_{\mathcal{J}}(x)$ and $U_{\mathcal{J}}(x)$ denote the set of all ideal lower and ideal upper bounds of the sequence $x = (x_k)$.

Also, it was shown that equality of \mathcal{J} -sup and \mathcal{J} -inf of the sequence $x = (x_k)$ is necessary but not sufficient for to existence of usual limit of the sequence.

On the other hand, equality of \mathcal{J} -sup and \mathcal{J} -inf is necessary and sufficient for to existence of ideal limit. It is also shown that \mathcal{J} -limit superior and \mathcal{J} -limit inferior of $x = (x_k)$ equal \mathcal{J} -sup and \mathcal{J} -inf of $x = (x_k)$, respectively.

Key Words: Statistical limit superior and inferior, statistical supremum and infimum, \mathcal{J} -limit.

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On Two-scale Convergence

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ABSTRACT

The concept of two-scale convergence emerged from questions of periodic homogenization. Homogenization is a mathematical theory, or more precisely, an asymptotic analysis theory that originates from material engineering, or more precisely, from understanding the way constitutive equation of composite material can be gotten from the constitutive equation of each component of the concerned material and from their topological and geometrical distributions.

Two-scale convergence can be regarded as a generalization of the usual weak convergence in the Hilbert space $L^2(\Omega)$ where Ω is a domain in \mathbb{R}^N . Let Ω be a domain of \mathbb{R}^N , and set $Y := [0,1]^N$. A bounded sequence $\{u_\varepsilon\}$ of $L^2(\Omega)$ is said two-scale convergent to $u \in L^2(\Omega \times Y)$ if and only if

$$\lim_{\varepsilon \rightarrow 0} \int_{\Omega} u_\varepsilon(x) \phi\left(x, \frac{x}{\varepsilon}\right) dx = \iint_{\Omega \times Y} u(x, y) \phi(x, y) dx dy$$

for any smooth function $\phi: \mathbb{R}^N \times \mathbb{R}^N \rightarrow \mathbb{R}$ that is Y -periodic w.r.t. the second argument.

The aim of this work is to present the details of the basic ideas in this theory and give an overview of the main homogenization problems which have been studied by this technique.

Key Words: homogenization, two-scale convergence, periodic.

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On λ - Statistical Convergence Of Order (α, β)

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ABSTRACT

The concept of statistical convergence was introduced by Steinhaus [1] and Fast [2] and later reintroduced by Schoenberg [3] independently. The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan [4] and after then statistical convergence of order α and strong p -Cesaro summability of order α studied by Çolak [5] and generalized by Çolak and Asma [6]. In this paper, the concept of λ -statistical convergence of order α is generalized to λ -statistical convergence of order (α, β) , and some inclusion relations between the set of all λ -statistically convergent sequences of order (α, β) and the set of all strong $w(p)$ -summability of order (α, β) are given.

Key Words: λ -statistical convergence, statistical convergence, strong $w(p)$ -summability.

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On λ -Statistical Boundedness of Sequences of Fuzzy Numbers

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ABSTRACT

The idea of statistical convergence was introduced by Fast [4] and Steinhaus [8]. Recently, this notion was generalized by Gadjiev and Orhan [5] as order of statistical convergence and studied by Çolak [1]; Çolak and Bektaş [2].

Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive real numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$, $X = (X_k)$ be a sequence of fuzzy numbers and $0 < \beta \leq 1$. A sequence $X = (X_k)$ is said to be λ -statistically convergent of order β if there exists a fuzzy number X_0 such that $\lim_{n \rightarrow \infty} \frac{1}{\lambda_n^\beta} |\{k \in I_n : d(X_k, X_0) \geq \varepsilon\}| = 0$.

Similarly, a sequence (X_k) is called λ -statistically bounded above of order β if there exists $u \geq 0$ such that $\lim_{n \rightarrow \infty} \frac{1}{\lambda_n^\beta} |\{k \in I_n : X_k > u\} \cup \{k \in I_n : X_k \not\sim u\}| = 0$, and a sequence (X_k) is said to be λ -statistically bounded below of order β if there exists $u \geq 0$ such that $\lim_{n \rightarrow \infty} \frac{1}{\lambda_n^\beta} |\{k \in I_n : X_k < u\} \cup \{k \in I_n : X_k \not\sim u\}| = 0$. If a sequence $X = (X_k)$ of fuzzy numbers is both λ -statistical bounded above of order β and λ -statistical bounded below of order β , then it is called λ -statistical bounded of order β . By $S_\lambda^\beta B(F)$, we shall denote the set of all λ -statistically bounded sequences of order β of sequences of fuzzy numbers.

In this study we examine the concept of λ -statistical boundedness of order β in sequences of fuzzy numbers and give some inclusion relations between λ -statistical boundedness of order β and statistical boundedness.

Key Words: Fuzzy sequence, Statistical boundedness, Statistical convergence

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Periodic Solutions for Nonlinear Boundary Value Problems

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ABSTRACT

In this study, we show that, the periodic boundary value problem

$$y''(t) + \lambda f(t, y(t)) = 0, \quad (0, T)$$

$$y(0) = y(T), \quad y'(0) = y'(T),$$

has at least one periodic solutions for λ in a compatible interval under suitable conditions on $f(t, y)$.

Key Words: Periodic solution, Boundary value problem, Existence result.

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Porosity Supremum-Infimum of Real Valued Sequences and Porosity Convergence

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ABSTRACT

The concept of porosity for sets was given by Denjoy [3] and Khintchine [5] under different terminologies in 1920 and 1924 respectively. Then, in 1967 independently of these studies porosity reappeared Dolzenko's work on the concept of cluster sets [4].

Let $A \subset \mathbb{R}^+ = [0, \infty)$, then the right upper porosity of A at the point 0 is defined as

$$p^+(A) := \limsup_{h \rightarrow 0^+} \frac{\lambda(A, h)}{h}$$

where $\lambda(A, h)$ denotes the length of the largest open subinterval of $(0, h)$ that contains no point of A [6]. By using the right upper porosity of a set at the point 0, the definition of the right upper porosity for the subsets of natural numbers at infinity was given by authors and Dovgoshey in [1] as follows:

Let $\mu : \mathbb{N} \rightarrow \mathbb{R}^+$ be a strictly decreasing function such that $\lim_{n \rightarrow \infty} \mu(n) = 0$, and let E be a subset of \mathbb{N}

$$\bar{p}_\mu(E) := \limsup_{n \rightarrow \infty} \frac{\lambda_\mu(E, n)}{\mu(n)}$$

where $\lambda_\mu(E, n) := \sup\{|\mu(n^{(1)}) - \mu(n^{(2)})| : n \leq n^{(1)} < n^{(2)}, (n^{(1)}, n^{(2)}) \cap E = \emptyset\}$. Then by using this new concept \bar{p}_μ -convergence was defined in [2] as follows:

Let $x = (x_n)$ be a real valued sequence. $x = (x_n)$ is \bar{p}_μ -convergent to l if for every $\varepsilon > 0$

$$\bar{p}_\mu(A_\varepsilon) > 0,$$

where $A_\varepsilon := \{n : |x_n - l| > \varepsilon\}$ and it is denoted by $x_n \rightarrow l(\bar{p}_\mu)$.

In this study, we will define porosity lower bound and porosity upper bound for real valued sequences as follows:

It is said that $l \in \mathbb{R}$ is a porosity lower bound of given sequence $x = (x_n)$ if the following

$$\bar{p}_\mu(\{n : x_n < l\}) > 0 \text{ (or } \bar{p}_\mu(\{n : x_n \geq l\}) = 0)$$

holds. The set of all porosity lower bounds of the sequence $x = (x_n)$ is denoted by $L_{\bar{p}_\mu}(x)$.

It is said that $m \in \mathbb{R}$ is a porosity upper bound of given sequence $x = (x_n)$ if the following

$$\bar{p}_\mu(\{n : x_n > m\}) > 0 \text{ (or } \bar{p}_\mu(\{n : x_n \leq m\}) = 0)$$

holds. The set of all porosity upper bounds of the sequence $x = (x_n)$ is denoted by $U_{\bar{p}_\mu}(x)$.

Let porosity infimum $\inf_{\bar{p}_\mu} x_n := \sup L_{\bar{p}_\mu}(x)$ and porosity supremum $\sup_{\bar{p}_\mu} x_n := \inf U_{\bar{p}_\mu}(x)$. Then we have following results:

Theorem: Let $x = (x_n)$ be a real valued sequence. Then,

$$\inf_{\bar{p}_\mu} x_n \leq \inf x_n \leq \sup x_n \leq \sup_{\bar{p}_\mu} x_n$$

holds.

Theorem: $x_n \rightarrow l(\bar{p}_\mu)$ if and only if $\sup_{\bar{p}_\mu} x_n = \inf_{\bar{p}_\mu} x_n = l$.

Key Words: Porosity convergence, limit infimum, limit supremum.

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Quasilinear Spaces with its Regular-Singular Dimensions and Some New Results

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ABSTRACT

In 1986, S. M. Aseev [1] launched a new branch of functional analysis by introducing the concept of quasilinear spaces which is generalization of linear spaces. He used the partial order relation to define quasilinear spaces and gave coherent counterparts of results in linear spaces. Aseev's approach provided suitable base and necessary tools to proceed algebra and analysis on normed quasilinear spaces just as in normed linear spaces. On the other hand, our studies showed that concepts of quasilinear dependence-independence and basis directly depend on the partial order relation on quasilinear space, [2] and [3].

In this paper, after giving the notions of quasilinear dependence-independence and basis presented in [3], we introduce the concepts of "regular and singular dimension of any quasilinear space" and "floor of an element in quasilinear spaces". Also, we define a new structure namely "proper quasilinear spaces" and obtain some results about the features of proper quasilinear spaces.

Key Words: Quasilinear spaces, regular dimension, singular dimension, floor of an element, proper sets, proper quasilinear spaces, normed proper quasilinear spaces.

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Random Modelling of Biochemical Reactions under Gaussian Random Effects

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ABSTRACT

In this study the Biochemical Reaction Model (BRM) is examined under random effects with normal distribution. The deterministic differential equations of the Biochemical Reaction Model are added normal random effects to form a system of random differential equations. The random system built this way is examined to determine the random behaviour of the Biochemical Reaction Model. Simulations of the deterministic equation system under random effects with normal distribution are performed to derive the numerical characteristics of the Biochemical Reaction Model. Characteristics such as the expected values, variances, standard deviations, first four moments, skewness and kurtosis are found for the random equation system under random effects with normal distribution. These characteristics are used to interpret the contribution of a random analysis on the mathematical modelling of Biochemical Reactions. Numerical results of the random analysis are used for a comparison with the results of the deterministic analysis in an effort to comment on the random behaviour of the components of the Biochemical Reaction Model and the differences that occur between the deterministic and stochastic analysis of the mathematical models of BRM. Numerical solution of the random Biochemical Reaction Model is used to identify the random actions of the variables of the Biochemical Reaction Model and comment on the possible results of the random analysis.

Key Words: Random Effect, Random Differential Equation, Biochemical Reaction Model, Expected Value, Random Characteristics.

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Rate of Approximation of the Bernstein-Type Polynomials With Respect to the Variation Seminorm

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ABSTRACT

The aim of this work is to study variation detracting property and rate of approximation of the Bernstein-type polynomials in the space of functions of bounded variation with respect to the variation seminorm.

The first research devoted the variation detracting property and the convergence in variation of a sequence of linear positive operators was due to Lorentz [2]. However, it was Bardaro, Butzer, Stens and Vinti [4]'s work that made the variation detracting property more important. For this, the present work is strongly motivated by the paper [4]. They pointed out that in order to obtain a convergence result in the variation seminorm, it is necessary and important to state the variation detracting property.

Key Words: Variation Detracting Property, Convergence in Variation Seminorm.

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Riesz Semiconservative FK Space

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ABSTRACT

Snyder and Wilansky give the definition of semiconservative FK space and investigate the properties of this space in [1], [2]. In those papers, an FK space X containing Φ is called semiconservative space if cs containing f dual of X holds. Here replacing cs by rs , we give a new definition called as Riesz semiconservative FK space. The purpose of this work is to investigate specific semiconservative space. In this paper we call an FK space X containing Φ Riesz semiconservative space if rs containing f dual of X holds. Moreover we give some characterizations of these spaces.

Key Words FK space, semiconservative FK space.

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Some Characterizations on Compact Subsets of Normed Quasilinear Spaces, Riesz Lemma and its an Application

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ABSTRACT

Aseev [1] started a new field of functional analysis by introducing the concept of normed quasilinear spaces which is a generalization of normed linear spaces. Then, we introduced the notions of regular and singular dimension of quasilinear space and gave the definition of proper quasilinear space. Also, we proved that these two kind dimensions are equivalent in proper quasilinear spaces and we obtained some results about the properties of proper quasilinear spaces, [2].

In this study, we try to explore some characterizations on compact subsets of finite and infinite regular dimensional normed quasilinear spaces. Moreover, in order to examine consistent and detailed some properties of normed proper quasilinear spaces, we classify it as solid floored and non-solid floored. After, we emphasize some features of solid floored quasilinear spaces which is a special proper quasilinear space. Also we give the counterpart in normed quasilinear spaces of Riesz lemma in normed linear spaces. Finally, we give an important result of Riesz lemma by using the advantages of features of solid floored quasilinear spaces and proper quasilinear spaces.

Key Words: Quasilinear spaces, floor of an element, normed proper quasilinear spaces, solid floored quasilinear spaces, Riesz lemma for normed quasilinear spaces.

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Some Companions of Ostrowski Type Inequalities for Twice Differentiable

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ABSTRACT

The main aim of this paper is to establish some companions of Ostrowski type integral inequalities for functions whose second derivatives are bounded. Moreover, some Ostrowski type inequalities are given for mappings whose first derivatives are of bounded variation. Some applications for special means and quadrature formulae are also given.

Key Words: Bounded Variation, Ostrowski type inequalities, Riemann-Stieltjes integral.

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Some Properties of the Modified Bleimann, Butzer and Hahn Operators Based on q -Integers

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ABSTRACT

In this work, sequence of q - Bleimann, Butzer and Hahn operators which is based on a continuously differentiable function τ on \mathbb{R}^+ , with $\tau(0)=0$, $\inf \tau'(x) \geq 1$ has been considered. Uniform approximation by such a sequence has been studied and degree of approximation by the operators has been obtained. Moreover, shape preserving properties of the sequence of operators have been investigated.

Key Words: Modified q -Bleimann, Butzer and Hahn operators, degree of approximation, shape preserving properties.

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Some Properties of Orthonormal Sets on Inner Product Quasilinear Spaces

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ABSTRACT

Aseev introduced the notion of quasilinear space and normed quasilinear space [1]. Then he gives some basic definition and theorems related to these spaces. We can see in [1] that, as different from linear spaces, Aseev used the partial order relation when he defined quasilinear spaces and so he can give consistent counterparts of results in linear spaces. This pioneering work has motivated us to introduce the concept of inner product quasilinear spaces and Hilbert

Quasilinear spaces which were given in [5]. Our research with these spaces continued to investigate the quasilinear counterparts of fundamental theorems in linear functional analysis. While working in this nonlinear spaces, we have noticed that the new definition is necessary. So, in this study, we give some new concepts related to inner product quasilinear spaces.

In this paper, we present some properties of orthogonal and orthonormal sets on inner product quasilinear spaces. We introduce solid floored quasilinear spaces to deal with a better and suitable kind of quasilinear spaces. To define some topics such as Schauder basis, complete orthonormal sequence, orthonormal basis and complete set and some related theorems this classification is crucial. Furthermore, we try to display some geometric differences of inner product quasilinear spaces from the inner product (linear) spaces.

Key Words: Quasilinear space, Inner product quasilinear space, Hilbert quasilinear Space, Orthogonality, Orthonormality.

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Subdivision of the Spectra for Certain Lower Triangular Double Band Matrices as Operators on C_0

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ABSTRACT

The generalized difference operator $\Delta_{a,b}$ has been defined by El Shabrawy:
 $\Delta_{a,b}x = \Delta_{a,b}(x_n) = (a_n x_n + b_{n-1} x_{n-1})_{n=0}^{\infty}$ with $x_{-1} = b_{-1} = 0$, where $(a_k), (b_k)$ are convergent sequences of nonzero real numbers satisfying certain conditions. The purpose of this study is to completely determine the approximate point spectrum, the defect spectrum and the compression spectrum of the operator $\Delta_{a,b}$ on the sequence space C_0 .

Key Words: generalized difference operator, approximate point spectrum, defect spectrum, compression spectrum.

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Sublinear Operators with Rough Kernel Generated by Calderón-Zygmund Operators and Their Commutators on Generalized Morrey Spaces

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ABSTRACT

The aim of this paper is to get the boundedness of certain sublinear operators with rough kernel generated by Calderón-Zygmund operators and their commutators on generalized Morrey spaces under generic size conditions which are satisfied by most of the operators in harmonic analysis, respectively. Also, Marcinkiewicz operator which satisfies the conditions of these theorems can be considered as an example.

Key Words: Sublinear operator, Calderón-Zygmund operator, rough kernel, generalized Morrey space, commutator, BMO.

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Subordination Results for Certain Subclasses of Analytic Functions

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ABSTRACT

For functions belonging to each of the subclasses $M^*(\alpha, \beta, \mu, \lambda, \delta)$ and $M(\alpha, \beta, \mu, \lambda, \delta)$ of normalized analytic functions in the open unit disk U , which are defined by Darus and Faisal [5], the authors derive several subordination results involving the Hadamard product of the associated functions. Furthermore, we have obtained subordination results for the classes $N^*(\alpha, \beta, \mu, \lambda, \delta)$ and $N(\alpha, \beta, \mu, \lambda, \delta)$. A number of interesting consequences of some of these subordination results are also discussed.

Let A denote the class of analytic functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (1)$$

in the unit disc $U = \{z: |z| < 1\}$ normalized by $f(0) = f'(0) - 1 = 0$. In [8], Darus and Faisal introduced the following differential operator: For $f \in A$,

$$\begin{aligned} D_{\lambda}^0(\alpha, \beta, \mu) f(z) &= f(z) \\ D_{\lambda}^1(\alpha, \beta, \mu) f(z) &= \left(\frac{\alpha - \mu + \beta - \lambda}{\alpha + \beta} \right) f(z) + \left(\frac{\mu + \lambda}{\alpha + \beta} \right) z f'(z) \\ D_{\lambda}^2(\alpha, \beta, \mu) f(z) &= D(D_{\lambda}^1(\alpha, \beta, \mu) f(z)) \\ &\vdots \\ D_{\lambda}^k(\alpha, \beta, \mu) f(z) &= D(D_{\lambda}^{k-1}(\alpha, \beta, \mu) f(z)). \end{aligned} \quad (2)$$

If f is given by (1) then from (2), it can be obtained

$$D_{\lambda}^k(\alpha, \beta, \mu) f(z) = z + \sum_{k=2}^{\infty} \left(\frac{\alpha + (\mu + \lambda)(k-1) + \beta}{\alpha + \beta} \right)^k a_k z^k \quad (3)$$

where $f \in A$; $\alpha, \beta, \mu, \lambda \geq 0$; $\alpha + \beta \neq 0$; $k \in \mathbb{N}_0$.

For special cases of the parameters of $D_{\lambda}^k(\alpha, \beta, \mu)$, it can be obtained the well-known differential operators in [1,2,3,7,9,10,11].

Also, in [5], Faisal et al. defined the following new integral operator: For $f \in A$,

$$\begin{aligned} C^0(\alpha, \beta, \mu, \lambda) f(z) &= f(z) \\ C^1(\alpha, \beta, \mu, \lambda) f(z) &= \left(\frac{\alpha + \beta}{\mu + \lambda} \right) z^{1 - \left(\frac{\alpha + \beta}{\mu + \lambda} \right)} \int_0^z t^{\left(\frac{\alpha + \beta}{\mu + \lambda} \right) - 2} f(t) dt \end{aligned}$$

$$\begin{aligned}
 C^2(\alpha, \beta, \mu, \lambda) f(z) &= \left(\frac{\alpha + \beta}{\mu + \lambda} \right) z^{1 - \left(\frac{\alpha + \beta}{\mu + \lambda} \right)} \int_0^z t^{\left(\frac{\alpha + \beta}{\mu + \lambda} \right) - 2} C^1(\alpha, \beta, \mu, \lambda) f(t) dt \\
 &\vdots \\
 C^m(\alpha, \beta, \mu, \lambda) f(z) &= \left(\frac{\alpha + \beta}{\mu + \lambda} \right) z^{1 - \left(\frac{\alpha + \beta}{\mu + \lambda} \right)} \int_0^z t^{\left(\frac{\alpha + \beta}{\mu + \lambda} \right) - 2} C^{m-1}(\alpha, \beta, \mu, \lambda) f(t) dt. \quad (4)
 \end{aligned}$$

If f is given by (1) from (4) it can be written as

$$C^m(\alpha, \beta, \mu, \lambda) f(z) = z + \sum_{k=2}^{\infty} \left(\frac{\alpha + \beta}{\alpha + (\mu + \lambda)(k-1) + \beta} \right)^m a_k z^k \quad (5)$$

where $f \in A$; $\alpha, \beta, \mu, \lambda \geq 0$; $\alpha + \beta \neq 0$; $\mu + \lambda \neq 0$; $m \in \mathbb{N}_0$.

For special cases of the parameters of $C^m(\alpha, \beta, \mu, \lambda)$, it can be obtained the well-known integral operators in [6,13,14].

Key Words: Subordination, integral operator, analytic function, factor sequences, convolution.

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Spectral Analysis of q -Sturm-Liouville Problem with the Spectral Parameter in the Boundary Condition

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ABSTRACT

This paper is deals with q -Sturm-Liouville boundary value problem in the Hilbert space with a spectral parameter in the boundary condition. We construct a self-adjoint dilation of the maximal dissipative q -difference operator and its incoming and outgoing spectral representations, which make it possible to determine the scattering matrix of the dilation. We prove theorems on the completeness of the system of eigenvalues and eigenvectors of operator generated by boundary value problem.

Key Words: Dissipative q -Sturm-Liouville operator, spectral parameter, Completeness of the system of eigenvectors and associated vectors.

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Spectral Properties of an Eigenparameter Dependent Singular Dirac Operator

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ABSTRACT

In this talk, we consider eigenparameter dependent singular Dirac operators with transmission conditions. We will give a completeness theorem of the system of eigenvectors and associated vectors of these operators.

Key Words: Dissipative Dirac operator; eigenparameter in the boundary condition; completeness of the system of eigenvectors and associated vectors.

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Stability Estimate for the Sturm-Liouville Operator with Coulomb Potential

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ABSTRACT

In this study, we consider stability inverse spectral problem associated with the singular Sturm-Liouville operator

$$L: -\frac{d^2}{dx^2} + \left(q(x) + \frac{A}{x} \right) \quad (1)$$

where the function $q(x) \in L^1[0, \pi]$ and A is a real constant. Firstly, the method which is used was given by Ryabushko [4] for regular Sturm-Liouville problem.

Let $\{\lambda_{1,k}\}$ and $\{\mu_{1,k}\}$ be the spectrums of the problems

$$L_1 y = -y'' + \left(q(x) + \frac{A}{x} \right) y = \lambda y, \quad 0 < x \leq \pi \quad (2)$$

$$\begin{aligned} y(0) &= 0, \\ y'(\pi) - Hy(\pi) &= 0, \end{aligned} \quad (3)$$

and

$$\begin{aligned} y(0) &= 0, \\ y'(\pi) - Hy(\pi) &= 0, \end{aligned} \quad (4)$$

respectively, where H, H are real constants and $\frac{y(x)}{x} \in C[0, \pi]$.

We consider the second equation

$$L_2 y = -y'' + \left(q(x) + \frac{A}{x} \right) y = \lambda y, \quad 0 < x \leq \pi \quad (5)$$

where the function $q(x) \in L^1[0, \pi]$. Let $\{\lambda_{2,k}\}$ and $\{\mu_{2,k}\}$ be the spectrums of the problems (5)-(3) and (5)-(4), respectively. The operator L is self adjoint on $L_2(0, \pi)$

and with (3),(4) boundary conditions have discrete spectrums $\{\lambda_k\}, \{\mu_k\}$. We show by $\{\rho_{1,m}\}, \{\rho_{2,m}\}$ spectral functions of the problem (2)-(3) and (5)-(3)

$$\rho_{1,m} = \sum_{\lambda_{1,m} < \lambda} \frac{1}{\alpha_{1,m}}, \quad \rho_{2,m} = \sum_{\lambda_{2,m} < \lambda} \frac{1}{\alpha_{2,m}},$$

respectively. When the eigenvalues $\{\lambda_{j,k}\}$ and $\{\mu_{j,k}\}, (j=1,2)$ coincide numbers of $N+1$ for $k=1,2,\dots,N+1$, we must evaluate the difference of the spectral functions of these problems. The main theorem in this study is following:

Theorem 1. When the eigenvalues $\{\lambda_{j,k}\}$ and $\{\mu_{j,k}\}, (j=1,2)$ coincide numbers of $N+1$ such that $\lambda_{1,k} = \lambda_{2,k}$ and $\mu_{1,k} = \mu_{2,k}$ for $k=1,2,\dots,N+1$, then the spectral functions $\rho_{j,m}(\lambda)$ satisfy

$$\text{Var}_{-\infty < \lambda < \frac{N}{2}} \left\{ \rho_{1,m}(\lambda) - \rho_{2,m}(\lambda) \right\} < \rho_{1,m} \left(\frac{N}{2} \right) \frac{8K \left(1 + \frac{2}{N} \right)}{N^2 \left(2 + \frac{9}{N} + \frac{13-4K}{N^2} + \frac{6-8K}{N^3} \right)} e^{\frac{8K \left(1 + \frac{2}{N} \right)}{N^2 \left(2 + \frac{9}{N} + \frac{13-4K}{N^2} + \frac{6-8K}{N^3} \right)}}$$

for $k > N+1$, $m < \frac{N}{2}$, where $K = \frac{1}{2\pi} \int_0^1 \{q(t) - q(t)\} dt + O\left(\frac{1}{k}\right)$.

Key Words: Stability problem, coulomb potential, spectral function.

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Statistical Convergence of Order α in Šerstnev Spaces

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ABSTRACT

Probabilistic normed spaces were introduced by Šerstnev [1] in 1963. Alsina and Schweizer] gave a new definition of PN- spaces which includes Šerstnev's [1] as a special case and leads naturally to the identification of the principle class of PN- spaces. This concept has been studied by many authors, see for more detail Alotaibi[3], Lafuerza and Rafi [4], Mursaleen *et al.*([5],[6]), Sempi [7] and Şençimen *et al.* ([8], [9],[10], [11]).

The idea of statistical convergence was given by Zygmund [12] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [13] and Fast [14] and later reintroduced by Schoenberg [15] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory, number theory, measure theory, trigonometric series, turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Fridy [16], Connor [17], Šalát [18], and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. The order of statistical convergence of a sequence was previously given by Gadjiev and Orhan [19] and after then statistical convergence of order α and strong p – Cesàro summability of order α studied by Çolak [20].

In this paper, we intend to make a new approach to introduce the notion of statistical convergence of order α , in Šerstnev spaces where α is a real number such that $0 < \alpha \leq 1$ and prove some results

Key Words: Statistical convergence, t-norm, probabilistic normed space

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Strong α -Deferred Cesàro Means of Double Sequences

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ABSTRACT

The concept of statistical convergence was first introduced by Fast [11] and also independently by Buck [5] and Schoenberg [9] for real and complex sequences. Further, this concept was studied by Šalát[13], Fridy [7], Connor [10] and many others. Some equivalence results for Cesàro submethods have been studied by Goffman and Petersen [12], Armitage and Maddox [6] and Osikiewicz [8]. In 1932, Agnew [1] defined the deferred Cesàro mean $D_{p,q}$ of the sequence $x = (x_k)$ by

$$(D_{p,q}x)_n := \frac{1}{q(n) - p(n)} \sum_{k=p(n)+1}^{q(n)} x_k$$

where $\{p(n)\}$ and $\{q(n)\}$ are sequences of positive natural numbers satisfying

$$p(n) < q(n) \quad \text{and} \quad \lim_{n \rightarrow \infty} q(n) = \infty.$$

In [2], the concepts of α -strongly deferred Cesàro summable have been examined. Also in [3], we defined deferred Cesàro mean $D_{\beta,\gamma}$ of the double sequence x as follows:

Let $x = (x_{kl})$ be a double sequence and $\beta(n) = q(n) - p(n)$, $\gamma(m) = r(m) - t(m)$. Then deferred Cesàro mean $D_{\beta,\gamma}$ of the double sequence x is defined by

$$(D_{\beta,\gamma}x)_{mn} = \frac{1}{\beta(n)\gamma(m)} \sum_{k=p(n)+1}^{q(n)} \sum_{l=t(m)+1}^{r(m)} x_{kl}$$

where $\{p(n)\}, \{q(n)\}, \{t(m)\}$ and $\{r(m)\}$ are sequences of nonnegative integers

satisfying the conditions $p(n) < q(n)$, $t(m) < r(m)$ and $\lim_{n \rightarrow \infty} q(n) = \infty$, $\lim_{m \rightarrow \infty} r(m) = \infty$, and we obtained some results for deferred Cesàro mean.

In this study, the concepts of α -strongly deferred Cesàro mean for double sequences are defined and studied by using deferred double natural density of the subset of natural numbers. Also we consider the case $\beta(n) = \lambda(n) - \lambda(n-1)$, $\gamma(m) = \mu(m) - \mu(m-1)$ for deferred Cesàro mean $D_{\beta,\gamma}$ where $\lambda = (\lambda(n))$ and $\mu = (\mu(m))$ are strictly increasing sequences of positive integers with $\lambda(0) = 0$ and $\mu(0) = 0$. Finally, we obtain some inclusion results for deferred Cesàro mean $D_{\beta,\gamma}$ of the double sequences.

The α -strongly deferred convergent sequence $x = (x_{kl})$ is an α -strongly Cesàro convergent only if $\left\{ \frac{p(n)}{\beta(n)} \right\}$ and $\left\{ \frac{t(m)}{\gamma(m)} \right\}$ are bounded.

Let $\{\lambda(n)\}_{n \in \mathbb{N}}$ $\{\mu(m)\}_{m \in \mathbb{N}}$ are increasing sequences of positive integers and $\lambda(0) = 0$, $\mu(0) = 0$. The α -strongly $C_{\lambda, \mu}$ convergent sequence can be an α -strongly $D_{\lambda, \mu}$ convergent only if

$$\liminf_{n \rightarrow \infty} \frac{\lambda(n)}{\lambda(n-1)} > 1 \quad \text{ve} \quad \liminf_{m \rightarrow \infty} \frac{\mu(m)}{\mu(m-1)} > 1.$$

Key Words: Statistical convergence, Deferred Cesàro mean, Double sequence, Deferred statistical convergence, α -strongly deferred Cesàro mean

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The Boundedness of Maximal and Calderon-Zygmund Operators on Local Morrey-Lorentz spaces

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ABSTRACT

In this talk the boundedness, including the limiting cases, of the Hardy-Littlewood maximal operator M , the Calderon-Zygmund operators T and the maximal Calderon-Zygmund operators T on the local Morrey-Lorentz spaces $M_{p,q;\lambda}^{loc}(R^n)$ will be proved. Further some applications of obtained results will be given.

Key Words: local Morrey-Lorentz spaces, Maximal operator, Calderon-Zygmund operators, maximal Calderon-Zygmund operators.

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The Modified Hadamard Products of Functions Belonging to the Class $P(j, \lambda, \alpha, n, z_0)$ of Starlike Functions

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ABSTRACT

The subclass $P(j, \lambda, \alpha, n)$ of starlike functions with negative coefficients by using the differential D^n operator and functions of the form $f(z) = z - \sum_{k=j+1}^{\infty} a_k z^k$ which are analytic in the open unit disk is considered. The subclass $P(j, \lambda, \alpha, n, z_0)$ for which $f(z_0) = z_0$ or $f'(z_0) = 1$, z_0 real is examined by Kiziltunc and Baba [1]. The modified Hadamard products of functions belonging to the class $P(j, \lambda, \alpha, n, z_0)$ has been obtained. As special cases, the results of my paper reduce to Aouf and Srivastava [2].

Key Words: Fixed point, Starlike, Salagean operator, Hadamard product.

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The Performance of the Test Statistic Based on Ranked Set Sampling For Two Population Mean Under Normality and Nonnormality

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ABSTRACT

Ranked Set Sampling (RSS) was first introduced by McIntyre (1952) and is a well-known sampling technique that has been used recently in fields such as the environment, ecology and agriculture. Very commonly in these fields, the measurement of the sampling units according to the variable of interest can be quite difficult or expensive in terms of cost, time and other factors. In these cases, RSS is more cost-effective than the Simple Random Sampling (SRS) technique for estimation of the parameters of the population.

RSS can also be used to test the hypothesis related with population parameters. In this study, we investigate the hypothesis test for the difference of means of two populations under RSS for normal distribution and unknown variances. Since the theoretical distribution of the sample mean cannot be derived in RSS, the critical values of the test statistics cannot be obtained theoretically. For this reason, the critical values are obtained for the different sample sizes by using Monte Carlo method. Type I errors and powers of test based on RSS are compared to the type I errors and powers of test based on SRS under normality. For non-normality, we also compare RSS and SRS according to type I error and power of test by using t distribution.

Key Words: Ranked Set Sampling, Hypothesis Test, Monte Carlo.

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The σ -statistical Convergence of Double Sequences of Order $\tilde{\alpha}$

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ABSTRACT

The idea of statistical convergence was proposed by Zygmund [11] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [9] and Fast [4] and later reintroduced by Schoenberg [7] independently. The idea of statistical convergence was later extended to double sequences by Tripathy [10], Mursaleen and Edely [6] and Moricz [5].

In 2013, R. Colak and Y. Altın . [1] defined new concepts of $\tilde{\alpha}$ -density and $\tilde{\alpha}$ -statistical convergence for double sequence of complex or real numbers.

In this study, we introduce concepts of σ -statistical convergence for double sequence of complex or real numbers, where $0 \leq \tilde{\alpha} \leq 1$. Also, some relations between σ -statistical convergence and strong σ -Cesaro summability of order $\tilde{\alpha}$ are given.

Key Words: Double Sequences, density, statistical convergence.

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Various Modes of Double Convergence in Measure

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ABSTRACT

The notion of convergence for double sequences was first introduced by Pringsheim [5]. A double sequence $x=(x_{mn})$ is said to be convergent in the Pringsheim's sense if for every $\varepsilon>0$ there exists $N\in\mathbb{N}$ such that $|x_{mn}-L|<\varepsilon$ whenever $j,k\geq N$, L is called the Pringsheim limit (or double limit) of x and denoted by $P - \lim x_{mn}=L$.

Double sequences that the elements are functions are defined in the same way as single sequences of real-valued functions. There are several possible modes of convergence for sequences of functions. The most important of these are the notions of pointwise and uniform convergence, but these convergence modes are too restrictive. In many cases, some of the function sequences do not convergence for every element in the domain. Therefore, the convergence modes in the weaker sense has been define; for example , almost everywhere convergence and convergence in measure and so on.

Throughout this note (Ω,Γ,μ) , or shortly Ω , denotes a measure space with a complete measure $\mu:\Gamma\rightarrow\mathbb{R}$ (and Γ is a σ -algebra of subsets of Ω measurable with respect to μ). E is always an element in Γ such that $\mu(E)<\infty$ [3], [6] .

A function $f: E\rightarrow\mathbb{R}$ is a measurable function if for each $c\in\mathbb{R}$, $\{x\in E:f(x)>c\}\in\Gamma$ or ,equivalently, $f^{-1}(B)\in\Gamma$ for each $B\in\mathcal{B}_{\mathbb{R}}$, where $\mathcal{B}_{\mathbb{R}}$ is a σ -algebra of Borel sets in \mathbb{R} .

It is well known, in probability and statistics, a probability measure is a measure with total measure one, i.e. $\mu(\Omega)=1$. A probability space is a measure space with a probability measure. A random variable is a measurable real function whose domain is the probability space [1], [2], [4].

In the present paper, we define some modes of convergence in the Pringsheim's sense for a double sequence (f_{mn}) of measurable functions. We also give the relations between modes of these convergences.

Key Words: Double sequence, double limit, convergence in measure.

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Zweier Matrix Method and Weakly Unconditionally Cauchy Series

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ABSTRACT

l_∞ and c_0 will be denoted the Banach spaces of bounded and convergent sequences endowed with sup norm, respectively. Let X be real Banach space, X^* is a dual space of X and $\sum_i x_i$ a series in X . A series $\sum_i x_i$ is called weakly unconditionally Cauchy (wuC) if $(\sum_{i=1}^n x_{\pi_i})_{n \in \mathbb{N}}$ is a weakly Cauchy for every permutation π of \mathbb{N} ; alternatively, $\sum_i x_i$ a wuC series if and only if $\sum_{i=1}^\infty |f(x_i)| < \infty$ for every $f \in X^*$. A series $\sum_i x_i$ is called unconditionally convergent (uc) if $\sum_{i=1}^\infty x_{\pi_i}$ is converges for every permutation π of \mathbb{N} [6].

Many researches have been worked on the behaviour of a series of the form $\sum_{i=1}^\infty a_i x_i$, where $(a_i)_i$ is a bounded sequence of real numbers. It is well known that ([2], [4], [5], [7]) a series $\sum_i x_i$ is uc if and only if the series $\sum_{i=1}^\infty a_i x_i$ is convergent for every $(a_i)_{i \in \mathbb{N}} \in l_\infty$, and $\sum_i x_i$ is wuC if and only if the series $\sum_{i=1}^\infty a_i x_i$ is convergent for every $(a_i)_{i \in \mathbb{N}} \in c_0$.

Let $Z = (\alpha_{ij})_{i,j \in \mathbb{N}}$ be a Zweier matrix and S be a subspace of l_∞ such that $c_0 \subseteq S$. Then, we will denote

$$X(S, Z) = \left\{ (x_k)_{k \in \mathbb{N}} \in X^{\mathbb{N}} : Z - \sum_{i=1}^\infty a_i x_i \text{ exists for every } (a_i)_{i \in \mathbb{N}} \in S \right\}$$

and

$$X_w(S, Z) = \left\{ (x_k)_{k \in \mathbb{N}} \in X^{\mathbb{N}} : wZ - \sum_{i=1}^\infty a_i x_i \text{ exists for every } (a_i)_{i \in \mathbb{N}} \in S \right\}$$

where

$$Z - \sum_{i=1}^\infty a_i x_i = \lim_{n \rightarrow \infty} (1 - \alpha) Z - \sum_{i=1}^{n-1} a(i) x^n(i) + \alpha \sum_{i=1}^n a(i) x^n(i).$$

and

$$wZ - \sum_{i=1}^\infty a_i x_i = w - \lim_{n \rightarrow \infty} (1 - \alpha) Z - \sum_{i=1}^{n-1} a(i) x^n(i) + \alpha \sum_{i=1}^n a(i) x^n(i),$$

respectively. In [1], the space $X(S)$ is defined and it is proved that this spaces are real Banach spaces with the norm

$$\|(x_k)_{k \in \mathbb{N}}\| = \sup \left\{ \left\| \sum_{i=1}^\infty a_i x_i \right\| : |a_i| \leq 1, i = 1, 2, \dots, n; n \in \mathbb{N} \right\}.$$

In [3], the spaces $X(l_\infty)$ and $X(c_0)$ is defined by $BMC(X)$ and $CMC(X)$, respectively; The spaces $X(l_\infty)$ and $X(c_0)$ can be considered as the space of uc and wuC series in X , respectively. In this paper, using Zweier matrix domain, we give some results about

completeness of the spaces $X(S, Z)$ and $X_w(S, Z)$. Also, we give necessary conditions for the operator

$$T_x: S \rightarrow X$$
$$a \rightarrow T_x(a) = Z - (wZ-) \sum_{k=1}^{\infty} a(k)x(k)$$

is continuous for $x = (x(k))_{k \in \mathbb{N}} \in X(S, Z)$ ($x = (x(k))_{k \in \mathbb{N}} \in X_w(S, Z)$).

Key Words: Zweier matrix, completeness, weakly unconditionally Cauchy series.

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$\lambda_{\{d\}}$ -Statistical Convergence in Metric Spaces

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ABSTRACT

In this paper, we give $\lambda_{\{d\}}$ -statistical convergence and strong $(V, \lambda)_{\{d\}}$ -summability of sequences in metric spaces. Also we establish some relations between the the set of $\lambda_{\{d\}}$ -statistically convergent sequences and the set of strongly $(V, \lambda)_{\{d\}}$ -summable sequences in a metric space.

Key Words: Statistical convergence, Strong summability, Metric Spaces.

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APPLIED MATHEMATICS

A Bernstein Collocation Method for Nonlinear Fredholm-Volterra Integral Equations

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ABSTRACT

Fredholm and Volterra integral equations that were firstly introduced by Vita Volterra (1860-1940) and Ivar Fredholm (1866-1927) are well known that linear and nonlinear integral equations arise in many scientific fields such as the population dynamics, spread of epidemics and semi-conductor devices. In this study, a collocation method is derivated for the solution of nonlinear Fredholm-Volterra integral equations (FVIEs) in the most general form. The method is based on the generalized Bernstein polynomials that the Bernstein polynomials defined on the interval $[0,1]$ generalize to the interval $[a,b]$ by considering transformation $t=((x-a)/(b-a))$. This method leads to solve the nonlinear integral equations iteratively by using technique of quasilinearization. The quasilinearization technique reduces the nonlinear equations to the linear equations. The main advantage of this developed method that it converges quadratically to the solution of the original equation. The other advantage of the method is easy applicability to nonlinear problems and computable as provide saving from the time. Uniqueness of the nonlinear Fredholm-Volterra integral equations is analized by considering the Banach fixed-point theorem. Quadratic convergence and error estimate of the proposed method are also produced. Moreover some examples of nonlinear integral equations are presented as tables and figures to show the accuracy, efficiency and applicability of the method. Numerical results are also compared with different methods.

Key Words: Bernstein polynomial approach; Nonlinear integral equations; Quasilinearization technique; Collocation method.

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A Case History in Singular Perturbations and Some Applications

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ABSTRACT

We shall consider singular perturbations problems P_ε consisting of a nonlinear ordinary differential equation of the second order,

$$\varepsilon y'' + f(t, y, y', \varepsilon) = 0, 0 \leq t \leq 1$$

containing a small positive parameter ε , and the boundary conditions

$$y(0, \varepsilon) \pm \alpha(\varepsilon), y(1, \varepsilon) = \beta(\varepsilon)$$

Here t is the independent variable, $y = y(t, \varepsilon)$ and primes denote differentiation respect to t . As $\varepsilon \rightarrow 0^+$ one would expect the solution of P_ε to approach a solution of the limiting differential equation,

$$f(t, y, y', 0) = 0, 0 \leq t \leq 1.$$

Now, this differential equation is one of the first order, and it cannot be expected that its solution will satisfy both of the limiting boundary conditions

$$y(0, 0) = \alpha(0), y(1, 0) = \beta(0).$$

Thus, the very nature of the limiting problem P_0 is at question here.

Key Words: Singular perturbations, ordinary differential equations,

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A Class of Polynomials Which are Orthogonal Over The Elliptic Domain

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ABSTRACT

The aim of this paper is to present two-variable analogues of Jacobi polynomials which are orthogonal over the elliptic domain and to investigate some properties of these polynomials such as recurrence relations and integral representations. Also, relationships between these polynomials and some other known polynomials are given.

Key Words: Jacobi polynomials, generalized Gegenbauer polynomials, recurrence relation, integral representation

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A Fuzzy Based Method to Predict Magnetic Field Pollution

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ABSTRACT

In this study, magnetic field levels near transmission lines have been obtained by the utilization of magnetic field measurement data in a novel model based on fuzzy logic algorithm. Up to the present, some studies are related to magnetic fields calculations based upon analytical methods. Some are related to evaluation and measurement of the magnetic field [1,2]. Magnetic field measurement readings on (x, y) coordinates (h=1.75m above the earth's surface) were used on the written computer program which consist of fuzzy logic algorithm. The measurement values were taken in fours so as to fit a curve with fuzzy operation. Thereafter, to create the surface, the generated curves were combined.

Fuzzy logic is extended version of the classic cluster display [3,4]. In the fuzzy set of assets; there is a degree of membership of each entity. Membership degree of the assets can have any value in the range of (0, 1). Membership function is indicated by the notation $\mu(x)$. A fuzzy system has three units. These units are; fuzzyfication unit, rule processing unit and defuzzification unit respectively. Rule processing unit stores rule processing information which based on the information database in the structure of "if ... and ... then ... else". In the last step; the results obtained according to the structure of a logical decision proposition problems is sent to the defuzzifier unit. In the defuzzifier unit, Each of the information is converted to a real number.

In the curve fitting process of the study, control points are chosen from measured field values and the curve is modeled by fuzzy logic. Every curve is composed using 4 measured field values.

There are lots of researches in the scientific journals which includes estimation of magnetic field by means of artificial intelligence ways. In the context of this scientific examination, the levels of magnetic fields in the near area of a transmission line are estimated with the help of a novel fuzzy logic model.

The method used in this study decreases the determination time of the investigation of magnetic field distribution near the transmission lines with low errors. The average prediction error by entering eight measurement points into the written program is found as 29.3 %. When numbers of measuring points are increased, the average prediction error is decreased. The developed model has been found to give accurate and satisfactory results for practical applications.

Key Words: Energy transmission lines, ELF field measurements, Fuzzy Logic, Curve fitting.

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A Geometric Approach for the One-Dimensional Hyperbolic Telegraph Equation

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ABSTRACT

Group preserving scheme for calculating the numerical solutions of the one-dimensional hyperbolic telegraph equation is given in this work. Stability of group preserving scheme for one-dimensional hyperbolic telegraph equation is displayed. Numerical results present the efficiency and power of this technique.

Key Words: Group preserving scheme, telegraph equation, stability analysis.

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A Hybrid Approach for Solving Singularly Perturbed Turning Point Problems Exhibiting Dual Layers

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ABSTRACT

In this study, we consider singularly perturbed second order ordinary differential equations exhibiting dual (twin) boundary layers with a turning point. An efficient and easy-applicable method so-called Successive Complementary Expansion Method (SCEM) is proposed in order to obtain a uniformly valid approximation to the solution of this kind of problems. To do this, the original domain of the problem is divided into sub-regions and the method is employed. Two illustrative examples are provided to show the application and to test the computational efficiency of the method. Furthermore, the results are compared with those that are obtained by other existing methods. We observe that SCEM approximations lead us highly accurate results which are consistent with the theoretical error estimates and that the method requires only a few iterations to achieve this.

Key Words: Asymptotic approximation, dual boundary layers, Singular perturbation

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A Lumped Galerkin Finite Element Approach for Generalized Hirota- Satsuma Coupled KdV and Coupled Modified KdV Equations

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ABSTRACT

In the current manuscript, lumped Galerkin finite element method has been applied to generalized Hirota Satsuma coupled Korteweg-de Vries (KdV) and coupled modified Korteweg-de Vries (mKdV) equations by using quadratic B-spline base functions. The numerical solutions of the splitted equations using lumped Galerkin finite element method have been obtained by fourth order Runge-Kutta method which is widely used for the solution of systems of ordinary differential equations. The numerical solutions obtained by various space and time steps have been compared with exact solutions using the error norms L_2 and L_∞ and they have been presented in tabular form. Lumped Galerkin finite element method is an effective method which can be applied to a wide class of nonlinear evolution equations.

Key Words: Finite element method, Galerkin, B-spline, Runge-Kutta, Hirota-satsuma generalized coupled KdV, Hirota-satsuma generalized coupled modified KdV.

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A Mathematical Programming Model for the Vehicle Routing Problem and a Case Study

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ABSTRACT

The capacitated vehicle routing problem (VRP) is one of the most important problems in combinatorial optimization with wide-ranging applications in practice. The objective of VRP which is classified as an NP-hard problem, is to determine the optimal set of routes for a set of vehicles to deliver customers subject to a set of constraints. The VRP is used in supply chain management in the physical transportation of goods and services. The VRP is so commonly studied because of its wide applicability and its significance in determining effective strategies for reducing total operational costs in distribution networks.

The vehicle routing problem has been extensively studied in the operations research and optimization literature. In the literature, solution algorithms for VRP are classified into exact methods, heuristics approaches, metaheuristics and hybrid methods. Moreover, exact methods are also classified into three main categories; direct tree search methods; dynamic programming and integer linear programming.

In this paper, a mixed integer linear programming formulation (MIP) which aims at minimizing the total transportation cost is presented. Firstly, developed mathematical model for the vehicle routing problem is introduced in detail. Then, the model is tested with a case study. Computational results show that the proposed mathematical formulation is promising quality routes in a reasonable computing time.

Key Words: Vehicle routing problem, mathematical programming, optimization, applied mathematics, operations research.

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A New Approach to Artificial Intelligence Optimization: Ant Lion Algorithm

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ABSTRACT

Artificial intelligence optimization algorithms are the methods that utilize a simple approach as a solution technique of search and optimization problems and are recently getting strong and becoming more popular. They provide general solution strategies that can be applied to the problem in case of concurrent different decision variables, objective functions and constraints and they do not depend on the solution space type, the number of decision variables, and the number of constraints. Furthermore, they do not require very well defined mathematical models that are hard to organize for system modeling and objective function and, that cannot be used due to high solution time cost even the mathematical model has been organized. Their computation power is also good and they do not require excessive computation time. Their transformations and adaptations are easy. They give efficacious solutions to the high-scale combinatorial and non-linear problems. They are adaptable in order to solve different types of search and optimization problems. Due to these advantages, these algorithms are densely being used in many different fields such as management science, engineering, computer, etc. and new versions of these algorithms have been proposed.

The Ant Lion Algorithm (ALA) is a relatively recent algorithm and mimics the hunting mechanism of ant lions in nature. ALA is implemented in five main steps: random walks of ants, building pits, entrapment of ants, catching preys, and lastly rebuilding pits. This paper explains the main steps of ALA and represents its comparative performance within the benchmark functions which are as complex as engineering search and optimization problems. Although ALA is very new and no optimization has been done in its parameters, promising results have been obtained

from the experimental results. Its more efficient multi-objective, distributed, and parallel versions may be proposed for future works.

Key Words: Artificial Intelligence, Ant Lion, Optimization.

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A New Approach with CSCE Method to Find the Wave Interactions

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ABSTRACT

Recently, some important studies have been done in order to determine the interactions between cnoidal waves and solitons for nonlinear physical problems. Some of them are respectively the consistent Riccati expansion (CRE) method, the consistent tanh expansion (CTE) method, CRE solvability based on symmetry analysis. In this research, a new approach including the nonlinear equation $R_\omega = \sin(R(\omega))$ is considered to find the interactions among the traveling waves of a wave equation. Also it is shown that the new method called as the consistent sine-cosine expansion (CSCE) method is very efficient, powerful and direct in obtaining the wave interactions of nonlinear partial evolution equations.

Key Words: CSCE method, exact solutions, interactions among waves.

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A New Generalization of Second Appell Hypergeometric Function F_2

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ABSTRACT

In this work, we first introduced a new generalization of second Appell hypergeometric function F_2 with the help of generalized Pochhammer symbol which defined in a very recent paper. Then, we presented a systematic study for investigating the fundamental properties of the generalized second Appell hypergeometric functions such as integral representations, derivative and summation formulas, etc.

Key Words: Gamma function, extended gamma function, Pochhammer symbol, generalized Pochhammer symbol, second Appell hypergeometric function, generalized second Appell hypergeometric function, generalized hypergeometric function.

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A New Mathematical Model for University Examination Timetabling Problem: a Case Study

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ABSTRACT

The university timetabling problems can be divided into two categories: course timetabling and examination timetabling [1]. In this study, the examination timetabling problem, which is a sub-section of university timetabling problems, is handled. The examination timetabling problem can be considered as an assignment problem where a set of examinations must all be scheduled within a limited number of time-slots while satisfying a predetermined set of constraints. The construction of an examination timetable is a challenging task and is quite often time consuming in relatively large universities.

The examination timetabling problem is one of the most difficult combinatorial optimization problems and is considered to be NP-Hard. Thus, metaheuristics and their integrations/hybridisations with a variety of techniques, including many of the early techniques have been commonly used in the previous studies [3]. In contrast to others, this paper describes a new mathematical programming approach for the optimization of examination timetables. Moreover, the assignment of classrooms and invigilators to exams is performed simultaneously in the proposed model.

Firstly, previous studies in this field are mentioned. Then, a novel 0-1 integer programming model is proposed for the problem. The proposed mathematical model is also tested with case studies from Firat University. As an outcome of the study, the assignment of the exams to the most suitable classes, days and time-slots are realized with the suggested mathematical model. In this way, the examination timetabling problem is solved automatically within a reasonable time, which was tried to be solved without computer and required huge amount of time and effort previously.

Key Words: Examination timetabling problem, mathematical programming, scheduling, applied mathematics, operations research.

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A New Method For The Time Fractional Reaction-Diffusion Equations: Residual Power Series Method

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ABSTRACT

In this article, the residual power series method(RPSM) which a powerful iterative method for solving nonlinear time fractional reaction-diffusion equations is introduced. Residual power series algorithm gets Maclaurin expansion of the solution. Reliability of the method is given graphical consequens and series solutions are made use of to illustrate the solution. The found consequens show that the method is a power and efficient method in determination of solution the time fractional reaction-diffusion equations.

Key Words: Residual power series method, Time fractional Fitzhugh-Nagumo equation, Time fractional non-homogeneous reaction diffusion equation, Two dimensional time fractional Fisher equation, Series solution.

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A Note on the Fractional Boundary Problem

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ABSTRACT

There has been a significant development in fractional differential equations involving derivatives of fractional order in recent years. Fractional differential equations arise in many scientific disciplines as the mathematical modelling of systems and processes in the fields of physics, chemistry, economy, electro-dynamics, etc. For details, see [1-6]. But many papers and books on fractional differential equations provide very little discussion on fractional Sturm-Liouville problems. Formulations for fractional Sturm-Liouville presented recently and investigated the properties of the eigenfunctions and the eigenvalues of the operator in detail in [7,8].

In this study, we define singular fractional Sturm-Liouville problem with Coulomb potential:

$$\begin{aligned} D_{\pi,-}^{\alpha} p(x) {}^c D_{0,+}^{\alpha} y_{\lambda}(x) + \left(\frac{A}{x} + q(x) \right) y_{\lambda}(x) + \lambda w_{\alpha}(x) y_{\lambda}(x) &= 0, \\ y_{\lambda}(0) &= 0, \\ c_1 y_{\lambda}(\pi) - c_2 I_{\pi,-}^{1-\alpha} p(\pi) {}^c D_{0,+}^{\alpha} y_{\lambda}(\pi) &= 0, \end{aligned}$$

where $p(x) \neq 0, w_{\alpha}(x) > 0, \forall x \in (0, \pi], w_{\alpha}(x)$ is a weight function and p, q are real-valued continuous functions in the interval $(0, \pi]$. We show that the fractional Sturm-Liouville operator with Coulomb potential is self adjoint and also prove spectral properties of spectral data for the operator.

Key Words: Fractional Calculus, Sturm-Liouville problem, Caputo derivative

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A Novel S-box Construction Method Based on Chaos and Cyclic Groups

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ABSTRACT

Cryptographic techniques have been used for many applications areas. Today, developments on computer and communication sciences have helped transferring a big amount of data through the long distance channels. These data must be protected in several ways to provide confidentiality, integrity and authentication. Cryptology, which is the intersection of mathematics, computer science and electrical engineering, is the science. The construction of a substitution box (S-box) is an important research area in cryptography. S-boxes are vector Boolean functions. Being the only nonlinear part of a block cipher, the S-box should be constructed such that, its nonlinearity is as high as possible. This study has been proposed chaos and cyclic group based S-box generator. Performance and security analyses have been demonstrated that the proposed system has strong cryptographic properties.

Key Words: Cryptography, S-boxes, chaos, cyclic group.

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A Quintic Trigonometric B-spline Finite Element Method for Solving the Nonlinear Schrödinger Equation

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ABSTRACT

The nonlinear Schrödinger equation (NLSE) describes many physical phenomena such as propagation of optical pulses, waves in water, waves in plasmas and self-focussing in laser pulses. The NLSE was solved exactly for an initial condition $U(x, t_0) = f(x)$ using the inverse scattering method provided that the initial condition $U(x, t_0)$ and its derivatives vanish for sufficiently large x [1-3]. Since this method is only applicable for initial data with compact support, theoretical solution of the NLSE is unknown for general initial conditions. Approximate-analytical solutions by using various methods are also given in previous studies.

The B-spline functions have been used to construct efficient numerical methods for finding solutions of nonlinear partial differential equations. B-splines of various degrees have been used to build up numerical methods to obtain solutions of the partial differential equations [4-6]. The quintic B-spline is one of less used splines in forming numerical methods since it increases the cost of the algorithm. But the results of calculations show that accuracy of solutions is improved when numerical formulation employing quintic B-spline functions is set up in obtaining numerical solutions of partial differential equations. The trigonometric B-spline functions are alternative to the polynomial B-spline functions [7]. The trigonometric B-splines have been used to fit curve and to approximate the surfaces. But few studies in which the differential equations have solved with the collocation method incorporated the trigonometric B-splines exist [8].

In this study, a method of collocation with quintic trigonometric B-splines is described for finding an approximate solution of the NLSE. The collocation method leads to a spatial discretization of the NLSE. Then, Crank-Nicolson scheme is opted for time integration of resulting global matrix system of ordinary differential equations.

Finally, the resulting nonlinear matrix system of a recurrence relationship for global parameters is solved by using Matlab packet program. A single soliton solution is studied to illustrate the robustness of the algorithm by comparing with both analytical and previous numerical results. Besides, interaction of two colliding solitons is presented in turn.

Key Words: Schrödinger Equation, Collocation, Quintic Trigonometric B-spline.

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A Quartic Trigonometric B-spline Collocation Method for Solving the Nonlinear Schrödinger Equation

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ABSTRACT

It is well known that nonlinear Schrödinger equation (NLSE) governs many physical phenomena in various fields such as propagation of optical pulses, waves in water, waves in plasmas and self-focussing in laser pulses. The NLSE was solved exactly for an initial condition $U(x, t_0) = f(x)$ using the inverse scattering method provided that the initial condition $U(x, t_0)$ and its derivatives vanish for sufficiently large x [1-3]. Since this method is only applicable for initial data with compact support, theoretical solution of the NLSE is unknown for general initial conditions. Therefore, the numerical methods play an important role in the research of the NLSE.

B-splines of various degrees have been used to construct efficient numerical methods for finding approximate solutions of NLSE [4-6]. Present work also concerns with B-spline finite element method involving a collocation scheme with quartic trigonometric B-spline. The quartic trigonometric B-spline functions are alternative to the polynomial B-spline functions [7].

In this study, we have proposed an algorithm for numerical solution of the NLSE, which is a finite element approach by using collocation method over finite element with quartic trigonometric B-spline interpolation functions. For the numerical procedure, time derivative is discretized in the Crank-Nicolson scheme. Solution and its principal derivatives over the subintervals are approximated by the combination of the quartic trigonometric B-splines and unknown element parameters. The resulting nonlinear matrix system is solved by using Matlab packet program after the boundary conditions are applied. In order to show the accuracy of the algorithm and make a comparison of numerical solution with exact one, we have studied the single soliton solution. Moreover, interaction of two colliding solitons is also studied.

Key Words: Schrödinger Equation, Collocation, Quartic Trigonometric B-spline.

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A Solution of Telegraph Equation Using Natural Decomposition Method

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ABSTRACT

In this paper, we develop an Algorithm for solving telegraph equation using Natural Decomposition Method (NDM). The natural decomposition method is based on natural transform method and Adomian decomposition method. We solved three examples to illustrate the effectiveness of the method in obtaining exact solution in a rapid convergence series.

Key Words: Natural transform method, Natural decomposition method, telegraph equation.

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A Trigonometric B-spline Finite Element Method for Equal Width Equation

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ABSTRACT

The Equal Width (EW) equation

$$U_t + \varepsilon U U_x - \mu U_{xxx} = 0,$$

where the subscripts t and x denote differentiation and ε, μ are positive parameters, was introduced by Morrison et al. [1] as a model equation to describe the nonlinear dispersive waves. This equation is an alternative form of nonlinear dispersive waves to the well-known Regularized Long Wave equation and Korteweg-de Vries equation. These equations have solitary wave solutions which are wave packets or pulses. These waves propagate in nonlinear media by keeping wave forms and velocity even after interaction occurs. Few analytical solutions of the EW equation are known. So numerical methods are useful tool for study for the EW equation. Main properties of those solutions are that solitary waves propagate in one direction with constant speed without changing their shapes and that the solitary waves pass through one another and emerge unaltered in shapes.

The B-spline functions are bases for piecewise polynomials and used to construct approximate solution in the finite element techniques. So approximation solutions with B-splines of the differential equations can be obtained by method of weighted residuals, of which Galerkin and collocation methods are particular cases [2-5]. The quintic trigonometric B-spline functions are alternative to the polynomial B-spline functions [6].

In the present work, we set up finite element solution by using quintic trigonometric B-splines as the element for the EW equation. Using the Crank-Nicolson scheme for nodal parameters and the time integration, the resulting system of the ordinary differential equations is discretised to lead to linear system of

algebraic equations. This system is solved by using Matlab packet program. Nonlinear part of the system is handled by using an inner iteration.

Key Words: EW Equation, Finite Element, Spline.

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An Application of Finite Element Method for a Moving Boundary Problem

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ABSTRACT

The class of moving boundary problems has phase change which is almost concerned with mathematical models of heat, diffusion, oxygen tension equations etc. The main trouble of these problems is that the position of moving boundary must be determined as part of the solution. For this reason moving boundary problems are accepted as non-linear problems. These problems commonly called as Stefan problems have limited analytical solution for special cases. Due to the shortage of the analytical solutions, some numerical methods have been developed to solve Stefan problems.

In this study, we concerned with the numerical solution of the one-phase Stefan problem called as solidification problem. After applying variable space grid method and boundary immobilisation method, we use finite element method based on cubic B-spline bases functions. The computational results for the position of moving boundary are compared with exact solutions and the other numerical solutions obtained by using automatic differentiation method and finite element method. It is seen that the present results are more accurate than the other numerical solutions for small element numbers. And all of the numerical results obtained for the position of moving boundary are in good agreement with the exact ones. The stability analysis of finite element methods constructed with variable space grid method and boundary immobilisation method are also presented.

Key Words: Stefan Problems, Variable Space Grid Method, Boundary Immobilisation Method, Collocation Finite Element Method.

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An Application of Improved Bernoulli Sub-Equation Function Method to the Nonlinear Time-Fractional Burgers Equation

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ABSTRACT

In this work, we study on the improved Bernoulli sub-equation function method. We apply this method to the nonlinear time-fractional Burgers equation. We obtain new analytical solutions of the equation for values of M, n and m . Two and three dimensional surfaces of analytical solutions are plotted by wolfram Mathematica 9. Finally, we submit a conclusion by giving important aspect of this study.

Key Words: Improved Bernoulli sub-equation function method, Time-fractional Burgers equation, Exponential function solution.

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An Approach to Volterra and Fredholm Integral Equations by Homotopy Perturbation Method

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ABSTRACT

In this study, an application of the homotopy perturbation method for Volterra and Fredholm integral equations is presented. The proposed method is based on the homotopy perturbation method, which consists in constructing the series whose sum is the solution of the problem considered. In this method, a homotopy with an imbedding parameter $p \in [0,1]$ is constructed. Furthermore, this integral equations are solved by Adomian Decomposition Method, Variational Iteration Method and Successive Approximations Method. The effectiveness and practicality of the homotopy perturbation method is evaluated according to analytical and numerical results.

Key Words: Integral equations, Homotopy perturbation method.

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An Indicator Operator Algorithm for Solving a Second Order Intuitionistic Fuzzy Initial Value Problem

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ABSTRACT

L. Zadeh [1] was the first who introduced the concept of fuzzy set theory as an extension of the classical notion of the set theory. He reminded people that things are not always black or white; there may be some grey colours in life. Hence he simply assigned so called the membership function to each element in a classical set and started the fuzzy set theory. However in some cases the membership concept in a fuzzy set is itself uncertain. This uncertainty may be because of the subjectivity of expert knowledge, complexity of data or imprecision of the models. Since the fuzzy set theory considers only the degree of membership, it does not involve the degree of uncertainty for the membership. To handle such situations, the generalized concepts of fuzzy set theory are used [1]-[5]. One of these generalizations is intuitionistic fuzzy set theory which was given by Atanassov [2]. Atanassov introduced intuitionistic fuzzy set concept by extending the definition of fuzzy set after analyzing the shortcomings of it. He defined the intuitionistic fuzzy set concept by introducing the non-membership function into the fuzzy set such that sum of both is less than one. And in his further researches, he showed the exclusive properties of intuitionistic fuzzy sets [6]-[13]. In the past decades, the intuitionistic fuzzy set theory has penetrated into different research areas, such as decision making [14]-[17], clustering analysis [18], medical diagnosis [19]-[20], pattern recognition [21]-[23]. In recent years the topic of fuzzy differential equations has been rapidly grown to model the real life situations where the observed data is insufficient [24]-[27]. Especially to describe the relation between velocity and acceleration, second order differential equations has greater importance in science. Therefore many approaches [27]-[29] were given to solve second order fuzzy differential equations. However there are only few works

[30]-[32] to observe the intuitionistic fuzzy differentialequations. In this work, we have examined the solution of the following second order intuitionistic fuzzy initial value problems given in Eq. (1)-(2) using intuitionistic Zadeh's Extension Principle [13].

$$y''(x) + a_1^i y'(x) + a_2^i y(x) = \sum_{j=1}^r b_j^i g_j(x)$$
$$y(0) = \gamma_0^i; y'(0) = \gamma_1^i$$

Here $\gamma_0^i; \gamma_1^i$ and \bar{b}_j^i ($j=1, 2, 3, \dots, r$) are intuitionistic fuzzy numbers. $g_j(x)$ ($j=1, 2, 3, \dots, r$) are continuous forcing functions on the interval $[0, \infty)$.

Key Words: Intuitionistic fuzzy sets, Zadeh's extension principle, Intuitionistic fuzzy differential equations.

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An Optimal Control Problem for Schrödinger Equation

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ABSTRACT

In this paper, an optimal control problem for linear Schrödinger equation is considered. The optimal control problem is formulated as follows: is to find the minimum of objective functional

$$J_{\alpha}(v) = \|\psi_1 - \psi_2\|_{L_2(\Omega)}^2 + \alpha \|v - w\|_{L_2(0,T) \times L_2(0,T)}^2$$

on the set of admissible controls

$$V = \{v = (v_0, v_1) : v_j \in L_2(0, T), |v_j(t)| \leq b_j \text{ for almost all } t \in (0, T), b_j = \text{const.} > 0, j = 0, 1\}$$

under the conditions

$$i \frac{\partial \psi_1}{\partial t} + a_0 \frac{\partial^2 \psi_1}{\partial x^2} - a(x)\psi_1 - v_0(t)\psi_1 - iv_1(t)\psi_1 = f_1(x, t)$$

$$\psi_1(x, 0) = \varphi_1(x), \quad x \in (0, l)$$

$$\psi_1(0, t) = \psi_1(l, t) = 0, \quad t \in (0, T)$$

and

$$i \frac{\partial \psi_2}{\partial t} + a_0 \frac{\partial^2 \psi_2}{\partial x^2} - a(x)\psi_2 - v_0(t)\psi_2 - iv_1(t)\psi_2 = f_2(x, t)$$

$$\psi_2(x, 0) = \varphi_2(x), \quad x \in (0, l)$$

$$\frac{\partial \psi_2(0, t)}{\partial x} = \frac{\partial \psi_2(l, t)}{\partial x} = 0, \quad t \in (0, T)$$

where $a_0, \alpha > 0$ are given numbers, $0 \leq x \leq l, 0 \leq t \leq T, \Omega = (0, l) \times (0, T), a(x)$ and φ_k, f_k for $k = 1, 2$ are given functions.

In the present study, the existence and uniqueness of solutions of considered optimal control problem in two different cases are proved. The differentiability of objective functional is shown by means of an adjoint problem and its gradient is obtained. Finally, a necessary optimality condition in the variational form is given.

Key Words: Optimal control, Schrödinger equation, Objective functional.

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Algorithmic a View to Codes of Global Power

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ABSTRACT

In this study it was calculated area of regular pentagon with sinüs law. Then, algorithm calculated by giving area of regular n edge . This algorithm When calculating, first; as center that accepts the center of the regular pentagon and using axis system from the center to corners drawn change according to differences in these axes of each triangle area were examined.

After then; these operations by applying to n polygon are written general algorithm. Ghazali's approach with an approach that is built on these algorithms were developed as a quantitative measure. Later; developed this quantitative measure was proposed as a mathematical measure covering global power.

Key Words: Global power, Algorithm of Global Power, Pentagon Approach, Codes of Global Power

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Algorithmic Music Composition With Stochastic Processes and Genetic Algorithms

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ABSTRACT

Algorithmic composition is a music composition system that models the human emotions based on computations that depend on musical rules. In this study, we used stochastic processes and genetic algorithms to create algorithmic compositions and the results are analyzed.

For stochastic process, the composed music is provided as input to the model and some preprocessing is implemented and the corresponding statistics are obtained. The system uses these statistical information to start producing the melody of the music piece. The octaves implement a crucial role in the process. We need to find out how frequently the particular octaves are used in the original piece; as a result the weighted music array is formed. With that, the basic structure of the composition is formed. The general flow processing for the composition is one of the most important contributors to the structure. In Turkish music, this is particularly visible. The frequencies of the octaves that follow each other are computed. This provides the backbone of the stochastic model of the melody formation of the composition. In this study, the melody structure is processed using "Markov Model" [1].

Also, in this study, the composition generation is implemented using genetic algorithms [4,5,6,7,8]. During the experimentations, it is observed that increasing the number of genes (longer tunes) reduces the quality of the melody and the composition and the overall performance decreases. Nowadays, only short music pieces and small sample spaces are used for evolutionary music generation. However, in the jazz music area in particular, good results are obtained [2,3].

Unless we model the human ear structure and emotions within the genetic algorithm modelling, it will be very difficult to obtain satisfactory results with genetic

algorithms. However if we limit ourselves only to a smaller subspace of the chords (such as C major) for producing a short music piece, genetic algorithms provide better results. Besides, if the associated rules are well-defined and genetic algorithms increase the chord space gradually starting from scratch, the results can be improved, but these studies are out of scope for the current study.

Key Words: Algorithmic composition, music composition, Markov model, stochastic process, genetic algorithms.

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Analytical Evaluation Of Self-Friction Two-Center Hybrid Integrals Over Slater Type Orbitals By Using Guseinov One-Range Addition Theorem

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ABSTRACT

For the calculation of self-friction two-center hybrid integrals we use Guseinov one-range two center addition theorems [1] and Löwdin- α radial function. The obtained formulae are in terms of the two-center overlap integrals over Slater type orbitals (STOs) with the same screening parameters and one center two electron integrals. The overlap integrals occurring in established formulae can be easily evaluated by using our obtained basic formula [2, 3]. As seen from the evaluation results, the analytical expressions of new formulae are useful for the electronic structure calculations.

Key Words: Self-friction two center hybrid integrals, Guseinov one range addition theorem, Löwdin- α radial function

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Application of Eadm and Evim to Hirota-Satsuma Coupled Kdv Equation

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ABSTRACT

We obtain series solutions of Hirota-Satsuma coupled KdV (HSCkdv) equation with initial condition by using extended Adomian decomposition method (EADM) and extended variational iteration method (EVIM) and we compare these solutions with the some solutions that are exist in the literature.

Key Words: EADM, EVIM, HSCkdv equation.

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Application of Gravitational Search Algorithm for Obtaining Numerical Solutions of IVPs

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ABSTRACT

Artificial Neural Networks (ANNs) are used to predict or approximate functions depending on a large number of inputs. Therefore, ANNs have wide variety of application areas. In addition, they are capable of solving differential equations numerically. In this study, a Feedforward Neural Network (FNN) model is constructed to estimate the solution of initial value problem (IVP) for ordinary differential equations (ODEs). For this aim, a trial solution based on neural network parameters of given differential equation, which keeps initial condition, is generated firstly. Then, a cost function depending on trial function is defined. Finally, the network is trained with a global optimization method known as Gravitational Search Algorithm (GSA) to minimize the cost function. In experiments, obtained results for a given initial value problem are compared with analytical solution of IVP and some numerical methods.

Key Words: Feed-forward neural network, Gravitational Search Algorithm, GSA, Global optimization.

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Asymptotic Distribution of Eigenvalues for Fourth-Order Boundary Value Problem With Transmission Conditions

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ABSTRACT

We investigate a fourth-order boundary value problem with discontinuous coefficients, functional and transmission conditions. In this problem, boundary conditions contain not only endpoints of the considered interval, but also a point of discontinuity, a finite number internal points and abstract linear functionals. We discuss asymptotic formulas for the eigenvalues of the problem.

In classical theory, boundary-value problems for ordinary differential equations are usually considered for equations with continuous coefficients and for boundary conditions which contain only end-points of the considered interval. However, this problem deals with one nonclassical boundary-value problem for ordinary differential equation with discontinuous coefficients and boundary conditions containing not only end-points of the considered interval, but also a point of discontinuity and internal points. These type problems are connected with different applied problems which include various transfer problems such as heat transfer in heterogeneous media. Naturally, transmission problems arise in various physical fields as the theory of diffraction, elasticity, heat and mass transfer (see, [1]).

The investigation of boundary value problem for which the eigenvalue parameter appears both in the equation and boundary conditions originates from the works of G.D. Birkhoff ([2]). There are many papers and books that the spectral properties of such problem are investigated ([7]). Some spectral properties in the differential equation and boundary conditions have been studied by O. Sh. Mukhtarov and others (see, [3], [4], [5], [6]).

In this study, we shall consider a fourth order differential equation

$$p(x)u^{(4)} + \varphi(x)u = \lambda^4 u, \quad x \in I$$

with the functional-transmission boundary conditions

$$L_k(u) = \sum_{s=0}^3 \lambda^{4-s} [\alpha_{ks} u^{(s)}(-1) + \beta_{ks} u^{(s)}(-0) + \delta_{ks} u^{(s)}(+0) + \gamma_{ks} u^{(s)}(1)] \\ + \int_{-1}^0 u^{(s)}(x) \phi_{ks}(x) dx + \int_0^1 u^{(s)}(x) \phi_{ks}(x) dx = 0, \quad k = 1, 2, \dots, 8,$$

where $I = I_1 \cup I_2 = [-1, 0) \cup (0, 1]$; and $q(x)$ are complex valued functions;

$p(x) = p_j(x)$ and $q(x) = q_j(x)$ for $x \in I_j$, $j = 1, 2$; $\alpha_{ks}, \beta_{ks}, \delta_{ks}, \gamma_{ks}$ are complex coefficients,

$a_{ksj}^i \in I_i$ internal points and $u^{(m_k)}(\mp 0)$ denotes $\lim_{x \rightarrow \mp 0} u^{(m_k)}(x)$.

Key Words: Fourth order problem, eigenvalue parameter, asymptotic distribution.

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Asymptotic Stability for a Generalized Linear Difference System with Two Delays

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ABSTRACT

Asymptotic stability of linear difference equations with constant coefficients have been studied by many authors such as Levin-May [1], Clark [2], Kuruklis [3], Matsunaga-Hara [4], Matsunaga [5] and Nagabuchi [6]. In this study generalized the study of Nagabuchi for $a \in [-1,1] - \{0\}$ and obtained new necessary and sufficient conditions for the asymptotic stability of a generalized linear delay difference system with two delays of the form

$$x_{n+1} - ax_n + A(x_{n-k} + x_{n-l}) = 0, \quad n \in \{0,1,2, \dots\}$$

where A is a 2×2 constant matrix, $a \in [-1,1] - \{0\}$ is a real number and k, l are positive integers.

Key Words: Stability, Asymptotic stability, difference equations.

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Balancing of Assembly Lines with Mathematical Programming Method

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ABSTRACT

Assembly lines are the most important components of flow-line production systems. An assembly line is a manufacturing process which consists of a set of workstations connected together by transport mechanisms such as conveyor systems. The problem of assigning assembly works (tasks) to this workstations so that the total time required at each workstation is approximately the same by considering some constraints about cycle time or precedence relationships is known as the assembly line balancing problem (ALBP). Assembly lines are divided into several groups with subject to their shapes and number of different products produced on the line. In terms of the line shape, assembly lines are classified as straight and U-type. By means of the number of produced product, they are also classified into three main categories; single model, mixed model and multi model assembly lines. Many exact, heuristic and metaheuristic approaches have been proposed for balancing varied assembly lines.

In this study, mathematical programming method is used to solve the single model assembly line balancing problem. Firstly, the mathematical programming model with the objective of minimizing the cycle time of the assembly line is presented to solve the problem. Then, a real-world example is presented to illustrate the mathematical model. Finally, some computational analysis are conducted to assess the performance of the model.

Key Words: Assembly line balancing, mathematical programming, optimization, applied mathematics, operations research.

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Basis Properties of Discontinuous Sturm-Liouville Problems

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ABSTRACT

In the recent years there has been an increasing interest in the spectral analysis of Sturm-Liouville type problems with eigenvalue-dependent boundary conditions. Such problems arise upon separation of variables in the varied assortment of physical problems. For instance, this kind of problems arise in vibrating string problems when the string loaded additionally with point masses, in thermal conduction problems for a thin laminated plate, in the diffusion of water vapour through a porous membrane and several electric circuit problems involving long cables. Note that discontinuous Sturm-Liouville problems with supplementary transmission conditions at the points of discontinuities have been investigated in [1, 2, 3] such properties as isomorphism, coerciveness with respect to the eigenvalue parameter, expansion in the series of the eigenfunctions, asymptotics of eigenvalues and eigenfunctions of same discontinuous boundary-value problems and its applications have been investigated in [4, 5, 6,7, 8] .

In this work we shall study some spectral properties of one discontinuous Sturm-Liouville problem for which the eigenvalue parameter takes part linearly in both differential equation and boundary-transmission conditions. At first this standard formulation is replaced by the “week formulation” by using the classical method (see, for example Ladyzhenskaia [9]). The concept of “week formulation” allows the considered problem to be reduced to an operator pencil equation of the form

$$L(\lambda)u := \sum_{i=0}^n \lambda^i A_i u = 0 \quad \text{with } n = 1 \text{ in suitable Hilbert space } H .$$
 It is shown that there is a

real constant $\lambda = \lambda_0$ such that the operator $L(\lambda_0)$ is positive and self-adjoint in H . By

using this result it is proven that the system of “weak eigenfunctions” of the considered problem form a Riesz basis of the Hilbert space H .

Key Words: Sturm-Liouville problems, eigenvalue, eigenfunctions, basis, .

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Black Scholes Model with Numerical Solutions

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ABSTRACT

In this study we will consider Black Scholes model which is very handy in finance. We will approach the solution using the Euler Maruyama and Milstein numerical methods. Also we will obtain exact solution for this model using Ito calculus. We will compare exact solution and numerical solution which we obtained. The aim of this study is to compare our result by solving sample paths with numerical simulations.

Key Words: Black Scholes model, stochastic differential equations, Euler Maruyama method, Milstein method.

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Blow Up Of Positive Initial-Energy Solutions For The Extensible Beam Equation

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ABSTRACT

In this work, we study the extensible beam equation with initial and boundary condition. Under suitable conditions on the initial datum, we prove that the solution blow up in finite time with positive initial-energy.

Key Words: Extensible beam equation, blow up, nonlinear damping term.

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Computation of Fundamental Matrix in Floating Point Arithmetic According to IEEE Standard

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ABSTRACT

Computers work by finite set of numbers. Thus, to use a computer to do arithmetic in field of real numbers can cause to failure [1]. Because real numbers is an infinite set and most of the elements of this set can not be represented in computers.

The representation of the numbers $z = \pm \gamma^{p(z)} m_{\gamma}(z)$, which are called *floating point numbers*, are stored in memory and used in computations. These numbers are also called *computer numbers* or *machine numbers* in some references [2]. The set $F = F(\gamma, p_-, p_+, k)$ is *Format*, where $\gamma, p_- \in \mathbf{Z}^-$, $k, p_+ \in \mathbf{Z}^+$ for $p_- \leq p \leq p_+$, $p \in \mathbf{Z}$, γ – base.

Format provides mathematical model to represent floating point numbers [3]. It is clear that F is finite and $F \subset \mathbf{Q}$. Floating point arithmetic is used in scientific applications for numerical calculations [4]. IEEE 754 standard of binary floating point arithmetic is accepted standards for floating point arithmetic [5, 6]. Many hardwares and softwares use the IEEE 754 standard. Most scientific and engineering computations on a computer are performed using floating point arithmetic. IEEE standard recommends 24 binary digits for a single precision, and 53 binary digits for a double precision.

Let A_n be an N-dimensional periodic matrix (T-period) and consider the following difference equation system $x_{n+1} = A_n x_n$. The matrix X_n is called the fundamental matrix of the system, and the matrix X_T is called the monodromy matrix of the system [7]. In this paper, the effects of IEEE floating point arithmetic on the

computation of monodromy matrix X_T were investigated. Error bounds were obtained for $\|X_T - Y_T\|$, where the matrix Y_T is the computed monodromy matrix. The obtained results were applied to the asymptotic stability of the system [8]. In addition, these results were shown with numerical examples.

The errors which occurred using IEEE floating point arithmetic in computations can affect results about evaluation on stability of system. For this reason, an asymptotically unstable system can be an asymptotically stable system or, in a similar vein, an asymptotically stable system can be an asymptotically unstable system. The results show that the format set should be considered to use in computing.

Key Words: floating point, IEEE Standard, asymptotic stability.

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Complementary Lidstone Boundary Value Problems for Fractional Differential Equations

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ABSTRACT

In this work, we are concerned with a class of complementary Lidstone boundary value problems for fractional differential equations. We will set up some sufficient conditions for existence of solutions to differential equations of fractional order with complementary Lidstone boundary conditions which are odd-order boundary conditions and actually have been solely used for boundary value problems for differential equations of integer orders up to now.

Key Words: Fractional integral and derivative, fixed point theorem, existence of solutions, Lidstone boundary condition.

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Computation of the Solution of a System of Linear Algebraic Equations with Iterative Decreasing Dimension Method in Floating Point Arithmetic

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ABSTRACT

Studying the solution of the systems of linear algebraic equations $Ax = f$ is a classical problem which is important in applied mathematics. For system $Ax = f$ where A is an $n \times n$ regular matrix, x and f are n -vectors to be solved for, an iterative decreasing dimension method (IDDM) is given in (Keskin and Aydın 2007). According to this method, the solution of the system $Ax = f$ is

$$x = \sum_{i=1}^n \left(\prod_{j=1}^{i-1} R^{(j)} \right) X_0^{(i)} .$$

Here $X_0^{(k)}$, $R^{(k)}$ and $\prod_{j=1}^{i-1} R^{(j)}$ symbols are used same as in the relevant article (Keskin and Aydın 2007).

When a computer is used for the computations of the solution of the problem $Ax = f$, errors occur naturally. Because the computers make the calculations with computer numbers. The computer numbers set (or format set) is defined by a set of $\mathbf{F} = \mathbf{F}(\gamma, p_-, p_+, k)$, where $p_- \in \mathbf{Z}^-$, $k, p_+ \in \mathbf{Z}^+$ and γ -base. The set \mathbf{F} is characterized by the characteristics $\varepsilon_0 = \gamma^{p_- - 1}$, $\varepsilon_1 = \gamma^{1-k}$, $\varepsilon_\infty = \gamma^{p_+} (1 - 1/\gamma^k)$ (Godunov et al 1993, Akın and Bulgak 1998).

The operator fl ($fl : \mathbf{D} \rightarrow \mathbf{F}$) converts the real numbers to floating point numbers with rounding or chopping errors, where $\mathbf{D} = [-\varepsilon_\infty, \varepsilon_\infty] \cap \mathbf{R}$. According to Wilkinson model the operator fl is defined as

$$z \in \mathbf{D} \Rightarrow fl(z) = z(1 + \alpha), |\alpha| \leq u,$$

where $u = \frac{\varepsilon_1}{2}$ – rounding and $u = \varepsilon_1$ – chopping (Wilkinson 1963, Goldberg 1991, Golub and Van Loan 1996, Shampine et al 1997, Bjoerck and Dahlquist 2008).

In this study, the effects of floating point arithmetic in the computation of solution of the matrix equation $Ax = f$ with IDDM are investigated. Some upper bounds are obtained for $\|y\|$ and $\|x - y\|$, where $fl(x) = y$ is the computed value of the value x in floating point arithmetic. The obtained results are supported by numerical examples.

Key Words: Iterative decreasing dimension method, floating point arithmetic, error analysis.

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Conservation Laws and Exact Solutions of Nonlinear partial Differential Equation

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ABSTRACT

It is known that conservation laws plays an important role in the solution of an equation or system of differential equations. All conservation laws of partial differential equations (PDEs) have not physical meanings, but they are necessary for studying the integrability and reduction of PDEs. At the mathematical level conservation laws are deeply connected with the existence of a variational principle which admits symmetry transformations. This very important case was fully acknowledged by Emmy Noether in 1918. There are lots of powerful methods obtaining conservation laws for PDEs for example characteristic method, variational approach, symmetry and conservation law relation, multiplier approach, direct construction method for conservation laws, partial Noether approach, Noether approach, conservation theorem. In this study we will deal with conservation theorem which was introduced by Ibragimov. This theorem is associated with Lie symmetries, formal Lagrangian equations and adjoint equations of PDEs. Every symmetry of PDEs always provides a conservation law. These conservation laws can be trivial or non-trivial.

Also much effort has been spent on the construction of exact solutions of nonlinear PDEs, for their determining role in understanding the nonlinear phenomena. In recent years, many powerful methods have been proposed, such as the tanh function method, the extended tanh method, the sine-cosine method, the homogeneous balance method, the exp-function method, the modified simple equation method, the first integral method, the extended trial equation method, the

extended trial equation method, the (G'/G) -expansion method, the $(G'/G, 1/G)$ -expansion method and the auxiliary equation method and so on. In this study, we will use the $\exp(-\Phi(\xi))$ method for obtaining exact solutions of partial differential equations.

Key Words: Conservation laws, exact solutions, partial differential equation

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Conservation Laws of Partial Differential Equations Using Two Different Method

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ABSTRACT

The conservation laws plays an important role in the solution and reduction of partial differential equations. The mathematical idea of the conservation laws comes from the formulation of physical laws such as for mass, energy, and momentum. There are many different methods to the construction of conservation laws. For example, characteristic method, variational approach, symmetry and conservation law relation, direct construction method for conservation laws , partial Noether approach, Noether approach, conservation theorem.

It is well known that a conservation laws which are obtained with partial Noether approach, are associated with each Noether symmetry of a Euler Lagrange differential equation. Noether 's theorem is related with the knowledge of Lagrangian or partial Lagrangian. The existence of mixed derivative term (a term involves derivatives of more than one independent variables) gives increase to some interesting divergence properties and conserved vectors are computed by Noether's theorem does not satisfy the divergence relationship. A number of extra terms arise that contribute to the trivial part of conserved vector and need to be adjusted to satisfy the divergence relationship.

Conservation theorem associated with Lie symmetry generators of partial differential equations. We should obtain formal Lagrangian, adjoint equation. Every partial differential always have formal Lagrangian, so they have adjoint equation. Then we should obtain solution of partial differential equation for obtaining infinite number of conservation laws. All Lie symmetries construct conservation laws for given equation, that can be trivial or non-trivial.

This study deals with conservation laws of partial differential equations. We used partial Noether approach and conservation theorem for finding conservation laws for these equations. All founded conservation laws are trivial conservation laws in this study.

Key Words: Conservation laws, partial differential equations, Lagrangian approach

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Coordinate based Learning Parameter Optimization for Gradient Descent with Momentum

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ABSTRACT

The choice of learning parameters can be problematic in most of the cases when training a neural network with gradient descent. Smaller learning rates lead to oscillations in the parameter space, whereas larger learning rates have the risk of divergence throughout the learning process. Care must be taken in order to ensure convergence of the learning algorithm while improving the convergence speed. Effective learning rate and momentum factor is proposed in [1] based on the largest and smallest eigenvalue of the Hessian. These learning parameters are scalars that calculated at the beginning of the training process. However, each coordinate of the parameter vector has different convergence characteristics. Therefore, in this study, in order to capture these characteristics, we try to determine a learning rate and a momentum factor for each coordinate of the parameter vector. In this way, each coordinate proceeds in its own direction governed by its own learning parameters. Consequently, we have a vector of learning parameters to be optimized through the process. Furthermore, these parameters can be updated at every or some specific steps of the learning process in order to capture information on different regions of the parameter space. This approach is tested on randomly generated problems which have different degrees of difficulty. Results demonstrated that the new algorithm converges faster than the conventional gradient descent algorithms.

Key Words: gradient descent, convergence speed, stability, learning rate, momentum

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Cubic Trigonometric B-Spline Galerkin Solution for the Equal Width Equation

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ABSTRACT

Consider the following equal width (EW) wave equation

$$u_t + \varepsilon uu_x - \mu u_{xxt} = 0, \quad a < x < b, \quad 0 < t < T, \quad (1)$$

with boundary conditions

$$\begin{aligned} u(a, t) &= u(b, t) = 0, \\ u_x(a, t) &= u_x(b, t) = 0, \end{aligned} \quad (2)$$

and initial condition

$$u(x, 0) = f(x), \quad (3)$$

where u is the dependent variable, ε and μ are positive parameters, t and x are the independent variables. The EW wave equation suggested by Morrison et.al. is used as a model partial differential equation for the simulation of one-dimensional wave propagation in a nonlinear medium with a dispersion process [1]. Nonlinear dispersive wave equations exhibit types of solutions such as solitary waves and solitons. Solutions of them are not analytically available for every boundary and initial conditions in general. Since only a few analytical solutions to the EW equation with some initial and boundary conditions have been known so far, numerical methods are a useful tool for studying the EW equation [2,3].

In this study, the EW equation will be solved numerically using the Galerkin finite-element method, based on Crank Nicolson method for time integration and cubic trigonometric B-spline for space integration. The proposed algorithm is tested on the solitary wave motion and interaction of two solitary waves test problems. The three conserved quantities of motion are calculated to determine the conservation properties of the proposed algorithms for both test problems.

The numerical results of this study demonstrate that the proposed algorithm exhibited high accuracy and efficiency in for numerical solution of the EW equation.

Key Words: Trigonometric B-spline, Solitary waves, EW equation

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Dark Soliton Solution of Nonlinear Partial Differential Equations Using Ansatz Method

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ABSTRACT

It is well known that soliton solutions of nonlinear partial differential equations (NPDEs) play an important role in the study of nonlinear wave phenomena. Many problems of fluid mechanics, plasma physics, elastic media, optical fibres, population dynamics, chemical kinetics, nonlinear optics etc. are described by nonlinear partial differential equations [1,2]. Once a phenomenon is modelled in terms of mathematical equations, one is generally interested to find exact analytic solutions of such nonlinear equations in order to predict and quantify the underlying features of the system under study.

Over the past two decades or so several methods for finding the exact solutions to NPDEs have been proposed, such as the homogeneous balance method [3], the trigonometric function series method [4], the sine-cosine method [5], the auxiliary equation method [6], the (G'/G) expansion method [7], the exp- function method [8], the Jacobi elliptic function expansion method [9], the transformed rational function method [10], the wave ansatz method [11], Lie transform perturbation method [12], the trial equation method [13], the extended tanh-coth function method [14] and so on.

In this study, we establish exact solutions for the nonlinear partial differential equations. By using solitary wave ansatz in terms of \tanh^p functions we find exact analytical dark soliton solutions for the considered model.

Key Words: The ansatz method, partial differential equation, exact solution.

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Decay Of Solutions For A Nonlinear Petrovsky Equation With Nonlinear Damping Term

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ABSTRACT

This work studies the initial boundary value problem for the Petrovsky equation. Under suitable conditions decay estimates of the solution are proved by using Nakao's inequality.

Key Words: Decay, Petrovsky equation, global existence

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Deriving the Discrete Optimal Control Problem of Stochastic Partial Differential Equations By Runge-Kutta Method with Numerical Applications

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ABSTRACT

Stochastic control problems have a crucial place in the fields of finance, physics, biology, economics, etc.. Many works have been done in the numerical solutions of stochastic differential equations [1, 3, 4]. Recently, numerical solutions of optimal stochastic control problems of stochastic partial differential equations (SPDEs) have been in interest. There are many approaches to find optimality conditions. The most popular of such approaches is the Pontryagin maximum principle. One can follow *optimize-then-discretize* and *discretize-then-optimize* to find the discrete scheme of an optimal control problem. When we apply the first approach to optimal control of SPDEs, Lagrange multipliers are obtained implicitly. In other words, the multiplier is coupled as (p, q) with $q = p_{xx}$. But, it may be difficult to decide the parameter q . The advantage of the second approach with Runge-Kutta scheme is that all Lagrange multipliers can be found explicitly in the optimality system. The discrete form of the parameter q is computed automatically.

As for space discretization, a finite difference scheme is used. Then, the state equation is discretized by Runge-Kutta method [3, 4]. A discrete Lagrange function is defined to obtain the discrete optimality conditions [2]. The coefficients of the obtained Runge-Kutta scheme are symplectic partitioned. In this work, it is the first time that SPDEs are transformed to a matrix equation by following finite difference scheme, and then Runge-Kutta method is applied to system of equations. Numerical solution of optimal control of SPDEs is obtained by Runge-Kutta scheme. Numerical results are presented for some applications.

Key Words: stochastic optimal control, Runge-Kutta scheme, SPDEs.

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Detection of Counterfeit Electronic Materials By Using Image Comparison Methods

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ABSTRACT

With the continuous development of information and technology era, the world electronics production volumes also grew. Increasing of electronic product requirement in 21st century also has provided the magnification of the electronic sector every day. Counterfeits electronic products are becoming a bigger problem every day with the sector growth. Counterfeits electronic products mostly constitutes a major risk in medical, telecommunications and military fields. Even though such materials work properly, constitute a major risk in terms of stability and service life. In this article, a study based on image processing methods for the detection of counterfeit electronic materials which constitute a great risk was accomplished. Original and counterfeit materials have been distinguished from each other by using image processing techniques.

Key Words: counterfeit electronic product; image processing.

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Development of an M2M Platform with a Responsive Design

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ABSTRACT

Nowadays it has been clearly observed that machines are more and more communicating with each other. An M2M (Machine to Machine) communication application developed for any sector needs an M2M platform in order to interact with the user. The aim of an M2M platform is to fulfill the functions such as collecting data, tracking the data, data management, and reporting and analysis.

M2M platforms are mostly developed as web-based systems, and the user is aimed to connect in any environment to the platform. In this direction, some of the M2M platforms are released as both desktop and mobile applications. However, in such cases, there is a need to develop different interface designs for mobile operating systems and screen size. This issue makes the interface developers' job harder. In this study, the M2M platform is designed with a responsive interface rather than developing different M2M interface designs for every mobile operating system and screen size. Thanks to this responsive design, it is no longer needed to develop different interfaces for desktop, web, and mobile systems. The M2M platform interface developed in this study. This platform can be used comfortably with the dynamic feature shaped according to the desktop and mobile screen sizes through web browsers.

Key Words: responsive design, M2M platform, responsive web design

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Development of Recommender Systems Based on the Curve Fitting Method

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ABSTRACT

Due to advances in network and information technologies, the amount of information on the World Wide Web is growing day by day with an awesome pace. This slows down the speed of individual user access to relevant and useful information. Recommender Systems (RS) have emerged as a technique that can analyze a large amount of data, and advice users in making decisions. Collaborative Filtering (CF) is a common RS approach that estimates the customer preferences on a product. The main assumption of CF methods is that the users will, in the future, exhibit behavior similar to their past behaviors. Judging from this assumption the problem reduces to the analysis of the data matrix. CF methods can be divided into two groups: memory-based CF methods and model-based CF methods. In memory-based methods; the similarity indices between the users are computed, and the predictions are made based on these similarity indices. A popular model-based CF method is Probabilistic Matrix Factorization (PMF). PMF method investigates the spectral features of the data matrix.

In this study, we propose a new algorithm for CF approach based on curve fitting methods. We compare the experimental results of our proposed algorithm with the results of other algorithms on the same datasets.

Key Words: Recommender System, Collaborative Filtering, Curve Fitting.

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Dirichlet Boundary Problem for the $p(x)$ Laplacian Equations with Dependence on Gradient

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ABSTRACT

The present deals with a quasilinear elliptic equation with variable exponent under homogenous Dirichlet boundary-value condition, where nonlinearity depends also on the gradient of the solution. By using an iterative method based on Mountain Pass techniques, the existence of a positive solution is obtained.

In the present we study the existence of solutions of the problem

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u + |u|^{p(x)-2}u) = f(x, u, |\nabla u|^{p(x)-2}\nabla u), & \text{in } \Omega \quad (P), \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $p(x) < N$ for any $x \in \bar{\Omega}$ and f is a continuous function which obeys some specific conditions. Since the nonlinearity f depends on the gradient of the solution, equation (P) is not variational. To build the variational frame, we firstly use a "freezing" technique whose formulation appears initially in [4,5]. This technique will help us change the problem (P) into a family of problems without dependence of ∇u . That is, for each $w \in W_0^{1,p(x)}(\Omega)$ fixed, we consider the "freezed" problem given by

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u + |u|^{p(x)-2}u) = f(x, u, |\nabla w|^{p(x)-2}\nabla w), & \text{in } \Omega \quad (P_w), \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

The nonexistence of a priori estimates, with respect to the norms of the gradient of the solution problem (P_w) , is the main difficulty for using the variational technique. Moreover, solution of the problem described in the variable exponent Sobolev spaces $(W^{1,p(x)}(\Omega))$. Now let us describe this spaces. Set $C_+(\bar{\Omega}) = \{h: h \in C(\bar{\Omega}), h(x) > 1 \text{ for all } x \in \bar{\Omega}\}$. $h^- = \min_{x \in \bar{\Omega}} h(x)$ and $h^+ = \max_{x \in \bar{\Omega}} h(x)$, $\forall h \in C_+(\bar{\Omega})$. For any $p \in C_+(\bar{\Omega})$, we define the variable exponent Lebesgue space by

$$L^{p(x)}(\Omega) := \left\{ u \mid u: \Omega \rightarrow \mathbb{R} \text{ is measurable, } \int_{\Omega} |u(x)|^{p(x)} dx < \infty \right\}.$$

Then $L^{p(x)}(\Omega)$ endowed with the norm

$$|u|_{p(x)} = \inf \left\{ \lambda > 0 : \int_{\Omega} \left| \frac{u(x)}{\lambda} \right|^{p(x)} \leq 1 \right\}.$$

The modular of $L^{p(x)}(\Omega)$ which is the mapping $\rho_{p(x)}: L^{p(x)}(\Omega) \rightarrow \mathbb{R}$ is defined by $\rho_{p(x)}(u) = \int_{\Omega} |u|^{p(x)} dx$ for all $u \in L^{p(x)}(\Omega)$. The variable exponent Sobolev space

$W^{1,p(x)}(\Omega)$ is defined by $W^{1,p(x)}(\Omega) = \{u \in L^{p(x)}(\Omega) : |\nabla u| \in L^{p(x)}(\Omega)\}$, with the norm

$$\|u\|_{1,p(x)} = |u|_{p(x)} + |\nabla u|_{p(x)},$$

for all $u \in W^{1,p(x)}(\Omega)$. Denote by $W_0^{1,p(x)}(\Omega)$ the closure of $C_0^\infty(\Omega)$ in $W^{1,p(x)}(\Omega)$; we know that $|\nabla u|_{p(x)}$ is an equivalent norm on $W_0^{1,p(x)}(\Omega)$. Moreover, it is well known that if $1 < p^- < p^+ < \infty$, then spaces $L^{p(x)}(\Omega)$, $W^{1,p(x)}(\Omega)$ and $W_0^{1,p(x)}(\Omega)$ are separable and reflexive Banach spaces. The detailed informations about variable exponent Lebesgue and Sobolev spaces ($L^{p(x)}$ and $W^{1,p(x)}$) have been given in [1,2,3].

To proof the following theorem, we state the assumptions imposed on the nonlinearity f , which appears in problem (P). Let $f: \bar{\Omega} \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}$ is a continuous function which satisfies the following conditions:

$$(f_1) f(x, t, |\xi|^{p(x)-2}\xi) = 0 \quad \forall t \leq 0, \forall (x, \xi) \in \bar{\Omega} \times \mathbb{R}^N.$$

$$(f_2) f(x, t, |\xi|^{p(x)-2}\xi) = o(|t|^{p(x)-1}) \text{ as } t \rightarrow 0 \text{ uniformly for } x \in \bar{\Omega} \text{ and } \xi \in \mathbb{R}^N.$$

$$(f_3) f(x, t, |\xi|^{p(x)-2}\xi) = o(|t|^{q(x)-1}) \text{ as } t \rightarrow \infty \text{ uniformly for } x \in \bar{\Omega} \text{ and } \xi \in \mathbb{R}^N,$$

where $p(x) < q(x) < p^*(x) \forall x \in \bar{\Omega}$, and $p^*(x)$ is the Sobolev critical exponent given by

$$p^*(x) = \begin{cases} \frac{Np(x)}{N-p(x)}, & p(x) < N, \\ +\infty, & p(x) \geq N. \end{cases}$$

(f₄) Ambrosetti-Rabinowitz's type condition holds, i.e., there exists $\theta > p^+$ such that

$$0 < \theta F(x, t, |\xi|^{p(x)-2}\xi) = \int_0^t f(x, \eta, |\xi|^{p(x)-2}\xi) d\eta \leq t f(x, t, |\xi|^{p(x)-2}\xi),$$

for all $t > 0, x \in \bar{\Omega}$ and $\xi \in \mathbb{R}^N$.

(f₅) There exist positive constants a and b such that

$$F(x, t, |\xi|^{p(x)-2}\xi) \geq at^\theta - b, \forall t > 0, \forall (x, \xi) \in \bar{\Omega} \times \mathbb{R}^N.$$

(f₆) There exist constants $L_1 = L_{\rho_1}$ and $M > 0$ such that

$$|f(x, t_1, |\xi|^{p(x)-2}\xi) - f(x, t_2, |\xi|^{p(x)-2}\xi)| \leq L_1 |t_1 - t_2|^{p(x)-1} - \frac{M}{|t_1 - t_2|}$$

for all $t_1, t_2 \in [0, \rho_1]$ ($t_1 \neq t_2$) and for all $|\xi| \leq \rho_2$.

(f₇) There exist constants $L_1 = L_{\rho_2}$

$$|f(x, t, |\xi_1|^{p(x)-2}\xi_1) - f(x, t, |\xi_2|^{p(x)-2}\xi_2)| \leq L_2 |\xi_1 - \xi_2|^{p(x)-1},$$

for all $t \in [0, \rho_1]$ and for all $|\xi_1|, |\xi_2| \leq \rho_2$, where ρ_1 and ρ_2 depend on p^+ and θ given in the previous assumptions.

Theorem: Assume the conditions (f₁) – (f₇) hold, then problem (P) has a positive solution provided

$$(L_1 L_3 p^- + L_2 L_4 p^+) < \frac{p^-(p^- - 1)}{2},$$

where $1 < p^- \leq p^+ < 2$ and $L_3, L_4 \geq 1$ are real numbers.

Remark 1: If the function p is log-Hölder continuous on $\bar{\Omega}$, i.e., there exists a positive constant L such that

$$|p(x) - p(y)| \leq (L / (-\log|x - y|)) \text{ for } x, y \in \bar{\Omega} \text{ with } |x - y| \leq 1/2,$$

then $u \in C^{0,\alpha}(\bar{\Omega})$ for some $\alpha \in (0, 1)$.

Remark 2: If the function p is Hölder continuous on $\bar{\Omega}$, i.e., there exists a positive constant H such that

$$|p(x) - p(y)| \leq H|x - y|^\alpha \text{ for } x, y \in \bar{\Omega}$$

then $u \in C^{1,\alpha}(\bar{\Omega})$ for some $\alpha \in (0, 1)$.

Key Words: Variable exponent Lebesgue-Sobolev spaces, $p(x)$ -Laplacian, Iteration methods, Mountain Pass theorem.

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Double-diffusive natural convection in porous cavity heated by an internal boundary

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ABSTRACT

Laminar double-diffusive natural convective flow of a binary fluid mixture in a rectangular cavity heated by an internal circular boundary, filled with a uniform porous medium is considered. Transverse gradients of mass are applied on two opposing walls of the cavity while three walls are kept adiabatic and two walls are assumed to be impermeable to mass transfer. The considered problem is expressed in term of the stream function-vorticity formulation. A numerical solution based on the finite element methodology is obtained. Numerical results illustrating the effects of the buoyancy ratio and Rayleigh number on the contour maps of the streamline, temperature, and concentration are reported. In addition, numerical results for the average Nusselt and Sherwood numbers are presented and discussed.

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Dynamics Between Immune System Response-Bacterial Load With Holling Response of Multiple Antibiotics

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ABSTRACT

Bacterial resistance to antibiotic treatment is a very problematic issue, since introduction of any new antibiotic is shortly followed by the occurrence of resistant bacterial isolates in the clinic. Indeed, it is often said that the host's immune system plays a primary role in the development of infections.

Accordingly, we proposed a mathematical model defining population dynamics of the specific immune cells produced according to the properties of these by host and the bacteria exposed to multiple antibiotics synchronically, presuming that resistance is gained through mutations due to exposure to antibiotic.

Model:

Let us denote by $S(t)$ and $R(t)$ the population sizes of sensitive and resistant bacteria against multiple antibiotics at time t , respectively, by $B(t)$ population sizes of immune cells at time t , and by $A_i(t)$ the concentration of the i -th antibiotic, for $i = 1, 2, \dots, n$ at time t . The parameters used in the model are as follows:

We assume that bacteria follow a logistic growth with carrying capacity T . Let β_S and $(1 - c)\beta_S$ the birth rate of sensitive and resistant bacteria, respectively. We quantify fitness cost as a reduction on the reproduction rate of the resistant strain, therefore $0 < c < 1$. Also, the immune cells proliferate proportionally (with a proportionality constant $\bar{\gamma}$) to the bacterial load.

Immune cells, sensitive and resistant bacteria have per capita natural death rates μ_B , μ_S and μ_R , respectively. During the administration of the i -th antibiotic, a number of resistant bacteria to it can show up due to mutations of exposed sensitive bacteria to such antibiotic, we model this situation by the term $\bar{\alpha}_i A_i S$ where $\bar{\alpha}_i$ is the mutation rate of sensitive bacteria due to exposure to i -th antibiotic. Sensitive and

resistant bacteria have per capita death rates by response of immune cells and this rates is η . We assume that the predation of the i -th antibiotic on sensitive bacteria follows a Holling function of the 2nd type. In this respect, sensitive bacteria also die due to the action of the i -th antibiotic, and we presume that the effect of the i -th antibiotic on susceptible bacteria for $i = 1, 2, \dots, n$ is modelled using a saturating response, $\frac{E_{max}^i A_i}{E_{50}^i + A_i}$, subject to a maximum killing rate E_{max}^i and the antibiotic concentration required for half maximum effect, E_{50}^i . Lastly, the i -th antibiotic concentration is supplied at a constant rate δ_i , and is taken up at a constant per capita rate μ_i .

Under the assumptions aforementioned, we obtain the following system of $(n + 3)$ ODE's:

$$\begin{aligned} \frac{dS}{dt} &= \beta_S S \left(1 - \frac{S + R}{T}\right) - \mu_S S - \eta S B - S \sum_{i=1}^n \frac{A_i E_{max}^i}{E_{50}^i + A_i} - S \sum_{i=1}^n A_i \bar{\alpha}_i \\ \frac{dR}{dt} &= (1 - c) \beta_S R \left(1 - \frac{S + R}{T}\right) - \mu_R R - \eta R B + S \sum_{i=1}^n A_i \bar{\alpha}_i \\ \frac{dB}{dt} &= \bar{\gamma} (S + R) B - \mu_B B \\ \frac{dA_i}{dt} &= \delta_i - \mu_i A_i, \quad \text{for } i = 1, 2, \dots, n \end{aligned} \quad (1)$$

In the qualitative analysis of the system (1), it is benefited from Routh-Hurwitz criteria for locally asymptotically stable of equilibrium points and Lyapunov theorem for globally asymptotically stable of equilibrium points. Also, the results of this analysis of system (1) have supported by numerical simulations using Matlab. Accordingly, among the treatment regimen recommended by WHO includes isoniazid, rifampicin, streptomycin and pyrazinamide for some bacterial infections caused by bacteria such as mycobacterium tuberculosis. In this respect, data related to the aforementioned bacteria and antibiotics have used in our numerical study.

Key Words: Ordinary differential equations systems, Equilibrium points, Immune system response, Bacterial resistance, Multiple antibiotic therapy.

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Energy Balance Method for Solving Force Nonlinear Oscillator

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ABSTRACT

This paper applies He's energy balance method (EBM) to a nonlinear oscillator with fractional potential $u^{1/3}$. The results show that the method is effective and convenient without the requirement of any linearization or small perturbation. The energy balance method can lead to adequately accurate solutions for nonlinear oscillators.

Key Words: Energy balance method, oscillators

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Exact Solutions of (2+1) Dimensional Hyperbolic Nonlinear Schrödinger Equation by Using New Version Of F -Expansion Method

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ABSTRACT

In this study, a new version of the F -expansion method is applied to find exact solutions of the (2+1) dimensional hyperbolic nonlinear Schrödinger equation. As a result, some new exact solutions such as single and combined non-degenerate Jacobi elliptic function solutions are obtained by using this method. The proposed new method has contributed to the further development of the family of solutions of nonlinear equations.

Key Words: A new version of F -expansion method, hyperbolic nonlinear Schrödinger equation, single and combined non-degenerate Jacobi elliptic function solutions.

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Exact Solutions of Unstable Nonlinear Schrödinger Equation by Use of a New Version of Generalized Kudryashov Method

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ABSTRACT

Kudryashov suggest a Q function method to solve nonlinear differential equations. In this study, we use different Q function to determine the exact solution of unstable nonlinear Schrödinger equation. As a result, we find some new function classes of solitary wave solutions by use of proposed method. This method is straightforward which can be applied to other nonlinear partial differential equations.

Key Words: A new version generalized Kudryashov method, unstable nonlinear Schrödinger equation, solitary wave solutions.

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Existence of Solutions for Higher Order Linear Hyperbolic Differential Equations With Damping Term

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ABSTRACT

The aim of the present paper is to study the initial-boundary value problem of higher order higher order linear partial differential equations:

$$Lu = \left(\frac{\partial^{2k}}{\partial x^{2k}} + -1^k \left(\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} \right) \right) u = f(x, t), \quad x, t \in D = \{x, t : 0 < x < p, 0 < t < T\},$$
$$u(x, 0) = u_x(x, 0) = 0, \quad 0 \leq x \leq p,$$
$$\frac{\partial^{2l} u}{\partial x^{2l}}(0, t) = \frac{\partial^{2l} u}{\partial x^{2l}}(p, t) = 0, \quad l = 0, 1, \dots, k-1, \quad 0 \leq t \leq T,$$

where $k \geq 1$ is fixed integer. Based on a priori estimate of solution the existence of a regular solution in the form of Fourier series is proved under suitable conditions.

Key Words: regular solution, existence, higher order differential equation.

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Existence of Solutions For Higher Order Linear Partial Differential Equations

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ABSTRACT

The aim of the present paper is to study the initial-boundary value problem of higher order higher order linear partial differential equations:

$$Lu = \left(t^m \frac{\partial^{2k}}{\partial x^{2k}} + -1^k \frac{\partial^2}{\partial t^2} \right) u = f(x, t), \quad x, t \in D = \{x, t : 0 < x < p, 0 < t < T\},$$
$$u(x, 0) = u_x(x, 0) = 0, \quad 0 \leq x \leq p,$$
$$\frac{\partial^{2l} u}{\partial x^{2l}}(0, t) = \frac{\partial^{2l} u}{\partial x^{2l}}(p, t) = 0, \quad l = 0, 1, \dots, k-1, \quad 0 \leq t \leq T,$$

where m is a positive number and $k \geq 1$ is fixed integer. Based on a priori estimate of solution the existence of a regular solution in the form of Fourier series is proved under suitable conditions.

Key Words: regular solution, existence, higher order differential equation.

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Exponential Growth of Solutions With Positive Initial Energy to Systems of Nonlinear Wave Equations

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ABSTRACT

In this work, we consider initial-boundary conditions for a coupled nonlinear wave equations with damping and source terms. We prove that the solutions of the problem are unbounded when the initial data are large enough in some sense.

Key Words: Exponential growth, systems of wave equations, nonlinear damping and source terms.

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Extended B-spline Differential Quadrature Method for Nonlinear Viscous Burgers' Equation

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ABSTRACT

A new differential quadrature method based on extended B-spline functions has been constructed to solve the nonlinear viscous Burgers' equation. The well-known initial boundary value problem representing the fadeout of an initial shock is solved by using the proposed method.

Firstly, the weighting coefficients required for the derivative approximations are determined by using the extended B-splines. Then, the Burgers' equation is reduced to a first order ordinary differential equation system by discretizing it in space via the differential quadrature method. The resultant system is integrated in time by Rosenbrock fourth order implicit method.

The accuracy of the method is determined by measuring the error between the analytical and the numerical solutions. A comparison with the results of some earlier studies is also reported.

Key Words: Differential quadrature method; Extended B-spline; Burgers' equation; Shock wave.

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Flatness Control Approach for Hydropower Plants

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ABSTRACT

To be able to give satisfactory response to the load frequency control of the power system, the speed governor should response the set point changes as quick as possible within the safe margins [1]. Due to unsatisfactory performance indices of conventional controllers, a new controller based on the differential flatness theory is designed for using in hydro power plant. The new controller is constructed by adding on a flatness-based feed forward part to the existing PI feedback controller, which results in improved performance than the conventional PI controller.

In order to determine the performance of the controller for the plant, the system model is constructed and simulation studies are implemented in Matlab/Simulink environment. The conventional PI controller is compared with a two degree of controller structure which consists of a flatness-based feed forward part and a PI feedback loop.

The flatness property is useful for both the analysis of and controller synthesis for nonlinear dynamical systems. It is particularly advantageous for solving trajectory planning problems and asymptotical set point following control [2-4].

For continuous-time systems differential flatness is defined as follows. The system

$$\frac{dx}{dt} = f(x, u); \quad x(t) \in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^m \quad (1)$$

is called differentially flat if there exists algebraic functions A , B , C , and finite integers α , β and γ such that for any pair (x, u) of inputs and controls, satisfying the dynamics (1), there exists a function z (of the same dimension as the control u), called a flat output, such that

$$\begin{aligned} x(t) &= \mathcal{A}(z, \dot{z}, \dots, z^{(\alpha)}) \\ u(t) &= \mathcal{B}(z, \dot{z}, \dots, z^{(\beta)}) \\ z(t) &= \mathcal{C}(x, u, \dot{u}, \dots, u^{(\gamma)}). \end{aligned} \quad (2)$$

When constructing open-loop controls for the state variable system (1) flatness of the system in the above sense facilitates the design. Then the control u and the state variable x are parameterized by the external function; the flat output z . Then the input-state pairs (u, x) (and the output y) can be obtained as functions of the parameter function z without solving of the original nonlinear ordinary differential equation system.

Key Words: Load frequency control, Modeling, Flatness-based controller

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Finite Difference and Generalized Taylor Series Methods for Space and Time Fractional Burgers Equation

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ABSTRACT

In this study, fractional differential operators are defined and analyzed. The space and time fractional Burgers equation with initial condition is considered. The extended finite difference method which is based on shifted Grünwald and Caputo formula are used. The fractional terms are approximated by using two fractional difference schemes. Time Fractional Burgers Equation, Space Fractional Burgers Equation and Space-Time Fractional Burgers Equation are discussed with an example and error estimates obtained for the Finite difference method (FDM) and Generalized Taylor Series Methods (GTSM). The numerical methods have been applied to solve a numerical example and results are compared with the exact solution. By these methods, the numerical solutions of space and time fractional Burgers equation are found. These results are presented in tables using the Mathematica software package when it is needed.

In recent years have seen a rapid development both in scientific theory and applications of the fractional calculus in technic science and applied science [1–4]. The exact solutions of the fractional differential equations may not be easily obtained, so we need numerical methods for fractional differential equations. One of them is finite difference method and it is one of the most popular methods of numerical solution of partial differential equations. Classical partial differential equations have been extended to the fractional partial differential equations. There are many applications of this equation in literature. The fractional partial differential equations have been used in applications such as fluid, flow, finance, hydrology and others [5–7]. In this paper we investigate finite difference numerical methods to solve the space and time fractional Burgers equation of the form [8]

$$\frac{\partial^\beta u(x,t)}{\partial t^\beta} + u(x,t) \frac{\partial^\alpha u(x,t)}{\partial x^\alpha} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0,$$

$$u(x,0) = u_0, \quad a \leq x \leq b \quad \text{and} \quad u(a,t) = u(b,t) = 0, \quad 0 < t \leq T,$$

where $0 < \alpha \leq 1$, $0 < \beta \leq 1$.

Key Words: Finite Difference Method; Generalized Taylor Series Method; Space and Time Fractional Burgers Equation; Grünwald Formula; Caputo Formula; Numerical Solutions; Fractional Partial Differential Equation

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Finite Difference Method the Multi-span Nonlinear Model

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ABSTRACT

In this study, the simple supported beam is considered. To show the general form, the equation for the simple support N is given. The problems containing this kind of discontinuity can be written as

$$y_i^{iv} + Py_i'' - \frac{1}{2} y_i'' \sum_{i=1}^N \left(\int_{a_{i-1}}^{a_i} y_i'^2 \right) dx = 0$$

where y is the displacement and P is the axially harmonic compressive load. We present the analytical and numerical solution for beams containing discontinuity with fixed–fixed boundary conditions. We determine the critical load in the beams with the multi-span nonlinear model. For nonlinear mathematical model, the multi-span Euler-Bernoulli beam is considered. We used the finite difference method for the numerical solution of the mathematical model. For the expansion of finite difference, the central difference is considered. The numerical results are compared with the analytical results.

Key Words: Multi-span beam, finite difference method, nonlinear model.

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Fitted Numerical Schemes for singularly perturbed delay differential equations

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ABSTRACT

In this study, we have investigated some different numerical schemes for singularly perturbed initial value problem for first order delay differential equation. Different difference schemes are constructed for this problem. The difference schemes are shown to be uniformly convergent to the continuous solution with respect to the perturbation parameter. Numerical results for a test problem are given using the presented methods.

Key Words: Delay differential Equations, Singularly perturbed problem, Difference scheme .

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Fractional Solutions of a Nonhomogeneous Gauss Equation

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ABSTRACT

By our fractional calculus operator N^η method we obtain the following solutions which contain the N-fractional calculus.

Let $\psi = \{\psi : 0 \neq |\psi_\eta| < \infty, \eta \in R\}$ and $\phi = \{\phi : 0 \neq |\phi_\eta| < \infty, \eta \in R\}$, then the non-homogeneous Gauss equation

$$\psi_2(r^2 - r) + \psi_1[r(m+n+1) - \gamma] + \psi mn = \phi \quad (r \neq 0,1)$$

has solutions of the form

$$\psi = \left\{ r^{m-\gamma} (r-1)^{\gamma-n-1} \left[(r^{\gamma-m-1} (r-1)^{n-\gamma} \phi_{-m})_{-1} \right] \right\}_{m-1} \equiv \psi^1$$

where $\psi_k = \frac{d^k \psi}{dr^k}$ ($k = 0,1,2$), $\psi_0 = \psi(r)$, m, n, γ are given constant.

Key Words: Gauss equation, Fractional calculus, ordinary differential equation.

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Global Existence and Nonexistence of Solutions for a Klein-Gordon Equation with Exponential Type Nonlinear Term and Arbitrary Positive Energy

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ABSTRACT

In this work we study the global existence and nonexistence of solutions for a nonlinear Klein-Gordon equation of the form

$$u_{tt} - \Delta u + u = f(u), \quad x \in \Omega, \quad t > 0, \quad (1)$$

where $f(u) = e^u - 1 + au$, $\Omega \subset \mathbb{R}^2$ is a bounded domain.

The Klein-Gordon equation is the relativistic generalization of Schrödinger equation in quantum mechanics for a particle with zero spin. It arises in a variety of physical situations, e.g., in modelling of dislocations in crystals, the propagation of waves in ferromagnetic materials, laser pulses in two state media [6]. Eq. (1) is mostly studied with polynomial nonlinearity [2,3].

The argument we use in this paper, the potential well method, was first introduced by Sattinger [5], and recently was developed by Kutev et. al. [1] to establish global existence of a Boussinesq-type equation in the case of high initial energy. Later, the modified method of Kutev et. al. was applied for proving global existence of some evolution equations. The question of global existence for problem (1)-(3) with both the exponential source term and high initial energy has not been treated previously.

Finite time blow-up of solutions for parabolic equations with arbitrary positive energy was considered in some papers by comparison principle. Due to the lack of comparison principle for hyperbolic equations, this method cannot be applied to wave equations.

The problem of blow up of solutions on a bounded domain for (1)-(3) was considered in the paper of Saanouni [4] under the following conditions:

$$(2) \quad I_0 < 0, J_\varepsilon < 0, \int_{\Omega} u_0 u_1 dx > 0, \|u_0\|^2 \geq \frac{2}{\varepsilon} (2 + \varepsilon E_0 + |b||\Omega|)$$

where I is Nehari manifold, J_ε is a modified potential energy functional, $b = \inf_{x \in \mathbb{R}} x f(x) - 2 + \varepsilon F(x)$ and F is the primitive of f . The second aim of this work is to prove the blow-up of solutions by imposing more general conditions than (2). The conditions depend both the initial displacement and initial velocity.

Key Words: Klein-Gordon equation, exponential nonlinearity, high initial energy level.

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Global Existence for the Weakly Dissipative Higher Order Camassa-Holm Equation

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ABSTRACT

In this work, we would like to consider the following dissipative higher order Camassa-Holm equation:

$$m_t + 2u_x m + u m_x + L(u) = 0, \quad t > 0, x \in R$$

where $m = u - u_{xx} + u_{xxx}$. $L(u)$ is a dissipative term, L can be a differential operator or a quasi-differential operator according to different physical situations. We are interested in the effect of the weakly dissipative term on the dissipative higher order Camassa-Holm equation. In particular, we study following weakly dissipative higher order Camassa-Holm equation:

$$m_t + 2u_x m + u m_x + \lambda m = 0, \quad t > 0, x \in R \quad (1)$$

$m = u - u_{xx} + u_{xxx}$ and $\lambda > 0$ is a constant, $u(t, x)$ describes the horizontal velocity of the fluid. When $\lambda = 0$ and $m = u - u_{xx}$, equation (1) becomes the classical Camassa-Holm equation, namely,

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad t > 0, x \in R.$$

This equation was derived by Camassa and Holm [1] as a model for the unidirectional propagation of shallow water waves over a flat bottom. This equation and its derivatives have been studied from various aspects (see [2, 3, 4, 5] and references therein).

In [6], the authors studied the local well-posedness and global existence of the equation (1) for $\lambda = 0$. In this work, we study the local well-posedness and global existence of the equation (1) for $\lambda > 0$. We used the Kato theory [7] to establish the local well-posedness. Then we obtained the global existence result.

Key Words: Global existence, weakly dissipative, higher-order Camassa-Holm equation.

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Global Nonexistence of Solutions For a System of Nonlinear Higher-Order Kirchhoff-Type Equations

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ABSTRACT

This work studies the initial boundary value problem for the Kirchhoff-type equations. We prove the blow up of the solution with negative initial energy by using the technique of [2] with a modification in the energy functional due to the different nature of the problems.

Also, we prove the blow up of the solution with positive initial energy by using the technique of [6] with a modification in the energy functional due to the different nature of problems. This improves earlier results in the literature [1, 3, 4, 5].

Key Words: Global nonexistence, Higher-order Kirchhoff type equations, Nonlinear damping and source terms.

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Global Nonexistence Of Solutions For A System Of Viscoelastic Wave Equations With Weak

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ABSTRACT

This paper deals with the initial boundary value problem for the viscoelastic wave equations in a bounded domain. We obtain the global nonexistence of solutions by applying a lemma due to Y. Zhou [4].

Key Words: Global nonexistence, viscoelastic wave equation.

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Gompertzian Stochastic Model for Tumor Growth

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ABSTRACT

In this paper we consider a stochastic model of solid tumor growth based on deterministic Gompertz's law that describes the tumor growth. We discuss parameter estimation of stochastic model. Furthermore we apply Euler-Maruyama method to simulate the solution of stochastic Gompertzian model. Finally some numerical results are obtained for the cervical cancer.

Key Words: Gompertzian stochastic model, Euler-Maruyama Scheme, cervical cancer, parameter estimation

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Green's Functions For Second-Order Eigenvalue Problems With Interface Conditions

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ABSTRACT

In this work we analyze the boundary value problems on finite interval, associated with Sturm-Liouville differential operators together with eigenparameter-dependent boundary-transmission conditions. We have found some spectral properties of the considered problem by using the Green's function method. By using some new approaches, we construct the Green's function in terms of the left and right solutions. Then we introduce adequate operator formulation in the suitable Hilbert space and prove the self-adjointness of the considered problem in this Hilbert space. Mathematically Green's function, is the kernel of an integral operator that represents the inverse of a differential operator; physically, it is the response of a system when a unit point source is applied to the system. The main advantage of the Green's function method is that it can be applied directly for continuous and discontinuous types of differential operators. Comparison with the results of others will also be presented. In developing the spectral theory of a two-point differential operator L in the Hilbert space $L_2[a,b]$, two key ingredients are the characteristic determinant and the Green's function (see, [4]). The theme of the development of the theory and applications of Green's Functions is skilfully used to motivate and connect clear accounts of the theory of distributions, Fourier series and transforms, Hilbert spaces, linear integral equations, spectral theory, and semi-linear partial differential equations. The Dirichlet Green's function is generally used for electrostatic problems where the potential is specified on bounding surfaces, while the Neumann Green's function is useful for finding temperature distributions where the bounding surfaces are heat insulated or have specified heat currents.

Key Words: Sturm-Liouville problem, Green function

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Higher Order Accurate Numerical Solution of Advection Diffusion Equation

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ABSTRACT

We consider the following one dimensional advection-diffusion equation

$$u_t + \alpha u_x - \mu u_{xx} = 0, \quad a < x < b, \quad 0 < t < T \quad (1)$$

with the boundary conditions

$$u(a, t) = u(b, t) = 0, \quad t \in [0, T] \quad (2)$$

and initial condition

$$u(x, 0) = f(x), \quad a \leq x \leq b \quad (3)$$

in a restricted solution domain over a space/time interval $[a, b] \times [0, T]$. In the one dimensional linear advection diffusion equation, α is advection (velocity) coefficient, μ is the diffusion coefficient and $u = u(x, t)$ is a function of two independent variables t and x , which generally denote time and space, respectively. The advection-diffusion equation is the basis of many physical and chemical phenomena, and its use has also spread into economics, financial forecasting and other fields [1]. Various numerical techniques have been proposed for solving the one dimensional advection-diffusion equation with constant coefficient [2,3].

In this study, the advection diffusion equation will be solved numerically using the quintic B-spline Galerkin finite-element method, based on second and fourth order single step methods for time integration. Two test problems are studied and accuracy of the numerical results are measured by the computing the order of convergence and error norm L_∞ for the proposed methods. The numerical results of this study demonstrate that the proposed two algorithms especially the fourth order single step method are a remarkably successful numerical technique for solving the advection-diffusion equation.

Key Words: Quintic B-spline, Advection diffusion equation, Galerkin method

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Impulsive Fractional Pantograph Differential Equations

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ABSTRACT

This work deals with impulsive fractional differential equations with pantograph delay. The model of pantograph equations has a very effective usage in the area of various scientific disciplines. It is owing to fact that pantograph delay has been considered in the ordinary differential equations of integer orders and fractional orders as well. However, it is the first time that it has been considered in an impulsive fractional differential equations. Therefore, we will find some sufficient conditions for the existence and uniqueness of solution to an initial value problem for impulsive fractional differential pantograph equations by obtaining an equivalent integral equation to the given problem and by making use of standard fixed point theorems.

Key Words: Fractional differential equation, Caputo fractional derivative, impulsive effect, pantograph delay.

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Isomorphism and Coerciveness of Fourth Order Discontinuous Boundary Value Problems with Transmission Conditions

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ABSTRACT

In this study, we have considered a nonclassical fourth order boundary value problem. This problem contains a fourth order ordinary differential equation with discontinuous coefficients and boundary conditions containing not only endpoints of the considered interval, but also a point of discontinuity, functionals and finite number of internal points. Namely, we consider the differential equation

$$L(\lambda)u := -a(x)u^4(x) + \lambda^4 u(x) = f(x), \quad x \in I$$

together with multipoint-transmission conditions

$$L_k u := a_k u^{(m_k)}(-1) + \beta_k u^{(m_k)}(-0) + \eta_k u^{(m_k)}(+0) + \gamma_k u^{(m_k)}(1) + \sum_{j=1}^2 \sum_{i=1}^{N_{jk}} \delta_{jki} u^{(m_k)}(x_{jki}) + S_k u = f_k, \quad k = 1, 2, \dots, 8,$$

where $I = I_1 \cup I_2$, $I_1 = [-1, 0)$, $I_2 = (0, 1]$; $a(x)$ is piecewise constant function, $a(x) = a_1$ at $x \in [-1, 0)$, $a(x) = a_2$ at $x \in (0, 1]$; λ -complex parameter; $a_1 (i = 1, 2)$, $a_k, \beta_k, \gamma_k, \delta_{kij} (k = 1, 2, \dots, 8; i = 1, 2, \dots, N_{jk}^j)$ are complex coefficients; $x_{1ki} \in (-1, 0)$, $x_{2ki} \in (0, 1)$ are internal points; $m_k \geq 0 (k = 1, 2, \dots, 8)$ are any integers; $S_k, k = 1, 2, \dots, 8$ are linear functionals in the space $L_q(-1, 1)$. We shall assume that $a_1 \neq 0$, $a_2 \neq 0$ and $|a_k| + |\beta_k| + |\eta_k| + |\gamma_k| \neq 0 (k = 1, 2, \dots, 8)$.

Nonclassical boundary-value problems with transmission conditions have become an important area of research in recent years because of the needs of modern technology, engineering, physics and various transfer problems arise after an application of the method of separation of variables to the varied assortment of physical problems, namely, in heat and mass transfer problems (see, for example,

[4]), in diffraction problems (for example, [1]), in vibrating string problems, when the string loaded additionally with point masses (see, [8]) and etc. Further, some problems with transmission conditions which arise in mechanics (thermal condition problem for a thin laminated plate) were studied in [9]. Spectral properties and coercive solvability of boundary value problems in Sobolev spaces can be found in some works of Ya. Yakubov, B. A. Aliev and V. B. Shakhmurov (for example [2], [7]). Some boundary–value problems for differential equations with discontinuous coefficients were investigated by M. L. Rasulov in monographs (see, [6]). Various spectral properties of such type problems and its applications were investigated by the authors of this study and some others (for example [3], [5]).

Key Words: Discontinuous problem, functional and transmission conditions, isomorphism, coerciveness, solvability.

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Jost Solution and Spectral Properties of Dirac System With Matrix Coefficient

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ABSTRACT

The aim of this paper is to get a polynomial-type Jost solution of a selfadjoint matrix-valued discrete Dirac system. We investigate analytical properties and asymptotic behavior of this Jost solution. Also, by using the Weyl compact perturbation theorem, we prove that matrix-valued discrete Dirac system has continuous spectrum filling the segment $[-2,2]$. Finally, we examine the properties of the eigenvalues of this system and we prove that it has a finite number of simple real eigenvalues.

Key Words: Discrete Dirac system, Jost solution, spectral analysis, eigenvalue.

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Mathematical Modelling in the Evaluation of Second Virial Coefficient with Sutherland Potential and Its Applications

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ABSTRACT

In this study, a new mathematical modelling is proposed to calculate of the second virial coefficient with Sutherland potential. The obtained formula is useful for the evaluation of thermodynamic properties (internal energy, free energy, specific heat capacity, sound of speed, Joule-Thomson coefficient,...) of fluids. The accuracy of proposed method is tested and their efficiency illustrated with practical application of molecules H_2, Ne, Xe and Kr . The obtained results are in good agreement with data available in the literature [1-4].

Key Words: Virial equation of state, Second virial coefficient, Sutherland potential.

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Mathematical Modelling of Immune System Response and Bacterial Resistance with Antibiotic Therapy

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ABSTRACT

Resistance developed bacteria to antibiotic treatment is a very important issue; introduction of any new antibiotic is shortly followed by the occurrence of resistant bacterial isolates in the clinic. Although these problems, antibiotic therapy is still the most common method used to combat bacterial infections. Actually a major role in the development of these infections is played by the immune system in host. Hence, we have proposed that a mathematical model describing population dynamics of the specific immune cells produced according to the properties of these by host and the the bacteria exposed to antibiotic, assuming that acquire of resistance is both through mutations due to antibiotic exposure and the conjugation including the transfer of genes between susceptible and resistant bacteria to antibiotic.

Model:

Let us denote by $S(t)$ and $R(t)$ the population sizes of susceptible and resistant bacteria against antibiotic at time t , respectively, by $B(t)$ population sizes of immune cells at time t , and by $A(t)$ the concentration of the antibiotic at time t . The parameters used in the model are as follows:

We assume that bacteria follow a logistic growth with carrying capacity T . Let β_S and $(1 - c)\beta_S$ the birth rate of sensitive and resistant bacteria, respectively. We quantify fitness cost as a reduction on the reproduction rate of the resistant strain, therefore $0 < c < 1$. Likewise, using a logistic style term, immune cells are recruited to the site of infection at rate β_B and carrying capacity of immune cells is Λ . Immune cells are lost through pathogen-induced apoptosis (at rate λ). We assume that antibiotic has administered in dose ($\alpha > 0$). Moreover, During the administration of

the antibiotic, a number of resistant bacteria to it can emerge due to mutations of exposed sensitive bacteria to such antibiotic, we model this situation by the term μSA where μ is the mutation rate of sensitive bacteria due to exposure to antibiotic. We represent interaction of bacteria through mass action kinetics with a conjugation rate, σ , being proportional to the levels of both susceptible and resistant bacteria to antibiotic in the population.

Sensitive and resistant bacteria have per capita death rates by response of immune cells and this rates is $\bar{\eta}$. Sensitive bacteria also die due to the action of the antibiotics, and we assume that the effect of the antibiotic on susceptible bacteria is modelled using a saturating response, $\frac{E_{max}A}{E_{50}+A}$, subject to a maximum killing rate E_{max} and the antibiotic concentration required for half maximum effect, E_{50} .

Under the assumptions aforementioned, we obtain the following system of four ODE:

$$\begin{aligned} \frac{dS}{dt} &= \beta_S S \left(1 - \frac{S+R}{T}\right) - \bar{\eta} SB - S \frac{E_{max}A}{E_{50}+A} - \mu SA - \sigma SR \\ \frac{dR}{dt} &= (1-c)\beta_S R \left(1 - \frac{S+R}{T}\right) - \bar{\eta} RB + \mu SA + \sigma SR \\ \frac{dB}{dt} &= \beta_B B \left(1 - \frac{B}{\Lambda}\right) - \lambda B(S+R) \\ \frac{dA}{dt} &= -\alpha A \end{aligned} \quad (1)$$

It is benefited from Routh-Hurwitz criteria and Lyapunov theorem in the qualitative analysis of the system (1). It has revealed that equilibrium points giving important ideas about the proliferation bacteria and immune cells. Also, the results of this analysis of system (1) have supported by numerical simulations using Matlab. To numerical study of system (1), data of different species of bacteria including *Staphylococcus aureus*, *Mycobacterium tuberculosis*, *Acinetobacter baumannii* and *E. coli* in host (each bacteria was evaluated separately in the model) and the ciprofloxacin as antibiotic were used.

Key Words: Ordinary differential equations systems; Equilibrium points; Immune system; Bacterial resistance; Antibiotic.

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Mathematics of Complex Networks

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ABSTRACT

Modelling and investigation of complex networks means evaluating them as interaction of numerous definite parts. Namely, one of the method among the methods which are possible to be used for the solution of complex problems in many areas is modelling of the system as graph. Appropriate solution methods are conducted by necessary procedures and details on graph structure defined. For instance while investigating such a biological phenomena like blood coagulation; proteins enzymes are feasible to be accepted as the points on graph and relations between these are possible to be assumed as connections. Similarly, to model global commercial flights, airports can be assumed as points than connecting the points which have flight between each other is possible. For highway and airways, routes can be determined by getting the cities as graph. Chemical bounds can be stated and structures of these chemical bounds can be exhibited by determining chemical molecules as graph. In sociology, representing the relationships between people can be shown similarly. However, graph structure is possible to be met in numerous fields like genetics, archeology, ecology and music. More similar instances are available in applications. Graph theory is used intensely and effectively with technological advancement in computer science. The reason which makes graph theory so frequent is its ease of use even in complex problems.

In this study mathematic of graph theory structure has been investigated and structure, criteria and metrics are noted. Serious similarities in structures of complex networks have been revealed as result of investigation on complex systems.

Key words: Complex networks, modelling of complex systems, complexity calculations, mathematic of networks, graph theory

Modified Cubic B-Spline Differential Quadrature Methods and Stability Analysis for Modified Burgers' Equation

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ABSTRACT

The one-dimensional Burgers' equation, a nonlinear partial differential equation of second order which was introduced by Bateman[1] and later treated by Burgers'[2], has the following form

$$U_t + UU_x - vU_{xx} = 0, \quad (1)$$

where v is a positive parameter and the subscripts x and t denote space and time derivatives, respectively.

The MBE discussed in this study is based upon the Burgers' equation of the following form

$$U_t + U^2U_x - vU_{xx} = 0. \quad (2)$$

The MBE has strong non-linear aspects and has been used in many practical transport problems such as non-linear waves in a medium with low-frequency pumping or absorption, turbulence transport, wave processes in thermoelastic medium, transport and dispersion of pollutants in rivers and sediment transport, ion reflection at quasi-perpendicular shocks.

As an efficient discretization technique to obtain accurate numerical solutions using considerably small number of grid points, Bellman *et al.*[3] first introduced DQM in 1972 in which partial derivative of a function with respect to a coordinate direction is expressed as a linear weighted sum of all the functional values at all mesh points along that direction [4].

In this article, modified cubic B-spline (MCB) differential quadrature methods (DQMs) have been applied to obtain the numerical solutions of the modified Burgers' equation (MBE).

Efficiency and accuracy of the MCB-DQMs are measured by calculating the maximum error norm L_∞ and discrete root mean square error L_2 . The obtained numerical results are compared with published numerical results and the comparison shows that the method is an effective numerical scheme to solve the MBE. A stability analysis has also been given.

Key Words: Differential quadrature method, modified Burgers' equation, modified cubic B-splines, strong stability-preserving Runge-Kutta method.

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Multilevel Inverter Topology for Energy Converting

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ABSTRACT

This paper presents a new multilevel inverter topology with a considerable reduction in the number of power switches connected negative side of DC source on inverter so that direct current energy forms can be converted to alternating energy forms. The proposed inverter topology is based on the created DC voltage levels at loads of the inverter according to increased number of switches on inverter. Two switches are added on positive side of DC sources when DC source is added on the inverter. The switch number on the negative side of the supply remains constant although the switches numbers on the positive side of source are increased. Hundred level inverter can be constructed by proposed a novel inverter method if hundred DC sources and hundred two switches are connected on inverter. Pulse width modulation is developed to generate the PWM signals of the power switches. Then, four-level, six levels, nine level inverters from peak to peak have been designed with proposed inverter topology for the study. The performances of the proposed inverters are investigated through MATLAB/SIMULINK simulations after the designed multi level inverters are simulated. The results demonstrate the satisfactory performance of the inverter and verify the effectiveness of a novel multi level inverter circuits.

Key Words: new multilevel inverter topology, sinus pulse width modulation

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N-Fractional Calculus Operator Method to Euler's Equation

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ABSTRACT

By using fractional calculus techniques, we find explicit solutions of Euler's equation. We use the N-fractional calculus operator method to derive the solutions of these equations.

In this work, the solutions to non-homogeneous Euler's equation

$$y_2 x^2 + y_1 \alpha x + y \beta = f, \quad x \neq 0$$

$$\left(y_0 = y(x), y_\nu = \frac{d^\nu y}{dx^\nu} \text{ for } \nu > 0, f = f(x) \right)$$

are discussed by means of N-fractional calculus operator.

Key Words: Euler equation, Fractional calculus, ordinary differential equation.

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New Applications of the $(G'/G, 1/G)$ -Expansion Method

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ABSTRACT

In this paper, we study general solutions of the new fifth order nonlinear evolution and the Burgers KP (BKP) equations with the aid of the two variables $(G'/G, 1/G)$ -expansion method. The kink, bell-shaped solitary wave, periodic and singular periodic solutions are obtained. Finally, the numerical simulations add to these obtained solutions.

Key Words: $(G'/G, 1/G)$ -expansion method; the new fifth order nonlinear evolution equation, the BKP equation; general solution.

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New Approach for White–Dwarf and Thomas–Fermi Equations

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ABSTRACT

The Thomas--Fermi model is one of the simplest approaches to the study of the potential and charge densities in a variety of systems, like, for example, atoms, molecules, atoms in strong magnetic fields, metals and crystals and dense plasmas [1]. For this reason there has been great interest in the accurate calculation of the solution to the Thomas--Fermi equation. We implement the reproducing kernel method to White–Dwarf and Thomas--Fermi equations in this work. A powerful method is demonstrated by reproducing kernel functions [2-4]. The numerical approximations to the exact solutions are computed. Some examples are given to illustrate the efficiency of the method.

Key Words: Reproducing kernel space, bounded linear operator.

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New Dark Soliton Structures to the Newell-Whitehead Equation

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ABSTRACT

Some mathematical models may include many specific models such as, Fisher, Newell-Whitehead, Allen-Cahn, Fitzhugh-Nagumo, Fisher's and the Burgers-Fisher equations. This model can be represented by

$$u_t = u_{xx} + au + bu^n.$$

For $a = -4$, $b = 4$ and $n = 3$, this equation becomes the Allen-Cahn equation. For $n = 2$ and $b = -a$, the equation reduces to the well-known Fisher's equation. If for $n = 3$, the coefficient b is replaced by $-b$, then the equation becomes the Newell-Whitehead Equation.

In this work, we apply the modified $\exp(-\Omega(\xi))$ -expansion function method to the Newell-Whitehead Equation. We obtain some new travelling wave solutions such as complex function, hyperbolic function and rational function solutions. We check whether all travelling wave solutions verify the Newell-Whitehead equation by using Wolfram Mathematica 9. Then, we plot two and three dimensional surfaces for all travelling wave solutions obtained in this paper by using algorithm proposed.

Key Words: Nonlinear parabolic equations, the modified $\exp(-\Omega(\xi))$ -expansion function method, the Newell-Whitehead Equation.

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New Soliton Solutions of Davey-Stewartson Equation with Power-Law Nonlinearity

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ABSTRACT

This study is related to new soliton solutions of Davey-Stewartson equation (DSE) with power-law nonlinearity. The generalized Kudryashov method (GKM) which is one of the analytical methods has been used for finding exact solutions of this equation. By using this method, dark soliton solutions of DSE have been found. Also, by using Mathematica Release 9, some graphical representations have been done to analyze the motion of these solutions.

Keywords: Davey-Stewartson equation, generalized Kudryashov method, dark soliton solutions, Mathematica Release 9.

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Non-linear Diffusion in the Keller-Segel Model

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ABSTRACT

This paper seeks to establish the stability of the non-linear diffusion for Chemotaxis in relation to the Keller-Segel model. Attention will be on mass criticality results applying to the Chemotaxis model. Afterwards, the analysis of the relative stability that stationary states exhibit is undertaken using the Keller-Segel system for the Chemotaxis having linear diffusion. Standard linearization and separation of variables are the techniques employed in the analysis. The stability or instability of the analysed cases is demonstrated by the graphics. By using the critical results obtained for the models, the graphics are then compared with the rest.

Key Words: Chemotaxis, Keller-Segel Model, Diffusion

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Nonlinear Partial Differential Equations Solved by Using Modified Sine-Cosine Method

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ABSTRACT

There are various nonlinear evolution equations that are studied in these years [1,2]. These equations (NLEEs) are widely used as models to describe complex physical phenomena in various fields of sciences, especially in plasma physics, solid state physics, fluid mechanics, plasma wave and chemical physics. In the present times there are many such equations that are studied with time-dependent coefficients.

There are various algorithms that are available for the numerical solutions of these NLEEs. However, still a closed-form analytical solution is always important. These analytical solutions help to obtain further analytical properties of the equations as well as solutions. There are various modern methods of integrability that has been developed in the past few decades. Some of the commonly studied methods are the tanh-sech method [3], extended tanh method [4], the exponential function method [5], the sine-cosine method [6], the homogeneous balance method [7], the (G'/G) expansion method [8], the wave ansatz method [9], the F-expansion method [10] and many other methods. Some of these techniques have been modified to solve NLEEs with variable coefficients and obtain their exact solutions.

In this study, we applied the modified sine-cosine method [11] to time-dependent coefficient nonlinear evolution equations and obtained some new exact periodic solutions.

Key Words: The modified sine-cosine method, exact solution, partial differential equation.

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Numerical Approach for The Two-Dimensional Fusion Problem with Convective Boundary Conditions

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ABSTRACT

A numerical method is developed for the solution of two-dimensional fusion problem with convective boundary conditions. In this study, we extended our earlier work on the solution of two-dimensional fusion problem by considering a class of time-split finite difference methods [1, 2]. Part of the derivatives are evaluated explicitly and part of them are computed implicitly using operator splitting [3]. The method is second order accurate in time and in (x, y) coordinates. Computational results obtained by present method are in totally consistent with the results reported previously by other researches.

Key Words: Flux limiters, LOD method, ADI method, moving boundary problems.

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Numerical Behavior and Stability Analysis of a Fractional Order Macrophage-Tumor Interaction Model

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ABSTRACT

In this paper, we consider a Macrophage-Tumor Interaction model introduced in [1]. This model describes interaction between three populations of macrophages, activated macrophages and tumor cells. This study is based on a tumor growth that is modeled such as

$$\begin{cases} \frac{dM(t)}{dt} = M(t)r_1 \left(1 - \frac{M(t)}{k_1}\right) - bM(t)A(t) - d_1M(t) + \varepsilon_1A(t) \\ \frac{dA(t)}{dt} = A(t)(bM(t) - d_2) \\ \frac{dT(t)}{dt} = T(t)r_2 \left(1 - \frac{T(t)}{k_2}\right) - aT(t)A(t) + cT(t) \end{cases} \quad (1)$$

where M , A and T denote respectively the concentrations of macrophages, activated macrophages and tumor cells. The parameters $k_1, k_2, d_1, d_2, \varepsilon_1, r_1, r_2, a, b$ and c denote positive numbers.

Starting from the integer-order Macrophage-Tumor Interaction model presented by (1), we introduce the fractional order derivatives by replacing the usual integer-order derivatives by fractional order Caputo-type derivatives to obtain the following fractional order system:

$$\begin{cases} \frac{d^\alpha M}{dt^\alpha} = M(t)r_1 \left(1 - \frac{M(t)}{k_1}\right) - bM(t)A(t) - d_1M(t) + \varepsilon_1A(t) \\ \frac{d^\alpha A}{dt^\alpha} = A(t)(bM(t) - d_2) \\ \frac{d^\alpha T}{dt^\alpha} = T(t)r_2 \left(1 - \frac{T(t)}{k_2}\right) - aT(t)A(t) + cT(t) \end{cases} \quad (2)$$

where $t \geq 0$ and $\alpha \in (0,1)$. Macrophages grow logistically with specific growth rate r_1 and carrying capacity k_1 . They undergo losses at a rate d_1 due to natural death. Macrophages are converted into active macrophages either by direct contact

with them or by contact with cytokines released by active macrophages. The corresponding activation rate is taken as b . The natural death rate of active macrophages are taken as d_2 . Tumor cells have a fixed input c resulting from conversion of normal cells to malignant ones. In addition, tumor cells undergo mitosis with logistic growth rate r_2 and carrying capacity k_2 . Loss of tumor cells occurs only due to attack by active macrophages following mass action kinetics with a as the rate of destruction. Active macrophages, after destroying the tumor cells, revert back to the passive state at a rate ε_1 referred to as the deactivation rate.

The fractional order systems are more suitable than integer-order in biological modeling due to the memory effects. The object of this study is to examine the stability properties for system (2) by deriving conditions under which the equilibria of this system is stable[2-4].

Keywords: logistic differential equations, local stability, fractional differential equation, Leslie-Gower prey-predator model, Tumor-immune system interaction.

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Numerical Scheme for Rosenau-KdV Equation Using Finite Element Method

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ABSTRACT

Interest in travelling wave solutions for nonlinear partial differential equations (NLPDEs) has grown rapidly in recent years because of their importance in the study of complex nonlinear phenomena arising in dynamical systems. Such nonlinear wave phenomena appears in various fields of sciences, particularly in fluid mechanics, solid state physics, plasma physics and nonlinear optics.

In 2009, Jin-Ming Zuo introduced the Rosenau-KdV equation to describe the dynamics of dense discrete systems. This physically interesting model reads

$$U_t + aU_x + bU_{xxx} + cU_{xxxxt} + d(U^2)_x = 0.$$

Here in this model, the dependent variable $U(x, t)$ represents the shallow water wave profile while the independent variables x and t represent the spatial and temporal variables, respectively. The coefficient of a represents the drifting term, the coefficient of b is the third order dispersion and the coefficient of c represents the higher order dispersion term. Finally, the last term with d is the nonlinear term.

In this work, numerical solutions for the Rosenau-KdV equation are studied by using finite element method. For temporal discretization, Crank-Nicolson and forward difference approach are used. Numerical results are obtained for five test problems. In order to apply the stability analysis, Rosenau-KdV equation is linearized by assuming that the quantity U in the non-linear term UU_x is locally constant. The results show that the present method is efficient and reliable.

Key Words: Finite element method, Rosenau-KdV equation, dispersion.

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Numerical Solution By Hermite Polynomials for Linear Complex Differential Equations in a Circular Domain

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ABSTRACT

In this paper, we have obtained the numerical solutions of complex differential equations by using the Hermite Polynomials and performed it two test problems. When we have compared exact solutions and numerical solutions of tables and graphs, we realized that our method is reliable, practical and functional.

In other words, using matrix relations between the Hermite polynomials and their derivatives, we will develop a method for solving linear complex differential equation.

$$\sum_{n=0}^m P_n(z) f^{(n)}(z) = g(z)$$

with the initial conditions $f^{(t)}(\alpha) = \vartheta_t \quad t = 0, 1, \dots, m - 1$.

Let us $f(z)$ is unknown function, $P_n(z)$ and $g(z)$ are analytical functions in the circular domain which $D = \{z = x + iy, z \in \mathbb{C}, |z - z_0| \leq r, r \in \mathbb{R}^+\}$, $\alpha, z_0 \in D, \vartheta_t$ is appropriate complex or real constant.

Key Words: Linear complex differential equations, Hermite Polynomials, Collocation Method, Numerical solution.

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Numerical Solution of Singularly Perturbed Convection-Diffusion Problems in the Reproducing Kernel Space

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ABSTRACT

In this research, a new approach is introduced for solving singularly perturbed convection-diffusion problems. We used the asymptotic expansion and reproducing kernel method for solution of problems. Firstly asymptotic expansion formed on inner region and then reduced terminal value problem solved via reproducing kernel method on outer solution. Some problems are solved by using the presented method. The numerical results show that the presented method is reliable and very effective for singularly perturbed convection-diffusion problems.

Key Words: Reproducing kernel method, Convection-diffusion problems, boundary layer, asymptotic expansion.

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Numerical Solution of the Korteweg-de Vries Equation by a Linearized Implicit Method

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ABSTRACT

In this study, we applied a linearized implicit numerical scheme based on finite difference method to obtain numerical solutions of the one dimensional Korteweg-de Vries (KdV) equation. Two test problems including the motion of a single solitary wave and the interaction of two solitary waves are solved to demonstrate the efficiency of the proposed numerical method, and its accuracy was examined by the error norms L_2 and L_∞ . The obtained numerical solutions show that proposed method is an accurate and efficient method at small times. The numerical solutions of the KdV equation are compared with both the exact solutions and other numerical solutions in the literature.

Key Words: KdV equation, finite difference, linearized implicit.

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Numerical Solution of Linear Transport Problem

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ABSTRACT

In the study, an initial boundary value problem modelling transport of an initial pulse constructed with advection-diffusion equation is solved numerically. In the model, the pulse propagates along the horizontal axis with constant velocity but can not keep its height and becomes smaller as time goes,

The numerical simulations are obtained by using the differential quadrature method based on trigonometric cubic B-splines for spatial discretization and integration the resultant ordinary differential equation system in time with various explicit and implicit techniques.

The error between the numerical and analytical solutions is measured by using discrete maximum norm.

Key Words: Advection-Diffusion Equation, Differential quadrature method; Trigonometric Cubic B-spline; Transport.

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Numerical Solutions of Boussinesq Equation by Using Galerkin Finite Element Method

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ABSTRACT

In this study, Galerkin finite element method has been applied to Good Boussinesq (GBq) and Bad Boussinesq (BBq) equations which are examples of Boussinesq type equations. GBq and BBq equations have been converted into coupled equations using a splitting technique. The Galerkin finite element formulations of the obtained equations have been formed using cubic B-Spline base functions. The solutions of the numerical schemes have been obtained using fourth order Runge-Kutta method. The error norms L_2 and L_∞ have been used to test how compatible obtained numerical solutions with those of exact ones. As numerical examples, travelling wave and interaction of wave problems have been investigated for both GBq and BBq equations and their simulations have been presents.

Key Words: Finite element method, Galerkin, B-spline, Runge-Kutta, Good Boussinesq equation, Bad Boussinesq equation, solitary wave, interaction.

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Numerical Solutions of Time Fractional Burgers' Equation by a Wavelet Collocation Method

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ABSTRACT

In recent years, it has been realized that fractional calculus allows a more realistic and effective approach for modelling a lot of problems from various branches of science. Owing to the appropriate applications of fractional differential equations in various fields of engineering and science, many researchers turn their directions to this topic [1-2]. In this study, a wavelet collocation method combined with finite differences approximation is developed for getting numerical solutions of time fractional partial differential equations. As a test model time fractional Burgers equation where fractional derivative is in the sense of Caputo is considered. Time discretization of the fractional derivatives are made by L1 discretization formula and space derivatives discretized by Chebyshev wavelet series. The mentioned time and space discretizations are utilized to put down time fractional Burgers equation to the solution of a linear system of equations. By solving this system of equations the Chebyshev wavelet series coefficients are obtained. Then plugging these coefficients into Chebyshev wavelet series expansion numerical solutions can be computed successively. L_2 and L_∞ error norms are used for measuring accuracy of the proposed method. Numerical results obtained by the aid of the proposed method are compared with numerical results already exist in literature as well as with exact solutions. The numerical results confirm the applicability of the proposed method for the considered problem.

Key Words: Chebyshev wavelet, Fractional Burgers' equation, Partial differential equations.

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Numerical Study on Fisher and Huxley Equations

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ABSTRACT

Most of the problems in various field as physics, chemistry, biology, mathematics and engineering modelled by nonlinear partial differential equations. The Fisher and Huxley equations are important nonlinear partial differential equations.

The Fisher equation defined on bounded interval or the whole line has multiple applications ranging from genes propagation and tissue engineering, to autocatalytic chemical reactions and combustion to neurophysiology. The Huxley equation which describes nerve pulse propagation in nerve fibres and wall motion in liquid crystals[1].

The explicit exponential finite difference method was originally developed by Bhattacharya for solving of the heat equation[2]. Bhattacharya[3] and Handschuh and Keith[4] used exponential finite difference method for the solution of Burgers equation. Bahadır solved the KdV equation by using the exponential finite difference technique[5]. Implicit, fully implicit and Crank-Nicolson exponential finite difference methods applied to the Burgers' equation by İnan and Bahadır[6,7]. Also, İnan and Bahadır[8,9] solved the Burgers' equation linearized by Hopf-Cole transformation with three different exponential finite difference methods. İnan and Bahadır[10] solved the generalized Burgers-Huxley equation by implicit exponential finite difference method.

In this work explicit exponential finite difference method is used to solve the non-linear Fisher and Huxley equation. The proposed scheme is tested on problems and the obtained numerical results are quite satisfactory in comparison to exact solutions for two equations.

Key Words: Exponential finite difference method, The Fisher equation, The Huxley equation.

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On Caesar Cipher

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ABSTRACT

In this work, we define multiple fragmented Caesar cipher that based on the fragmented Caesar cipher. The fragmented Caesar cipher is defined by Çağman et al [1] and it based on the Caesar cipher [2]. In the fragmented Caesar cipher has more possibility then the classical Caesar cipher because of the fragmented alphabet is used to cipher. The multiple fragmented Caesar cipher is a polyalphabetic cipher method that is constructed by Aydoğın [3]. In the multiple fragmented Caesar cipher, plaintext are encrypted by multiple encryption alphabets which are obtained by using the fragmented Caesar cipher.

Key Words: Encryption, Decryption, Caesar Cipher, Fragmented Caesar Cipher, Multiple Fragmented Caesar Cipher.

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On Higher Order Linear Partial Differential Equations

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ABSTRACT

The aim of the present paper is to study the initial-boundary value problem of higher order higher order linear partial differential equations:

$$Lu = \left(\frac{\partial^{2k}}{\partial x^{2k}} + \frac{\partial^{2k}}{\partial t^{2k}} \right) u = f(x, t), \quad x, t \in D = \{x, t : 0 < x < p, 0 < t < T\},$$
$$\frac{\partial^{2l} u}{\partial t^{2l}}(x, 0) = \frac{\partial^{2l} u}{\partial t^{2l}}(x, T) = 0, \quad 0 \leq x \leq p,$$
$$\frac{\partial^{2l} u}{\partial x^{2l}}(0, t) = \frac{\partial^{2l} u}{\partial x^{2l}}(p, t) = 0, \quad l = 0, 1, \dots, k-1, \quad 0 \leq t \leq T,$$

where $k \geq 1$ is fixed integer. Based on a priori estimate of solution the existence of a solution in the form of series is proved under suitable conditions.

Key Words: regular solution, existence, higher order differential equation.

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On Fourier Transforms

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ABSTRACT

The Fourier Transform, one of the gifts of Jean-Baptiste Joseph Fourier to the world of science, is an integral transform used in many areas of mathematics, physics and engineering. For example, generalized integrals, integral equations, ordinary differential equations, partial differential equations can be solved by using the Fourier transform. Another example of its applications could be; voice of every human being can be expressed as the sum of sine and cosine. Since the electromagnetic spectrum of the frequency of each voice is different, the frequency of each sine and cosine sum will be different. In this way, a voice record can be found belongs to whom using the Fourier transform. In fact, our ear automatically runs this process instead of us.

In this study, solutions of homogeneous differential equations with constant coefficients have been investigated. Firstly, Dirac Delta function in place of zero in the second part of these equations has been used. Later, the Fourier transforms of these equations have been taken. Finally, the inverse Fourier transform has been taken. Thus, we have seen that a special solution can be found in this way. Therefore, satisfying initial conditions of these solutions which have been obtained by using Fourier transforms can be interesting for readers. Finally two results have been obtained.

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On Inverse Problem for a String Vibrational Equation

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ABSTRACT

This paper addresses a variational method for the solution of an inverse problem for a string vibrational equation. The problem is reformulated as optimal control problem which aims to minimize the cost functional. The existence and uniqueness of solution for the problem are investigated and then a necessary condition for this solution is given.

Key Words: Optimal control, Inverse problem, Variational method.

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On Killing-Yano Tensors and Their Applications

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ABSTRACT

Killing-Yano tensors were investigated intensively during the last decades from both theoretical and applied viewpoints [1-7]. These tensors are crucial to investigate the geodesic motion on a curved space. In the studies [3-5] of the author, some new geometries were reported by adding a suitable term to the corresponding free Lagrangians. In this way some manifolds were reported and the related Killing – Yano tensors were listed. Also, the classification of the invariants of two – dimensional superintegrable systems was investigated and the hidden symmetries associated to the existence of Killing - Yano tensors were presented.

As it is known, the fractional calculus is an emerging topic in mathematics and it has very interesting applications in various branches of science and engineering. The fractional operators are non-local operators and they describe better the dynamics of processes possessing memory effects [8]. One of the open problems in this area is how to introduce the notion of a manifold by using the fractional operators.

On this line of thought, the fractional Killing-Yano tensors in some one-dimensional and two- dimensional curved space will be discussed.

In this talk, I will present some new trends in obtaining these tensors for the superintegrable systems which admit Killing-Yano tensors.

Key Words: Killing-Yano tensors, fractional calculus

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On Numerical Solution Of Fractional Order Boundary Value Problem With Shooting Method

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ABSTRACT

In this study the shooting method is used to calculate of the second order boundary value problem with fractional order. This method is found to be useful during the application and the accuracy of the shooting method is tested and then some examples are given to illustrate the efficiency of the method with respect to different value of fractional orders.

Key Words: Boundary value problem, shooting method, numerical solution, fractional order boundary value problem.

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On Solution of Complex Differential Equations by using Laplace Transform

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ABSTRACT

A method which is used for solution of linear differential equations is integral transform. A integral transform is form that

$$F(s) = \int_a^b K(s, t).f(t)dt.$$

Here, the function f which is given transform to F . $K(s, t)$ is kernel of transform. Laplace transform using several areas of mathematics is integral transform. Let $F(t)$ be a function for $t > 0$. Laplace transform of $F(t)$ is that:

$$\mathcal{L}(F(t)) = f(s) = \int_0^{\infty} e^{-st} F(t) dt \quad t > 0$$

We can solve ordinary differential equations, system of ordinary differential equation, integral equations, integro differential equations, difference equations, integro difference equations and can calculate some generalized integrals with Laplace transform. For example, $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$ can be seen with Laplace transform. Moreover, we can use Laplace transform in electrical circuits.

In this study, using Laplace transform we have given solutions of first order complex differential equations in general state. These equations are form that

$$A(z, \bar{z}) \frac{\partial w}{\partial z} + B(z, \bar{z}) \frac{\partial w}{\partial \bar{z}} + C(z, \bar{z})w = F(z, \bar{z}).$$

Here $w = u + iv$ is complex function, also $\frac{\partial}{\partial z} = \frac{\partial}{\partial x} - i \frac{\partial}{\partial y}$ and $\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial x} + i \frac{\partial}{\partial y}$ are complex derivative operators, respectively.

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On Spectral Expansions of Eigenfunctions System of One Boundary Value Problem

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ABSTRACT

Consider the boundary value problem

$$-y'' + q(x)y = \lambda y, \quad 0 < x < 1,$$
$$y(0) \cos \beta = y'(0) \sin \beta, \quad \frac{y'(1)}{y(1)} = a\lambda + b - \frac{c}{\lambda - d},$$

where λ is a spectral parameter, $q(x)$ is real-valued continuous function on $[0, 1]$; β, a, b, c, d are real constants and $0 \leq \beta < \pi, a \geq 0, c > 0$.

We investigate the uniform convergence of the spectral expansions of the continuous functions in the system of eigenfunctions of above spectral problem.

Let $\lambda_n (n = 0, 1, \dots)$ be eigenvalues of above problem and $y_n(x)$ be eigenfunction corresponding to λ_n .

Theorem. Suppose that r, s are different nonnegative integers and $f(x) \in C[0, 1]$.

i. Let $a = 0$ and $\beta = 0$. If the function $f(x)$ has a uniformly convergent Fourier series expansion in the system $\left\{ \sqrt{2} \sin \left(n - \frac{1}{2} \right) \pi x \right\}_{n=1}^{\infty}$ on the interval $[0, 1]$, then this function can be expanded in Fourier series in the system $y_n(x) (n = 0, 1, \dots; n \neq r)$ and this expansion is uniformly convergent on $[0, 1]$.

ii. Let $a \neq 0$ and $\beta = 0$. If the function $f(x)$ has a uniformly convergent Fourier series expansion in the system $\left\{ \sqrt{2} \sin n\pi x \right\}_{n=1}^{\infty}$ on the interval $[0, 1]$, then this function can be expanded in Fourier series in the system $y_n(x) (n = 0, 1, \dots; n \neq r, s)$ and this expansion is uniformly convergent on every interval $[0, b]$ $0 < b < 1$. If $(f, y_i) = 0, i = r, s$, then the Fourier series of $f(x)$ in the system $y_n(x) (n = 0, 1, \dots; n \neq r, s)$ is uniformly convergent on $[0, 1]$.

iii. Let $a=0$ and $0 < \beta < \pi$. If the function $f(x)$ has a uniformly convergent Fourier series expansion in the system $\{\sqrt{2} \cos n\pi x\}_{n=1}^{\infty}$ on the interval $[0,1]$, then this function can be expanded in Fourier series in the system $y_n(x) (n=0,1,\dots;n \neq r)$ and this expansion is uniformly convergent on $[0,1]$.

iv. Let $a \neq 0$ and $0 < \beta < \pi$. If the function $f(x)$ has a uniformly convergent Fourier series expansion in the system $\{\sqrt{2} \cos(n - \frac{1}{2})\pi x\}_{n=1}^{\infty}$ on the interval $[0,1]$, then this function can be expanded in Fourier series in the system $y_n(x) (n=0,1,\dots;n \neq r,s)$ and this expansion is uniformly convergent on every interval $[0,b]$ $0 < b < 1$. If $(f, y_i) = 0, i = r, s$, then the Fourier series of $f(x)$ in the system $y_n(x) (n=0,1,\dots;n \neq r,s)$ is uniformly convergent on $[0,1]$.

Note the papers [1]-[3] to which the present work is related.

Key Words: Differential Operator, Eigenvalues, Uniform Convergence of Spectral Expansion.

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On Stability of Fourth Order of Accuracy Difference Scheme for Hyperbolic Multipoint NBVP with Neumann Conditions

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ABSTRACT

In this paper, the multipoint nonlocal boundary value problem

$$\left\{ \begin{array}{l} \frac{\partial^2 u(t,x)}{\partial t^2} - \sum_{r=1}^m (a_r(x) u_{x_r})_{x_r} = f(t,x), \\ x = (x_1, \dots, x_m) \in \Omega_h, \quad 0 < t < 1, \\ u^h(0,x) = \sum_{j=1}^n \alpha_j u^h(\lambda_j, x) + \phi^h(x), \quad x \in \bar{\Omega}_h, \\ u_t^h(0,x) = \sum_{j=1}^n \beta_j u_t^h(\lambda_j, x) + \psi^h(x), \quad x \in \bar{\Omega}_h, \\ \left. \frac{\partial u^h(t,x)}{\partial \vec{n}} \right|_{x \in S} = 0, \quad x \in S. \end{array} \right. \quad (1)$$

in a Hilbert space H with self-adjoint positive definite operator is considered. Here, $a_r(x)$, ($x \in \Omega$), $\phi(x), \psi(x)$ ($x \in \bar{\Omega}_h$) and $f(t,x)$ ($t \in (0,1)$, $x \in \Omega_h$) are given smooth functions and $\bar{a}_r(x) \geq a > 0$, $a_r(x) \geq 0$, $\Omega = (0,l) \times \dots \times (0,l)$ is the open cube in the n -dimensional Euclidean space with the boundary $S = S_1 \cup S_2$, $\bar{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants. Fourth order of accuracy stable difference scheme for the approximate solution of (1) is presented. Here, we consider fourth order of accuracy in x and second order of accuracy in t difference scheme generated by the integer power of A approximately solving (1).

Stability estimates are obtained for the solution of the difference scheme without any assumptions in respect of the grid steps. Applying difference method, linear system with matrix coefficients is obtained and modified Gauss elimination method is used for solving this system. Several examples are given using MATLAB in order to verify theoretical statements.

Key Words: stability, nonlocal and multipoint BVP's, numerical analysis.

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On the Fifth-Degree Hermite Curves

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ABSTRACT

High-order degree curves are very important for geometric modelling and computer graphics. For this aim, the piece defined functions have been determining for each solution region on the curves. These functions are continuous and give better results compared to the all areas expressed with respect to single function. Continuity on the curve is defined between the solution regions and continuity c^0 , c^1 , and c^2 is reflected in the model by evaluating the solution. This paper focuses on a particular fifth degree curves. The aim in the equations are to constitute a structure that will support a variety of effects on the curve with the help of fifth-degree equations. The curve structure with the c^2 continuity is basically called as an improved model the Hermite curve. Hermite curve is known as geometric Hermite curve extending from the standard Hermite technique. Hermite blending functions are represented by polynomials according to parametric continuity. These functions represent how to blend the boundary condition to create any position on the curve. Derivative continuity is ensured at first and final points on curve and also recognized the impact bending and torsion effects.

Key Words: Fifth-degree curve, Hermite curve, c^1 and c^2 continuity.

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On the Mathematical Model of the Sequential Partially Covering Problem

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ABSTRACT

The Sequential Partially Covering Problem (SPCP) is a sub-problem of the Band Collocation Problem (BCP) [1]. The BCP was proposed by revising the Bandpass Problem (BP) which cannot be applicable to the recent technological developments [2]. The BCP aims to minimize the hardware costs by organizing the fiber optic network traffic that uses the dense wavelength division multiplexing system [3]. For more detailed information about the BP and the BCP, we encourage the reader to refer to [1-4]. Nuriyeva proposed a dynamic programming algorithm to solve the SPCP [5]. The SPCP has many applications such as job sharing in different workplaces and promotion campaigns of companies. In this paper, we present a binary integer linear programming model of the SPCP.

The definition of the SPCP. Let A be a binary array of length m and 2^k be the length of a B_k -Band, where $k=0,1,\dots,t=\lfloor \log_2 m \rfloor$. A group of consecutive 2^k entries of the array A forms a B_k -Band. All 1's must be covered using B_k -Bands. However, several B_k -Bands cannot have a common element. A B_k -Band may include zero elements. Let c_k be the cost of forming a B_k -Band. The SPCP is to minimize the total cost of B_k -Bands so that all 1's are covered.

A binary integer linear programming model of the SPCP. For a given binary array A and the costs c_k of B_k -Bands, we define two decision variables as follows:

$$y_i^k = \begin{cases} 1, & \text{if the entry } i \text{ is the first element of a } B_k\text{-Band} \\ 0, & \text{otherwise} \end{cases} \quad i = 1, \dots, m \text{ and } k = 1, \dots, t$$

$$x_i^k = \begin{cases} 1, & \text{if } a_i \text{ is an element of a } B_k - \text{Band} \\ 0, & \text{otherwise} \end{cases} \quad i = 1, \dots, m \text{ and } k = 1, \dots, t$$

We can formulate the binary integer linear programming model of the SPCP as follows:

$$\text{Minimize } \sum_{k=0}^t \sum_{i=1}^{m-2^k+1} c_k y_i^k \quad (1)$$

subject to

$$2^k y_l^k \leq \sum_{i=l}^{l+2^k-1} x_i^k, \quad k = 0, 1, \dots, t, \quad l = 1, \dots, m - 2^k + 1 \quad (2)$$

$$\sum_{i=l}^{l+2^k-1} y_i^k \leq 1, \quad k = 0, 1, \dots, t, \quad l = 1, \dots, m - 2^k + 1 \quad (3)$$

$$\sum_{k=0}^t x_i^k \geq a_i, \quad i = 1, \dots, m, \quad (4)$$

$$\sum_{k=0}^t x_i^k \leq 1, \quad i = 1, \dots, m, \quad (5)$$

$$\sum_{k=0}^t \sum_{i=1}^{m-2^k+1} 2^k y_i^k \geq \sum_{i=1}^m a_i, \quad (6)$$

$$\sum_{i=1}^{m-2^k+1} 2^k y_i^k = \sum_{i=1}^m x_i^k, \quad k = 0, 1, \dots, t, \quad (7)$$

$$x_i^k \in \{0, 1\}, y_i^k \in \{0, 1\}, \quad i = 1, \dots, m, \quad k = 0, 1, \dots, t. \quad (8)$$

The objective function (1) represents the total cost of forming $B_k - \text{Bands}$. Constraints (2) guarantee to find the coordinates of $B_k - \text{Bands}$. Constraints (3) guarantee that no two bands may have a common element. Constraints (4) say that each non-zero entry of the array has to be an element of a $B_k - \text{Band}$. Constraints (5) guarantee that any entry of the array belongs to at most one $B_k - \text{Band}$. Constraint (6) ensures that the total length of all $B_k - \text{Bands}$ cannot be less than the number of

1's. In other word, all 1's must be covered. Finally, constraints (7) express that the number of entries which are covered is equal to total length of the $B_k - Bands$. In this model all decision variables are binary as in (10). The binary integer linear model (1)-(8) finds the $B_k - Bands$ with their coordinates that cover all 1's with the minimum cost.

Key Words: Bandpass problem, band collocation problem, mathematical modelling, integer linear programming problem.

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On The New Fractional Derivative And Application To Nonlinear H1N1 Model

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ABSTRACT

We presented the nonlinear H1N1 model with new fractional derivative. We derived the special solution using an iterative method. The stability of the iterative method was presented using the fixed point theory. The uniqueness of the special solution was presented in detail using some properties of the inner product and the Hilbert space. We presented some numerical simulations to underpin the effectiveness of the used derivative and semi-analytical method.

Key Words: nonlinear H1N1 model, special solution, fixed point theorem, iterative method.

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On the Properties of Shape Functions of Moving Least Square Method for Two Dimensional Problems

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ABSTRACT

In this study we consider two dimensional shape functions, used in Element Free Galerkin method [1], obtained by Moving Least Square Method [2]. We extend our results presented in [3] to two dimensional case, thus optimize the procedure of obtaining shape functions for two dimensional problems. More precisely, we show that if Ω is the domain of definition for a two dimensional problem then all the shape functions with complete Ω support has translation invariant property in both directions. Later, we use this property to obtain shape functions for a given problem using relatively small number of operations independent of the size $N \times M$ of computational domain as opposed to $O(NM)$ operations needed in the conventional method. We perform some numerical tests to demonstrate the efficiency of the optimization procedure that we propose.

Key Words: Element Free Galerkin, Moving Least Square, Shape functions.

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On the solution of differential equations with three-derivative Runge-Kutta methods

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ABSTRACT

We present the methods based on Runge-Kutta methods extended by using the second and third derivatives of the solution for solving first order ordinary differential equations. Employing higher derivative information in the formulation of the method increases the order of accuracy of the solution. These methods are named as three-derivative Runge-Kutta methods and are a special class of multiderivative Runge-Kutta methods studied in [4, 5].

In this study, we obtain higher order methods using fewer stages compared with the classical Runge-Kutta methods. We investigate stability analysis for the proposed methods. For comparisons with other standard methods, the proposed methods are applied to some standard problems. Numerical results indicate that the proposed methods can be better than the classical Runge-Kutta methods.

Key Words: Two-derivative Runge-Kutta methods, Multiderivative Runge-Kutta methods, Stability region

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On The Solution of Multiplicative Linear Differential Equations without Right-Hand Side

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ABSTRACT

In recent years, Non-Newtonian calculus allowed scientists to look from a different point of view to the problems encountered in science and engineering. In the period from 1967 till 1970 Non-Newtonian calculus consisting of the branches of geometric, anageometric and biogeometric calculus was studied by Michael Grossman and Robert Katz. Fundamental definitions and concepts related to Non-Newtonian calculus are given in [1]. Subsequently geometric calculus was referred to as multiplicative calculus by Dick Stanley [2]. Multiplicative calculus uses "multiplicative derivative" and "multiplicative integral". Thus it is an alternative to the classical calculus of Newton and Leibniz (also referred as Newtonian calculus), which has an additive derivative and an additive integral. In [3], Bashirov and friends gave further concepts and applications to the properties of derivative and integral operators of the multiplicative calculus. Some of the problems in science and engineering can be solved by the multiplicative calculus in a more practical way as a consequence of recently done researches [4-8].

Here solutions of homogeneous differential equations with constant exponentials in multiplicative analysis are obtained by taking solutions of homogeneous differential equations with constant coefficients in classical analysis as a basis,. Thus, solutions for these equations, having a class of non-linear differential equations as a correspondence in the classical sense, are stated.

Key Words: Multiplicatively linear independent functions, multiplicatively Wronskian, multiplicative homogeneous linear differential equations

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On The Solutions Of Nonlinear Fractional Klein–Gordon Equation By Means Of Local Fractional Derivative Operators

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ABSTRACT

In this paper, an application of local fractional decomposition method (LFDM) to search for approximate analytical solution of nonlinear fractional Klein–Gordon equation is analyzed. The fractional derivatives are described in Jumarie’s modified Riemann–Liouville sense. A new application of local fractional decomposition method (LFDM) is extended to derive analytical solutions in the form of a series for this equation. The behavior of the solutions and the effects of different values of fractional order α are indicated graphically. It is shown that the solutions obtained by the LFDM are reliable, convenient and effective for nonlinear partial equations with modified Riemann–Liouville derivative.

Key Words: Local Fractional Decomposition Method, Jumarie’s modified Riemann–Liouville derivative, Klein–Gordon Equation, Approximate Analytical solution.

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Optical Solitons In a Type Of Nonlinear Directional Couplers

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ABSTRACT

This study focuses on exact solutions in a type of nonlinear directional optical couplers. The governing nonlinear Schrödinger's equation is studied with four forms of nonlinearity. The nonlinearities that are considered in this paper are the Kerr law, power law, parabolic law and dual-power law.

We consider the generalized nonlinear Schrödinger's equation with spatiotemporal dispersion (STD) and group velocity dispersion (GVD), that governs soliton propagation through optical fibers, in this study. The Jacobi elliptic functions are used to get exact solutions of this equation. Jacobi elliptic function solutions and also bright and dark optical soliton solutions are obtained for each law of the governing equation for optical couplers.

Key Words: Solitons, Jacobi elliptic functions, optical couplers.

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Optimal Homotopy Asymptotic Method to Solve Nonlinear Electrostatic Differential Equations

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ABSTRACT

Many problems of science and engineering lead to nonlinear differential equations. Except a few number of nonlinear differential equation problems, most of them cannot be solved analytically using traditional methods. Therefore these problems are often handled by the most common methods; Adomian decomposition method [1], homotopy decomposition method [2], differential transform method [3], homotopy perturbation method [4], variational iteration method [5,6] etc. Perturbation methods are based on small or large parameters and they have problem in dealing with strong nonlinearity in the problems. Adomian decomposition method and differential transform method are nonperturbation methods and can deal with strongly nonlinear problems but they have also problem about the convergence region of their series solution. These regions are generally small according to the desired solution. Vasile Marinca et al introduced Optimal Homotopy Asymptotic Method (OHAM) which is straight forward and reliable method for the approximate solution of nonlinear problems [7,8]. This method provides us with a convenient way to control the convergence of approximation series and adjust convergence regions.

In this work, we use the Optimal Homotopy Asymptotic Method (OHAM) to solve the Poisson-Boltzmann equation which is a nonlinear electrostatic differential equations arising in the research areas of physics and chemistry such as electro kinetics and soil remediation. We consider specifically the capillary systems in a planar geometry and in a cylindrical geometry. The obtained results show the evidence of the usefulness of the Optimal Homotopy Asymptotic Method for obtaining approximate analytical solutions for these kinds of nonlinear equations.

Key Words: Electrostatic differential equation, Poisson-Boltzmann equation, optimal homotopy asymptotic method.

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Oscillation Criteria for Fractional Differential Equation with Functional Term

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ABSTRACT

Recently, research for oscillation of various equations including differential equations, difference equations, and dynamic equations on time scales, has been a hot topic in the literature, and much effort has been done to establish new oscillation criteria for these equations so far. In these investigations, we notice that very little attention is paid to oscillation of fractional differential equations. By using fractional derivative which is known as the modified Riemann-Liouville derivative, we can establish a correlation between fractional order derivative and integer order derivative order of a function. Based on a certain variable transformation, by using generalized Riccati transformation and averaging technique, we establish new oscillation criteria. For illustrating the validity of the established results, we also presented applications.

Key Words: Fractional derivative, oscillation, Riccati transformation. Modified Riemann- Liouville, damping term, functional term.

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Perturbation-Iteration Technique and Optimal Homotopy Asymptotic Method for solving Nonlinear Equations

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ABSTRACT

Pakdemirli and co-workers have modified well-known perturbation method to construct perturbation iteration method. It has been efficiently applied to some strongly nonlinear systems and yields very approximate results [1-4]. We shall give some information about perturbation iteration algorithms. They are classified with respect to the number of terms in the perturbation expansion (n) and with respect to the degrees of derivatives in the Taylor expansions (m). Briefly, this process has been represented as PIA (n, m).

Vasile Marinca et al developed optimal homotopy asymptotic method for solving many different types of differential equations. This method is straight forward and reliable for the approximate solution of nonlinear problems [5-8]. OHAM also provides us with a convenient way to control the convergence of approximation series and adjust convergence regions.

In this study, we use Optimal Homotopy Asymptotic Method (OHAM) and Perturbation Iteration Method (PIM) to solve random nonlinear differential equations. Both of these methods are new and very effective for solving differential equations. Nevertheless, our comparison shows that PIM is more precise, easier to construct and requires less effort than OHAM. We give some numerical examples to check the convergence of the proposed methods. These illustrations have also been used to support our claims.

Key Words: perturbation iteration method, optimal homotopy asymptotic method, nonlinear differential equations.

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Properties of the Two-Dimensional Exponential Integral Functions

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ABSTRACT

The aim of this paper is to show uniform convergences of the two-dimensional exponential integral functions. Also, we investigate the continuity, integrability and asymptotic behaviour of these functions.

Key Words: Multidimensional radiative transfer, isotropic scattering, two-dimensional exponential integral functions, uniform convergence.

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Quintic Trigonometric B-spline Collocation Method to the Numerical Solution of the Burgers' Equation

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ABSTRACT

Bateman first introduced the quasi linear parabolic differential equation known as the one-dimensional Burgers' equation [1] and gave two of the steady solutions as well as mentioning the worth of the study. Due to applications in engineering and environmental sciences, it is used as a mathematical formulation describing the relation between convection and diffusion. So equation has been found application in fields as number theory, gas dynamics, heat conduction, elasticity etc. Burgers' equation is solved exactly for an arbitrary initial and boundary conditions [2-4]. Some of the solutions include serious solutions. These exact solutions are impractical for the small values viscosity constant due to slow convergence of serious solutions. which was illustrated in the study of Miller [5]. Thus many numerical studies are constructed to have solutions of the Burgers' equation for small values of viscosity constant which corresponds to steep front in the propagation of dynamic wave forms.

Spline functions can be integrated and differentiated due to being piecewise polynomials and since they have basis with small support, many of the integral that occur in the numerical methods are zero. Thus, spline functions are adapted to numerical methods to get the solution of the differential equations. Numerical methods with spline functions in getting the numerical solution of the differential equations lead to band matrices which are solvable easily with some algorithms in the market with low cost computation. The trigonometric B-spline functions are alternative to the polynomial B-spline functions [6]. The trigonometric B-splines have been used to fit curve and to approximate the surfaces. But few studies in which the differential equations have solved with the collocation method incorporated the trigonometric B-splines exist [7,8].

In this study, we add this series of the papers with the B-splines of five order known as quintic trigonometric B-spline. Achievement of the quintic trigonometric B-spline in the collocation methods will be illustrated by comparing the previous results.

Key Words: Burgers' Equation, Collocation, Quintic Trigonometric B-spline.

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Regularized Index-tracking Optimal Portfolio Selection

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ABSTRACT

The foundations of the modern portfolio theory were laid by Harry Markowitz in 1950s. In the Markowitz mean-variance portfolio theory, a mean-variance efficient portfolio is the one that has minimum variance for acceptable expected return or maximum expected return for acceptable variance among the possible portfolios. Markowitz model, allowing a unique efficiency frontier, aims to minimize a function in order to obtain a mean-variance efficient portfolio where the variance is used as a risk measure. Real world restrictions have led to the development of Markowitz mean-variance portfolio theory and proposal of new alternative models. Some of these models applied by adding constraints such as buy-in threshold (quantity constraint), cardinality constraint and budget constraint (investment constraint), or by adding other additional constraints. Other studies tried to deal with problems related to computational complexity. Edirisinghe (2013) considered the variance between the market index return and the return on the portfolio by dealing with index-tracking problem in portfolio optimization. In our study, we will recast the index-tracking optimization problem, by applying a form of regularization on the norm of the portfolio weights. A small sample of stocks is used from Borsa Istanbul (BIST) for portfolio selection in order to demonstrate how the proposed optimization model performs for different values of regularization constant.

Key Words: Index-tracking, mean-variance efficient portfolio, portfolio optimization, portfolio selection.

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Review of Link Prediction Methods in Social Networks

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ABSTRACT

Social networks are social communication platforms in virtual environment which individuals pose their own attitudes by showing symbolic actions presenting day life jest and mimics. Their swift development and wide usage proportion rises social networks' necessity and popularity. People communicate with each other and share some data like idea, thinking, photo, video and location. When this data is processed, some very important information is possible to be gathered belong to users and some forecasts become possible. Graph modelling of network which will be analysed, properties of graph structure, criterions and metrics should be calculated. Some forecast is possible to be done concerning the network by applying data mining methods to the date gathered.

Machine learning methods proposed for communication estimation is researched dividing into two division as controlled and uncontrolled methods. In uncontrolled methods one of the similarity criteria is determined, points of web node pairs are calculated respect to this criteria and possibility ratio of probable further connection with these pairs and probable connection brake are estimated. In controlled methods, classification is done according to different properties of network. In this study, social network link prediction methods have been investigated and calculation criteria and mathematical equations have been examined. In this scope, recent studies have been reviewed and new actions which can be done in the future have been proposed.

Keywords: Social Networks, Complexity Analysis, Mathematics of Networks, Link Prediction

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Slide Routing by Using Motion Detection and Laser Pointer

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ABSTRACT

In computer vision, real-time object tracking is an important issue, and is often used in the industrial and defense areas and robotic applications during the recent years. In this study, it has detected laser pointer over PowerPoint slides acquired by webcam from projection with developed Andrew Kirillov motion detection algorithm. It is aimed replacing slides for forward and backward with detection the movement of the laser pointer in the horizontal direction. Thus, it is not necessary to use the remote control for presentation.

Motion detection algorithms are divided into three groups according to the difference of two frames (scene), background modeling and counting. Our study is based on two frames difference. First on the images taken from the camera, the filtering process is performed with Microsoft's DirectShow library. Then the pointer is determined over the filtered images in conformity with the differences between the two scenes. Recording spots that pass through x and y coordinates of the detected pointer, whether the movement is made or not and it is used in determining the direction of movement, if any. Mentioned approach is required a threshold calculation for the difference of scenes never is zero. Respective threshold values is given a knowledge whether the movement is how intense. In addition, the direction of the pointer y is negligible. Looking to the x direction, the forward or backward orientation of the slides is provided. As a result, effectively a different perspective to the literature with the use of laser pointers has gained in educational resources

Key Words: Motion detection; object tracking; laser pointer.

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Some Estimations About N-Fractional Method

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ABSTRACT

N -fractional calculus has an important place in field of applied mathematics. This method is applied to the singular equation. Note that fractional solutions can be obtained for kinds of singular equation via this method [3-8]. The fractional calculus operator (Nishimoto's operator) N^ν be defined by [3-5],

$$N^\nu = \left(\frac{\Gamma(\nu+1)}{2\pi i} \int_c \frac{dt}{(t-z)^{\nu+1}} \right) \quad (\nu \notin \mathbb{Z}^-)$$

with

$$N^{-n} = \lim_{\nu \rightarrow -n} N^\nu \quad (n \in \mathbb{Z}^+).$$

We consider the non-homogeneous Bessel-type equation as follows [1,2];

Let $y \in \{y : 0 \neq |y_\nu| < \infty; \nu \in \mathbb{R}\}$ and $f \in \{f : 0 \neq |f_\nu| < \infty; \nu \in \mathbb{R}\}$,

$$L[y, x, \lambda, p] = y_2 + y \left[\lambda - \frac{p^2 - \frac{1}{4}}{x^2} \right] = f.$$

In this study, our aim is to apply N-fractional method for singular Sturm-Liouville equation with Bessel potential and obtain fractional solutions of this equation. Furthermore, we give some applications.

Key Words: Fractional calculus; Nishimoto's Operator; Riemann-Liouville operator.

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Some new Hermite-Hadamard type inequalities for twice differentiable convex mappings using green function and its applications

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ABSTRACT

In this study, using functions whose second derivatives absolute values are convex, we establish new inequalities that are connected with the right-hand side of Hermite-Hadamard inequality. Then we give some more estimates for trapezoidal quadrature formula by using these inequalities. Finally, we obtain some applications of these inequalities for special means are provided.

Key Words: Hermite-Hadamard inequality, convex function, green function, Hölder inequality, Trapezoidal formula, special means.

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Some Novel Exponential Function Structures to the Cahn-Allen Equation

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ABSTRACT

In this manuscript, we consider the Bernoulli sub-equation function method for obtaining some new exponential prototype structures to the Cahn-Allen mathematical model. We obtain some new results by using this technique. We plot two and three dimensional surfaces of the analytical results by using Wolfram Mathematica 9. At the end of this manuscript, we submit a conclusion in the comprehensive manner.

Keywords: Cahn-Allen Equation, Bernoulli sub-equation function method, Exponential function solution.

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Some Ostrowski Type Inequalities for the Co-Ordinated Convex Functions

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ABSTRACT

In this study, we give new some inequalities of Hermite-Hadamard's and Ostrowski's type for convex functions on the co-ordinates defined in a rectangle from the plane. Our established results generalize some recent results for functions whose partial derivatives in absolute value and partial derivatives in q .th power of absolute value are convex on the co-ordinates on the rectangle from the plane.

Key Words: Convex function, co-ordinated convex mapping, Ostrowski inequality, Hermite-Hadamard inequality.

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Some Variants of Multiplicative Newton Method with Third-Order Convergence

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ABSTRACT

Solving non-linear equation and system of equations is highly important in science and engineering. Besides, finding approximately a real root of a function is substantial in numerical analysis. As is known, the Newton's method is the best known and the most widely method which use to approximate a real root of a function. The classical Newton's method is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \dots$$

That is, the classical Newton's method has an iterative procedure which contains function and its derivative. In the literature, there are some variants of Newton's method such as trapezoidal Newton's method [1], harmonic mean Newton's method [2] and local fractional Newton's method [3]. Likewise, to accelerate the convergence of Newton's method, there have been many studies [4-6]. The multiplicative Newton method is also used to find a real root of equation $f(x)=1$. In literature [7], the multiplicative iterative formula is given as

$$x_{n+1} = x_n - \frac{\ln f(x_n)}{\ln f^*(x_n)}, \quad n = 0, 1, \dots$$

Since we work in multiplicative analysis, we use equation $f(x)+1=1$ to find a simple root equation $f(x)=0$. As can be noted in the above formula, it is seen that in the interval of convergence of the function must be defined positive.

In this paper, we develop some modifications of the multiplicative Newton's method which are third-order convergence. We use multiplicative Newton theorem to present these new modifications of multiplicative Newton method. Using multiplicative

Taylor expansion, we give also convergence analyses of these new methods. Furthermore, we compare the multiplicative Newton's methods with the classical Newton's methods in details.

Key Words: Multiplicative Newton method, Third-order convergence, Iterative method.

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Solution Analysis of Black-Scholes Differential Equation By Finite Element and Finite Difference Methods

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ABSTRACT

Black-Scholes partial differential equation (see [1]) is one of the most famous equations in mathematical finance and financial industry (see [2]). In this paper, numerical solution analysis is done for Black-Scholes partial differential equation using finite element method with linear approach and finite difference method. The methods used are known from literature. The numerical solutions are compared with Black-Scholes formula for option pricing. The numerical errors are determined for the finite element and finite difference applications to Black-Scholes partial differential equation. The goal of this work is to examine the behavior and source of the corresponding errors under various market situations.

In finite difference method, the oscillations are observed when the initial stock price is close to exercise price under low volatility conditions. This result is compatible with ([3]). Generally, Black-Scholes model is more useful when financial market is stable. We observe that the number of oscillations increases under high volatility conditions. The oscillations in finite element method are similar to finite difference method as the initial stock price is close to exercise price. The errors in finite element method with linear approach are greater than those of finite difference method. Also, better results may be obtained for selected different element approximation functions (See [4]).

We tested the effectiveness of the mesh refinement for the finite element method with linear-element approximation. As a result of the mesh refinement, it is seen that there are different cases for the finite element method with linear approximation and more efficient results are obtained in the case of low exercise price.

Key Words: Black-Scholes partial differential equation, finite element method, mathematical finance.

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Solution of the Fractional HIV Infection of CD4+T Cells Models by Laplace Adomian Decomposition Method

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ABSTRACT

In this paper, Laplace Adomian Decomposition Method (LADM) has been proposed for solving the Fractional model for HIV infection of $CD4^+T$ cells. Recently, several methods have been utilized to solve numerically the HIV infection model of $CD4^+T$ cells in literature. In this paper we solve fractional order of this model by using LADM. There are two important and well studied fractional derivatives, namely, the Riemann–Liouville and the Caputo derivatives. We will adopt Caputo derivative in this paper, which is a modification of the Riemann–Liouville definition. In the last decade, LADM method is attracted the many scientists attention The main advantage of LADM is its capability of combining the two powerful methods for obtaining exact solutions for nonlinear equations. Numerical simulations are given to demonstrate the main results.

Key Words: HIV infection of $CD4^+T$ cells, Fractional, Laplace Adomian Decomposition Method.

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Solutions of Quadratic Riccati Differential Equation by Means of Homotopy Perturbation Method and Adomian's Decomposition Method

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ABSTRACT

In this study, the quadratic Riccati differential equation is solved by means of analytic techniques, namely the Homotopy perturbation method (HPM) and Adomian decomposition method (ADM). Comparisons between Homotopy analysis method (HAM), Homotopy Perturbation Method, Adomian Decomposition Method and exact solution are made. Then solutions graph and tables are constructed and necessary comparisons are obtained. The results reveal that these methods are very effective and powerful. Furthermore, they indicate that only a few terms are sufficient to obtain accurate solutions.

We consider the quadratic Riccati differential equation:

$$\frac{dY(t)}{dt} = 2Y(t) - Y^2(t) + 1,$$

with the initial condition

$$Y(0) = 0.$$

The exact solution of above equation was found to be

$$Y(t) = 1 + \sqrt{2} \tanh \left[\sqrt{2} t + \frac{1}{2} \log \left(\frac{\sqrt{2}-1}{\sqrt{2}+1} \right) \right].$$

The Homotopy perturbation method, which provides an analytical approximate solution, is applied to various nonlinear problems. It may be concluded that the methods are powerful and efficient techniques for finding exact as well as approximate solutions for wide classes of linear and nonlinear differential equations. They provide more realistic series solutions that converge very rapidly in real physical

problems. The study shows that the techniques require less computational work than existing approaches while supplying quantitatively reliable results. Also the high agreement of the numerical results so acquired between the Homotopy perturbation method, Adomian decomposition method and Homotopy Analysis Method. Consequently, numerical results show their validity and great potential.

Key Words: Quadratic Riccati Differential Equation, Homotopy Perturbation Method, Adomian's Decomposition Method.

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Solving Irregular Boundary Value Problems with Transmission Conditions

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ABSTRACT

In the study, we consider boundary value problem with transmission conditions constructed on Regge problem

$$L(\lambda, D)u = \lambda^2 u(x) - u''(x) + \varphi(x)u(x) = g(x), \quad x \in (-1, 0) \cup (0, 1)$$

$$L_1(\lambda)u = \lambda(u(-1) + u(+1)) + u'(-1) = f_1$$

$$L_2 u = u(-0) = f_2$$

$$L_3 u = u(+0) = f_3$$

$$L_4 u = u(-1) + u(+1) + u'(-1) + u'(1) = f_4$$

where, $g(x) = g_1(x)$, $\varphi(x) = \varphi_1(x)$ at $x \in [-1, 0)$ and $g(x) = g_2(x)$, $\varphi(x) = \varphi_2(x)$ at $x \in [0, 1]$ are given functions and f_1, f_2, f_3, f_4 are given complex numbers. The conditions second and third are called the transmission conditions. Below $W_p^k(a, b)$ is a usual Sobolev space of functions $u(x)$ which have generalized derivatives up to the k -th order inclusive on (a, b) and the norm

$$\|u\|_{W_p^k(a,b)} = \left(\sum_{n=0}^k \int_a^b |u^{(n)}(x)|^p dx \right)^{1/p}$$

we shall define the direct sum $W_q^k(-1, 0) \oplus W_q^k(0, 1)$ as

$$W_q^k(-1, 0) \oplus W_q^k(0, 1) = \left\{ u = \begin{cases} u_1(x), & \text{for } x \in (-1, 0) \\ u_2(x), & \text{for } x \in (0, 1) \end{cases} \mid \begin{array}{l} u_1 \in W_q^k(-1, 0) \\ u_2 \in W_q^k(0, 1), \|u\| = \|u_1\|_{W_q^k(-1, 0)} + \|u_2\|_{W_q^k(0, 1)} \end{array} \right\}$$

In this study we investigated the solution of the boundary value problem with discontinuous coefficient and transmission conditions at point zero in $[-1,1]$ for Regge problem on which S. Yakubov has suggestion a new method about the solution in $[0,1]$ (see, [7]).

The boundary value problem studied here differs from the standard boundary value problems,such that the studied boundary value problem contains eigen value parameter in one of the boundary conditions and two new conditions called transmission conditions.

Some boundary value problems with discontinuous coefficients and transmission conditions dealing with spectral properties are investigated in the papers (see, [1], [2], [3], [4], [5], [6]).

Key Words: Boundary value problem, eigenvalue parameter, Regge problem, transmission conditions.

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Spline Method for Solving the Linear Time Fractional Klein-Gordon Equation

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ABSTRACT

In this study, we will deal with the problem of applying cubic parametric spline functions to investigate a numerical method for obtaining approximate solution of the linear time fractional Klein-Gordon Equation. We will prove the solvability of the proposed method. A numerical example is included to illustrate the practical implementation of the proposed method.

In recent years, there has been a growing interest in the field of fractional calculus. Fractional differential equations have attracted increasing attention because they have applications in various fields of science and engineering. Many phenomena in fluid mechanics, chemistry, physics, finance and other sciences can be described very successfully by using mathematical tools from fractional calculus. Most of the applications are given in the book of Oldham and Spanier, the book of Podlubny, and the paper of Metzler and Klafter, Bagley and Torvik.

There are several definitions of a fractional derivative of order $\alpha > 0$. Two most commonly used are the Riemann-Liouville and Caputo. The difference between the two definitions is in the order of evaluation. In this work we will use the Caputo fractional derivative.

The truncation error of the method will be theoretically analyzed. Computed results will be compared with the other methods.

Key Words: Time Fractional Klein-Gordon, Spline Functions, Stability

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Stability Analysis of NSE Model Obtained by Adding a Nonlinear Time Relaxation Term

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ABSTRACT

Time relaxation method is an approach to regularize the flow problems. In this study, we consider a non-linear time relaxation model which consists of adding a term " $\kappa|u-u'|(|u-u'|)$ " to the NSE. An algorithm depend on backward-Euler method is introduced. Then, by using finite element method, we obtain approximate solutions of the model. We prove the stability of the finite element solution. Some numerical experiments for FEM solutions of the model are presented. Numerical experiments is done by using FreeFem++.

Key Words: Non-linear time relaxation, Finite element method, Navier-Stokes equations, Time-filtering regularization.

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Stability Criterion For Volterra Type Delay Difference Equations Including Generalized Difference Operator.

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ABSTRACT

In this study by using Krasnoselskii fixed point theorem we investigate the asymptotic stability of zero solution of Volterra type difference equation including generalized difference operator of the form

$$\Delta_a[x(n) - b(n)x(n - \sigma)] - c(n)x(n) - \sum_{u=n-\sigma}^{n-1} k(u, n)h(x(u), x(u - \tau)) = 0$$

where $b(n): \mathbb{Z} \rightarrow \mathbb{R}$ and $c(n): \mathbb{Z} \rightarrow \mathbb{R}$ are discrete bounded functions, $k(u, n): \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{R}^+$, $h: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, σ and τ are nonnegative integers with $\lim(n - \sigma) = \infty$ and $\lim(n - \tau) = \infty$ and we obtain some new stability results.

The difference operator Δ and the generalized difference operator Δ_a are defined as

$$\Delta x(n) = x(n + 1) - x(n)$$

and

$$\Delta_a x(n) = x(n + 1) - ax(n), a > 0$$

respectively. We assume that $h(0,0) = 0$ and

$$|h(x_1, y_1) - h(x_2, y_2)| \leq K \max\{|x_1 - x_2|, |y_1 - y_2|\}$$

for some positive constant K .

Key Words: Stability, Volterra Difference Equations

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Step Sinus Pulse Width Modulation Inverter Application

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ABSTRACT

Renewable direct current sources and other direct current (dc) sources are converted to alternating current (ac) sources so that induction motors and some ac loads can be driven. Therefore, power electronic circuits were used to convert electric energy for loads driven in scientific articles. Switching function inverter circuit model is good choosing to show dc /ac inverter structure in the power electronic circuits. So, the paper deals with application and designing of Step Sinus Pulse Width Modulation (SSPWM) method for inverter controlling of three-phase R (resistive) and L (inductive) loads. The new mathematical model of three-phase inverter is presented after new pulse width modulation method is developed for one and three-phase loads in the study. Mathematical models are calculated for SSPWM driving three phase R and L loads while groups of three switches on inverter are operated by SSPWM as six times. Then, Direct Current (DC)/Alternating Current (AC) inverter can be identified for high performance inverter. After that, three-phase loads connecting inverter is simulated at the Matlab Simulink. According to observed results, demanded results can be achieved with presented mathematical model in power circuit.

Key Words: High performance inverter, step-sinus pulse width modulation, mathematical model

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The Discrete Homotopy Perturbation Method for Solving Fractional Partial Differential Equations

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ABSTRACT

In this paper, the discrete Homotopy Perturbation Method (DHPM) is proposed to solve the linear as well as nonlinear fractional partial differential equations. DHPM is extended to find the solution of fractional discrete diffusion equation, nonlinear fractional discrete Schrödinger equation and nonlinear fractional discrete Burgers' equation.

Key Words: Discrete Homotopy Perturbation Method, Caputo fractional derivative, fractional discrete diffusion equation, fractional discrete Schrödinger equation, fractional discrete Burgers' equation.

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The Numerical Solution of Modified Burgers' Equation

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ABSTRACT

The one-dimensional generalized Burgers' equation is in the form

$$u_t + u^p u_x - \nu u_{xx} = 0 \quad a \leq x \leq b, \quad p = 1, 2$$

in which $\nu > 0$ is a constant representing the kinematics viscosity of the fluid. It is known as Burgers' equation and modified Burgers' equation for $p = 1$ and $p = 2$, respectively. The equation we deal with is:

$$u_t + u^2 u_x - \nu u_{xx} = 0 \quad a \leq x \leq b$$

where x and t are independent variables, $u = u(x, t)$ and ν is the viscosity parameter. In the numerical examples, boundary conditions are chosen in the following form:

$$u(a, t) = \beta_1, \quad u(b, t) = \beta_2, \quad t \geq t_0.$$

In this study, explicit exponential finite difference schemes based on using four different linearization techniques are given for the numerical solutions of the Modified Burgers' equation. A model problem is used to verify the efficiency and accuracy of the methods that we proposed. Also comparisons are made with the relevant ones in the literature. It is shown that all results are found to be in good agreement with those available in the literature. L_2 and L_∞ error norms are calculated. The obtained error norms are sufficiently small during all computer runs. The results show that the present method is a successful numerical scheme to solve the Modified Burgers' equation.

Key Words: Modified Burgers' equation, Explicit exponential finite difference method

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The Sumudu Alternative for Cubic Polynomials

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ABSTRACT

In this research paper we establish the pattern effect of applying the Sumudu and its powers for detecting cubic polynomial roots. We establish limits at infinity criteria and their bearing on the actual roots of the cubic. Furthermore we show the Sumudu influence on Vieta's cubics related substitution, as well as Cardano and Lagrange methods, in this regard.

Key Words: Sumudu transform, cubic polynomial roots.

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Trichotomy of Nonoscillatory Solutions to Second-Order Neutral Difference Equation with Generalized Difference Operators

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ABSTRACT

In this study we investigate the trichotomy of nonoscillation solution of the nonlinear second-order neutral difference equation with generalized difference operators of the form

$$\Delta_a(p_n(\Delta_a(x_n - ax_{n-1}))^\alpha) + r_n x_{n-\sigma}^\beta = 0, \quad n \geq n_0.$$

We transform this equation a third-order non-neutral difference equation, by suitable substitution. Later we investigate the asymptotic properties of the neutral difference equation by using results obtained for the new equation. Finally, we present necessary and sufficient conditions for the existence of solutions to both considered equations being asymptotically equivalent to the given sequences.

We consider only solutions which are nontrivial for all large n . A solution to (x_n) is called nonoscillatory if it is eventually positive or eventually negative. Otherwise it is called oscillatory.

Key Words: Nonoscillation; Neutral difference equation; Generalized difference operators.

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$$x_{n+1} = \frac{x_{n-2}}{1 + x_n x_{n-1}} \quad \textbf{Solutions of Difference Equations}$$

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ABSTRACT

In this study,

$$x_{n+1} = \frac{x_{n-2}}{1 + x_n x_{n-1}}, \quad n = 0, 1, 2, \dots \quad (1)$$

the behaviors of the solutions of the differential equation with the initial conditions $x_{-2}, x_{-1}, x_0 \in (0, \infty)$ are investigated.

Recently there has been a lot of studies on the periodic nature of nonlinear difference equations (1, 2, 6, 7).

In the study, Simsek et al (2006) $x_{n+1} = \frac{x_{n-3}}{1 + x_{n-1}}$ by choosing a positive initial conditions the solution of the behaviour of the difference equations are examined. In the study, Simsek et al (2006) $x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}}$ by choosing a positive initial conditions the solution of the behaviour of the difference equations are examined.

In the study, Simsek et al (2006) $x_{n+1} = \frac{x_{n-5}}{1 + x_{n-1}x_{n-3}}$ by choosing a positive initial conditions the solution of the behaviour of the difference equations are examined.

In the study, Stevic (2002) $x_{n+1} = \frac{x_{n-1}}{g(x_n)}$ by choosing a positive initial conditions the solution of the behaviour and stability of the difference equations is examined.

Key Words: Difference Equation, 3 Periodic Solutions

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$$x_{n+1} = \max \left\{ \frac{1}{x_{n-3}}, \frac{y_n}{x_n} \right\}; y_{n+1} = \max \left\{ \frac{1}{y_{n-3}}, \frac{x_n}{y_n} \right\}$$

Solution of the Maximum of Difference Equations

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ABSTRACT

In this study, periodicity and behavior of the solution of the following difference equations were investigated.

$$x_{n+1} = \max \left\{ \frac{1}{x_{n-3}}, \frac{y_n}{x_n} \right\}; y_{n+1} = \max \left\{ \frac{1}{y_{n-3}}, \frac{x_n}{y_n} \right\} \quad (1)$$

The initial conditions $x_{-3}, x_{-2}, x_{-1}, x_0, y_{-3}, y_{-2}, y_{-1}, y_0 \in (1, +\infty)$ and are selected consecutively.

Recently, there has been a great interest in studying the periodic nature of nonlinear difference equations. Although difference equations are relatively simple in form, it is, unfortunately, extremely difficult to understand thoroughly the periodic behavior of their solutions. The periodic nature of nonlinear difference equations of the max type has been investigated by many authors. See for example [1-8].

In the study Mishev et al (2002), $x_{n+1} = \max \left\{ \frac{A}{x_n}, \frac{B}{x_{n-2}} \right\}$ on the periodicity of difference equations; assuming the initial conditions as positive number values with parameters A, B all positive solutions of equation sooner or later have been proven to be periodic.

In the study Cinar et al (2005); $A, B > 0$ for initial conditions to be different from 0, $x_{n+1} = \min \left\{ \frac{A}{x_n}, \frac{B}{x_{n-2}} \right\}$ periodicity of positive solutions of difference equations have been examined.

Key Words: Difference Equations, Maximum Operations, Semi-Cycle

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GEOMETRY AND TOPOLOGY

A Note on the New Set Operator ψ_r

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ABSTRACT

Many mathematicians made on local function used in ideal topological spaces can be found in related literature. In general, the researchers prefer using the generalized open sets instead of topology in ideal topological spaces. Obtaining a Kuratowski closure operator with the help of local functions is an important detail in ideal topological space. However, it is not possible to obtain a Kuratowski closure operator from many of these local functions. In order to address the lack of such an operator, the main goal of this work is to introduce another local function to give possibility of obtaining a Kuratowski closure operator. So we have obtained regular local function and then with the help of regular local functions we have obtained ψ_r -operator. Additionally, a new topology extracted from ψ_r -operator is given. The regular compatibility between the topology and ideal in the ideal topological space are also included by this work together with the obtained results presented under the title namely " ψ_r -C sets definition" as the research findings.

Many theorems in the literature have been revised according to the definition of ψ_r -operator. The revised new theorems and other derived results are also included in this work.

Key Words: ideal topological spaces, regular local function, ψ_r -operator, codense ideal, regular open sets.

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A Study on AW(k)- type Curves in the Geometry of Simple Isotropic Space

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ABSTRACT

In[1], Arslan and West defined the notion of AW(k)-type submanifolds. In[2], Arslan and Özgür studied curves and surfaces of AW(k)-type. Since then, many works have been done related to AW(k)-type curves[1,2,4,5,7].

Two curves which, at any point have a common principal normal vector are called Bertrand curves. The notion of Bertrand curves was discovered by J. Bertrand in 1850. Over years many mathematicians have studied Bertrand curves in different areas.

In[4], Külahcı and Ergüt focused on Bertrand curves of AW(k)-type in Lorentzian space. Yılmaz and Bektaş gave general properties of Bertrand curves in Riemann-Otsuki space[6].

In the study of the characterizations of space curves, the corresponding relations of space curves between the curves are the very interesting and important problem. The well-known Bertrand curve is characterized as a kind of such corresponding relation between the two curves.

Another kind of associated curves, called Mannheim curve and Mannheim mate (partner curve). If there exists a corresponding relationship between the space curves c and c^* such that, at the corresponding points of the curves, the principal normal lines of c coincides with the binormal lines of c^* , then c is called a Mannheim curve, and c^* a Mannheim partner curve of c . The pair $\{c, c^*\}$ is said to be a Mannheim pair.

In this paper, AW(k)-type curves are studied in the Geometry of the Simple Isotropic Space I_3^1 . In addition, Bertrand and Mannheim curves are investigated in the geometry of the Simple Isotropic space I_3^1 .

Key Words: AW(k)-type curve, Bertrand curve, Mannheim curve.

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A Survey On New Classes Of Submanifolds In The Pseudo-Euclidean Spaces Of Index 2.

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ABSTRACT

In this talk, we present a survey of some new classes of submanifolds in the pseudo-Euclidean spaces that we obtained after our recent studies. Firstly, we study a class of quasi-minimal surfaces with some prescribed geometrical properties in pseudo-Euclidean space of dimension 4 with neutral metric, [1]. Then, we will present our recent result on biconservative hypersurfaces in pseudo-Euclidean 5-space, [2, 3]. Finally, we obtain a class of hypersurfaces with vanishing Gauss-Kronecker curvature in the 4-dimensional pseudo-Euclidean space and describe some of its geometrical properties (See for instance [4]).

Key Words: Biconservative hypersurfaces, quasi-minimal surfaces, Gauss-Kronecker curvature.

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Action Leibniz Groupoids

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ABSTRACT

Groupoid actions of a groupoid G on a set S via a function w from S to the set of objects of G , can be considered as functors from the groupoid G to the category SET of sets. Any groupoid action defines a groupoid which is called the action groupoid. It has been shown by various authors that the category of action groupoid of a groupoid G and the category of covering groupoids of G are equivalent. Similar equivalences for the cases of topological spaces, groups, topological groups, etc. has been shown. Covering groupoids have applications in groupoid theory. It is a well known fact that in any category C with pullbacks one can refer the covering groupoid in that category.

In the light of these studies, first of all we introduce the notion of Leibniz groupoid, which is an internal groupoid in the category of Leibniz algebras. Further, since the category of Leibniz algebras has pullbacks we introduce the notion of covering Leibniz groupoid and action groupoid, under the name of action Leibniz groupoid, in the category of Leibniz algebras and investigate the equivalence between these structures. Moreover we introduce the notion of universal covering Leibniz groupoid and give some properties of action Leibniz groupoids.

Key Words: Leibniz algebra, internal groupoid, action groupoid.

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Anti-Invariant Submersions From Almost Paracontact Riemannian Manifolds

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ABSTRACT

We introduce anti-invariant Riemannian submersions from almost paracontact Riemannian manifolds onto Riemannian manifolds. We give an example, investigate the geometry of foliations which are arisen from the definition of a semi-Riemannian submersion and check the harmonicity of such submersions.

Given a submersion π from a Riemannian manifold (M_1, g_1) onto a Riemannian manifold (M_2, g_2) , there are several kinds of submersions according to the conditions on it: e.g. Riemannian submersion ([1], [3]), anti-invariant submersion ([4]), slant submersion [2], etc.

Let M be a $(2m+1)$ -dimensional manifold. If there exist on a $(1, 1)$ type tensor field F , a vector ξ , and 1-form η satisfying $F^2 = I - \eta \otimes \xi$, $\eta(\xi) = 1$ (1) then M is said to be an almost paracontact manifold, where \otimes , the symbol, denotes the tensor product. In the almost paracontact manifold, the following relations hold good:

$$F\xi = 0, \eta \circ F = 0, \text{rank}(F) = 2m.$$

An almost paracontact manifold M is said to be an almost paracontact metric manifold if Riemannian metric g on M satisfies $g(FX, FY) = g(X, Y) - \eta(X)\eta(Y)$ (2) for all $X, Y \in \Gamma(TM)$ [5]. From (1) and (2), we can easily derive the relation $g(FX, Y) = g(X, FY)$.

Let (M_1, g_1) and (M_2, g_2) be two Riemannian manifolds. A surjective C^∞ -map $\pi: M_1 \rightarrow M_2$ is a C^∞ -submersion if it has maximal rank at any point of M . Putting $V_x = \ker \pi_* x$, for any $x \in M_1$ we obtain an integrable distribution V , which is called vertical distribution and corresponds to the foliation of M determined by the fibres of π . The complementary distribution H of V , determined by the Riemannian metric g , is called horizontal distribution. A C^∞ -submersion $\pi: M_1 \rightarrow M_2$ between two Riemannian manifolds (M_1, g_1) and (M_2, g_2) is called a Riemannian submersion if, at each point x of M_1 , π_{*x} preserves the length of the horizontal vectors.

Definition 1. Let (M_1, F, η, ξ, g_1) be an almost paracontact Riemannian manifold and (M_2, g_2) a Riemannian manifold. Suppose that there exists a Riemannian submersion $\pi: M_1 \rightarrow M_2$ such that $\ker \pi_*$ is anti-invariant with respect to F , i.e., $F(\ker \pi_*) \subseteq (\ker \pi_*)^\perp$. Then we say π is an anti-invariant Riemannian submersion.

Let $R^{2m+1} = \{(x_1, \dots, x_m, y_1, \dots, y_m, t) : x_i, y_i, t \in \mathbb{R}, i = 1, 2, \dots, m\}$. The almost paracontact Riemannian structure (F, ξ, η, g_1) is defined on R^{2m+1} in the following way: $F(\frac{\partial}{\partial x_i}) = \frac{\partial}{\partial x_i}$, $F(\frac{\partial}{\partial y_i}) = -\frac{\partial}{\partial y_i}$, $F(\frac{\partial}{\partial t}) = 0$, $\xi = \frac{\partial}{\partial t}$, $\eta = dt$.

If $Z = a_i(\frac{\partial}{\partial x_i}) + b_i(\frac{\partial}{\partial y_i}) + v(\frac{\partial}{\partial t}) \in T(R^{2m+1})$, then we have $g_1(Z, Z) = \sum_{i=1}^m a_i^2 + \sum_{i=1}^m b_i^2 + v^2$.

From this definition, it follows that

$$\eta(Z) = v, g_1(FZ, FZ) = g_1(Z, Z) - \eta^2(Z), F\xi = 0, \eta(\xi) = 1$$

for an arbitrary vector field Z . Thus $(R^{2m+1}, F, g_1, \eta, \xi)$ becomes an almost

paracontact Riemannian manifold, where g and $\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial y_i}, \frac{\partial}{\partial t}\}$ denote usual

inner product and standard basis of $T(R^{2m+1})$, respectively.

Now, we give an example of anti-invariant Riemannian submersions.

Example 1. Let $\pi: R^5 \rightarrow R^2$ be a map defined by $\pi(x_1, x_2, y_1, y_2, t) = (\frac{x_1 + y_1}{\sqrt{2}}, \frac{x_2 + y_2}{\sqrt{2}})$.

Then, π is an anti-invariant Riemannian submersion such that ξ is vertical.

Key Words: Riemannian submersion, almost paracontact Riemannian manifold, anti-invariant Riemannian submersion.

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Applications of Geometry In the Classroom Using Geometry Expressions Dynamic Geometry Software

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ABSTRACT

In new Curriculum it is aimed that students should think about the features of geometric objects and relations among them. The drawings which are made by hand or ruler are static drawings. In this study it is aimed for the students to be able to observe and discover the new metric relations and formulas by dynamic geometry software Geometry Expressions v 0.3. We visualized some theorems such as heights of triangle, barycenter, triangle forming conditions, some collinearity problems, metric relations in triangle, incenter theorems, symmedians, are equalities etc. Another purpose of these activities prepared is to enable the students not only solve the problem but also to write their own problem and discover new relations. Thus, the students willingness and interest towards Geometry lesson is expected to increase. 40 in-class activities have been prepared using by Geometry Expressions software and some activities have been applied in the classroom. The students have more interested in the dynamic drawings rather than the static drawings and it was observed a significant increase in geometry lesson.

Key Words: Geometry Expressions, Dynamic Geometry, Education, Technology

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Cauchy Riemannian Lightlike Submanifolds of Indefinite Kaehler Manifolds

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ABSTRACT

In this talk, we survey lightlike submanifolds of indefinite almost Hermitian manifolds which are inspired from CR-submanifolds introduced in [1]. We first define CR-lightlike submanifolds introduced in [4] and show that such lightlike submanifolds exclude holomorphic and totally real lightlike submanifolds. Then, we introduce Screen Cauchy Riemann Lightlike submanifolds defined in [5] and show that these class include totally real cases and holomorphic cases. We also investigate geometry of Generalized Cauchy Riemann Lightlike submanifolds introduced in [6]. We show that this class is an umbrella of invariant, screen real and CR-lightlike submanifolds. Besides, we investigate necessary and sufficient conditions for distributions which are arisen from definitions of such submanifolds of indefinite Kaehler manifolds to be integrable and parallel. We investigate the existence (or non-existence) of such lightlike submanifolds and find certain conditions on the sectional curvature of indefinite complex space form. We also study minimal SCR and GCR-lightlike submanifolds and give examples each of these submanifolds. Then, we prove some characterization theorems on the existence of totally umbilical and CR minimal lightlike submanifolds. Also, we give examples each of a non-totally geodesic proper minimal SCR and GCR lightlike submanifolds of an indefinite Kaehler manifolds. Finally, we study mixed geodesic SCR and GCR lightlike submanifolds.

Key Words: Lightlike submanifold, CR submanifold, SCR lightlike submanifold, GCR-lightlike submanifold, Complex space form

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Closure Operators in Filter Convergence Spaces

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ABSTRACT

Moore and Smith in 1922 [6] had introduced convergence of nets, and Cartan in 1937 [2] provided the notion of filter. In 1954 [5], Kowalsky gave a filter description of convergence. In 1964 [4], Kent introduced Kent convergence spaces (there it is called convergence functions) by further weakening of the convergence axioms. In 1979, Schwarz [7] introduced the filter convergence spaces.

Baran in [1] introduced the notion of "closedness" and "strong closedness" in set-based topological categories and it is shown that these notions form appropriate closure operators in the sense of Dikranjan and Giuli [3] in case some well known topological categories. In this study, the characterization of closed and strongly closed subobjects of an object in category of filter convergence spaces, [7], is given and it is shown that they induce a notion of closure which enjoy the basic properties like idempotency, (weak) hereditariness, and productivity in the category of filter convergence spaces.

Key Words: Filter convergence spaces, strongly closed subsets, closure operators.

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Contact Pseudo-Slant Submanifolds of a Sasakian Manifold

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ABSTRACT

The differential geometry of slant submanifolds has shown an increasing development since B-Y. Chen [5, 6] defined slant submanifolds in complex manifolds as a natural generalization of both the invariant and anti-invariant submanifolds. Many research articles have been appeared on the existence of these submanifolds in different known spaces. The slant submanifolds of an almost contact metric manifolds were defined and studied by A. Lotta[10]. After, these submanifolds were studied by J.L Cabrerizo et.al [8] of Sasakian manifolds. Recently, in [1]. M. Atçeken studied slant and pseudo-slant submanifold in $(LCS)_n$ -manifolds. Cabrerizo et al. studied and characterized slant submanifolds of K- contact and Sasakian manifolds and have given several examples of such submanifolds. Recently, Carizzo [7, 8] defined and studied bi-slant immersions in almost Hermitian manifolds and simultaneously gave the notion of pseudo-slant submanifold in almost Hermitian manifolds. The contact version of pseudo-slant submanifolds has been defined and studied by V. A. Khan and M. A Khan [9]. The present paper is organized as follows.

In this paper, the geometry of the contact pseudo-slant submanifolds of a Sasakian manifold were studied. The necessary and sufficient conditions were given for a contact pseudo-slant submanifold to be contact pseudo-slant product.

Let \tilde{M} be a $(2m+1)$ - dimensional almost contact metric manifold together with a metric tensor g a tensor field φ of type $(1,1)$ – a vector field ξ and a 1-form η on \tilde{M} which satisfy

$$\varphi^2(X) = -X + \eta(X)\xi, \quad (1)$$

$$\varphi\xi = 0, \quad \eta(\varphi X) = 0, \quad \eta(\xi) = 1, \quad \eta(X) = g(X, \xi) \quad (2)$$

and

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \quad g(\varphi X, Y) = -g(X, \varphi Y) \quad (3)$$

for any vector fields X, Y on \tilde{M} If in addition to above relations

$$(\tilde{\nabla}_X \varphi)Y = g(X, Y)\xi - \eta(Y)X \quad (4)$$

then, \tilde{M} is called Sasakian manifold, where $\tilde{\nabla}$ is Levi-Civita connection of g .

We have also on a Sasakian manifold \tilde{M}

$$\tilde{\nabla}_X \xi = -\varphi X \quad (5)$$

for any $X, Y \in \Gamma(T\tilde{M})$.

Definition1: Let M be a slant submanifold of an almost contact metric manifold \tilde{M} . M is said to be contact pseudo-slant of \tilde{M} if there exist two orthogonal distributions D^\perp and D^θ on M such that:

- i) TM has the orthogonal direct decomposition $TM = D^\perp \oplus D_\theta$, $\xi \in \Gamma(D_\theta)$.
- ii) The distribution D^\perp is anti-invariant, that is, $\varphi(D^\perp) \subset T^\perp M$.
- iii) The distribution D_θ is a slant, that is, the slant angle between of D_θ and $\varphi(D_\theta)$ is a constant.[9]

A contact pseudo-slant submanifold M of Sasakian manifold $(\tilde{M}, \varphi, \eta, \xi, g)$ is said to be D_θ -geodesic (resp. D^\perp -geodesic) if $h(X, Y) = 0$ for any $X, Y \in \Gamma(D_\theta)$ (resp. $h(W, Z) = 0$ for any $W, Z \in \Gamma(D^\perp)$). If for any $X \in \Gamma(D_\theta)$ and $Z \in \Gamma(D^\perp)$, $h(X, Z) = 0$ the M is called mixed -geodesic submanifold.

Given a proper contact pseudo-slant submanifold M of a Sasakian manifold $(\tilde{M}, \varphi, \eta, \xi, g)$, if the distributions D_θ and D^\perp are totally geodesic in M , then M is said to be contact pseudo-slant product.

The following theorems characterize the contact pseudo-slant product in Sasakian manifold.

Theorem1: Let M be a proper contact pseudo-slant submanifold of a Sasakian manifold $(\tilde{M}, \varphi, \eta, \xi, g)$. Then, either M is a mixed-geodesic or an anti-invariant submanifold.

Theorem2: Let M be a proper contact pseudo-slant submanifold of a Sasakian manifold $(\tilde{M}, \varphi, \eta, \xi, g)$. Then, either M is a D^\perp geodesic or an anti-invariant submanifold of $(\tilde{M}, \varphi, \eta, \xi, g)$.

Theorem3: Every proper contact pseudo-slant submanifold M of a Sasakian manifold $(\tilde{M}, \varphi, \eta, \xi, g)$ is a contact pseudo-slant product .

Key Words: Sasakian manifold, contact pseudo-slant manifold, contact pseudo-slant product.

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Curvature Properties of Surfaces in Isotropic Space

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ABSTRACT

The work of surfaces in three dimensional isotropic space I^3 has important applications in several applied sciences, e.g., computer science, Image Processing, architectural design and microeconomics, see [2,4].

Differential geometry of such spaces have been introduced by K. Strubecker [7], H. Sachs [5] and others.

I. Kamenarovic [3], Z. M. Sipus [6] and M.E. Aydin [1] have studied several classes of surfaces in I^3 , such as ruled surfaces and translation surfaces, etc.

The three dimensional isotropic space I^3 is obtained from the three dimensional projective space $P(\mathbb{R}^3)$ with the absolute figure which is an ordered triple (p, l_1, l_2) , where p is a plane in $P(\mathbb{R}^3)$ and l_1, l_2 are two complex-conjugate straight lines in p (see [6]).

In this talk, we study the some surfaces with special curvature properties in three dimensional isotropic space I^3 . In particular, we classify the rotational surfaces in three dimensional isotropic space I^3 satisfying Weingarten conditions in terms of the relative curvature K (analogue of the Gaussian curvature) and the isotropic mean curvature H . Furthermore, we obtain some results for such surfaces in three dimensional isotropic space I^3 fullfilling that the ratio of the relative curvature K and the isotropic mean curvature H/K is a constant in I^3 .

Key Words: Isotropic space, rotational surface, weingarten surface

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Differential Geometric Approaches to Production Theory in Economics

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ABSTRACT

A production function is a mathematical expression in economics which denotes the relations between the output generated by a firm, an industry or an economy and the inputs that have been used in obtaining it.

The most famous one among production functions is Cobb-Douglas production function, introduced in 1928 by C. W. Cobb and P. H. Douglas [3]. In original form, it is given as

$$Y=bL^kC^{1-k}$$

where b presents the total factor productivity, Y the total production, L the labor input and C the capital input.

As a generalization of Cobb-Douglas production function, in 1961, K. J. Arrow (winning Nobel-Prize Economics in 1972), et al. [4] introduced a two-factor CES (constant elasticity of substitution) production function given by

$$Y=A(aK^r+(1-a)L^r)^{1/r}$$

where Y is the output, A the factor productivity, a the share parameter, K and L the primary production factors, $r=(s-1)/s$, and $s=1/(1-r)$ is the elasticity of substitution.

Each production function can identified with a non-parametric graph hypersurface of any ambient space. In this presentation, we study some production functions with arbitrary inputs including Cobb-Douglas and CES production functions in terms of the geometry of their associated graph hypersurfaces in several ambient spaces. We obtain some classification results for the graph hypersurfaces with constant curvature associated to such production functions.

Key Words: Generalized Cobb-Douglas production function, generalized ACMS production function, graph hypersurface.

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Dual-Split Semi-Quaternions And Planar Motions

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ABSTRACT

In this study, firstly the basic structure of dual-split semi-quaternions will be considered. Afterwards, the planar motions in semi-dual three-space will be expressed by dual-split semi-quaternions.

Key Words: Dual split semi-quaternion, boost, quasi-elliptic motion.

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Edge and Gram Matrices of de Sitter n -Simplex

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ABSTRACT

Two questions of W. Fenchel raised in his book, *Elementary Geometry in Hyperbolic Space*, (De Gruyter, Berlin, 1989, p. 174) which are “a necessary and sufficient condition for given $(n(n+1))/2$ positive real numbers to be the dihedral angles and edge lengths of a hyperbolic n -simplex. First and second problem was solved by Feng Luo (*Geom. Dedicata* **64** (1997), 277–282) and Karliga (*Geom. Dedicata* **109** (2004), 1–6).

In this talk, we give a necessary and sufficient condition for given $(n(n+1))/2$ positive real numbers to be edge lengths of a non-degenerate de Sitter n -simplex. We also give similar condition for the dihedral angles of the simplex.

Key Words: de Sitter, edge matrix, Gram matrix, simplex.

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Frenet Apparatus of The Quaternionic Curves in Euclidean Space

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ABSTRACT

In this paper, we study frenet apparatus of quaternionic curves in Euclidean spaces. Then, we give some characterization of the Frenet vectors and curvatures of any curve parametrized with the arc length.

Key Words: Frenet apparatus, Euclidean spaces, Quaternion algebra.

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Fuzzy Co-Hopf Spaces

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ABSTRACT

Zadeh [8] introduced the concepts of fuzzy sets and fuzzy set operations. Up to now, many researchers have given significant results on fuzzy sets. Fuzzy groups were defined in [7], fuzzy topological groups were given in [3] and fuzzy topological groups on the fuzzy continuous function space were presented in [4]. Gumus and Yildiz [5] introduced the pointed fuzzy topological spaces. Fuzzy homotopy and fuzzy fundamental group were given in [1]. In 2014, fuzzy Hopf groups and some their properties were studied [2].

Hopf [6] gave the notion of Hopf space. A co-Hopf space (X, ϕ) consists of a topological space X and a continuous function $\phi: X \rightarrow X \vee X$ which is called comultiplication such that $\pi_1 \circ \phi \simeq 1_X \simeq \pi_2 \circ \phi$ where π_1, π_2 are the projections $X \vee X \rightarrow X$ onto the first and second summands of the wedge product. Co-Hopf spaces have important role in Algebraic Topology because a binary operation could be defined through co-Hopf spaces on $[(X, x_0), (Y, y_0)]$ which is a homotopy class.

In this talk, we introduce fuzzy co-Hopf spaces. We prove that a fuzzy retract of a fuzzy co-Hopf space is a fuzzy co-Hopf space. It is shown that a pointed fuzzy topological space having a fuzzy co-Hopf space structure is fuzzy contractible. We show that a pointed fuzzy topological space having the same homotopy type as a fuzzy co-Hopf group is a fuzzy co-Hopf group.

Key Words: Fuzzy pointed topological space, co-Hopf space, fuzzy homotopy.

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General Properties of Bertrand curves with Constant Precession

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ABSTRACT

Bertrand curve theory is based on the question that for a principal normal of a curve, whether a second curve that has a linear relationship with constant coefficient exist between curvature and torsion of the first one or not. The pair of this kind curves called Bertrand mates or conjugate Bertrand curve.

On the other hand, a constant precession curve is a curve which has property that is transversed with a unit speed, its centrodes (Darboux vector field)

$$w = \tau T + \kappa B$$

revolves about a fixed axis with constant angle and speed. If one describes this Darboux vector field in terms of an alternative moving frame, this vector provides the following conditions

$$D\Delta N = N', \quad D\Delta C = C', \quad D\Delta W = W'$$

Then we call it C-constant precession curve.

In this paper, we study it C-constant precession curve and define Bertrand curve of constant precession in terms of this alternative frame. Using well-known properties of Bertrand curves, we obtain some characterizations about this type curves.

Key Words: C-constant precession curve, Darboux vector fields, Bertrand mates.

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Generalized Biharmonic Submanifolds

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ABSTRACT

Harmonic maps have an important area of study as a generalization of important ideas like geodesics and minimal submanifolds. A significant literature has been created in the last decade including relationships between harmonic maps and the other disciplines namely theoretical physics.

A map $\psi : (M, g) \rightarrow (N, h)$ between Riemannian manifolds is called harmonic if it is the critical point of energy functional given by

$$E(\psi) = \frac{1}{2} \int_{\Omega} |d\psi|^2 v_g,$$

for any compact domain of M . Therefore harmonic maps are the solutions of the corresponding Euler-Lagrange equation which is characterized by the vanishing of the tension field

$$\tau(\psi) = \text{trace} \nabla d\psi.$$

As introduced by J. Eells and J. H. Sampson in [2], bienergy of a map ψ is defined by

$$E_2(\psi) = \frac{1}{2} \int_{\Omega} |\tau(\psi)|^2 v_g,$$

and ψ is said to be biharmonic if it is critical point of bienergy. In [1], the first and second variation formula for the bienergy were derived by G. Y. Jiang, showing that the Euler-Lagrange equation associated to E_2 is

$$\tau_2(\psi) = -\Delta \tau(\psi) - \text{trace} R^N(d\psi, \tau(\psi))d\psi = 0.$$

The equation $\tau_2(\psi) = 0$ is called biharmonic equation and it is clear that any harmonic map is biharmonic.

The concept of f -biharmonic maps have been introduced by W-J. Lu [4,5] as

a generalization of biharmonic maps. A differentiable map between Riemannian manifolds is said to be f -biharmonic if it is a critical point of the f -bienergy functional defined by integral of f times the square-norm of the tension field, where f is a smooth positive function on the domain. If $f = 1$ then f -biharmonic maps are biharmonic.

In the present paper, we study generalized biharmonic submanifolds. By using the f -biharmonic equation some results on f -biharmonic hypersurfaces and f -biharmonic submanifolds are given.

Key Words: Harmonic maps, Biharmonic submanifolds, f -Biharmonic submanifolds.

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Inextensible Ruled Surface which Generated by First Principle Direction Curve in E^3

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ABSTRACT

In this paper, we study inextensible flows of Ruled surface which generated by first principle direction curve in Euclidean 3-space E^3 . Furthermore, we give some new characterizations of this surfaces.

Key Words: Ruled Surface, Inextensible flow, first principle direction curve.

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Lifting of Leibniz Crossed Modules

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ABSTRACT

Crossed modules are first introduced by J.H.C. Whitehead in late of forties, as a model for connected homotopy 2-types, i.e. connected spaces with $\pi_n(X, x) = 0$ for $n > 2$. While he was working on the second relative homotopy groups he discovered some algebraic properties of the boundary map and the natural action of fundamental group on the second relative homotopy group. This algebraic structure is called crossed module. After the invention of crossed modules many authors have worked on crossed modules for various algebraic structures such as Lie, Leibniz, commutative, associative algebras. Crossed modules have important roles in many areas of mathematics such as homology and cohomology theory, homotopy theory, algebraic K-theory, combinatorial group theory, differential geometry, etc.

Recently, Mucuk and Şahan defined the notion of lifting for crossed modules, over a group homomorphism having the same domain with the boundary map, as a model for action group-groupoids.

In this study, lifting of crossed modules in the category of Leibniz algebras over an algebra homomorphism is defined. Some properties of this lifting are investigated and for a fixed crossed module the category of lifting crossed modules is constructed. Finally the notion of universal lifting of a crossed module is defined and existence of universal lifting under some conditions and some properties of universal liftings are given.

Key Words: Leibniz algebra, crossed module, lifting, universal lifting.

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Motions of Curves in 4-Dimensional Galilean Space G_4

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ABSTRACT

In this work we investigate the flows of curves and its equiform geometry in the 4-dimensional Galilean space G_4 . We obtain the evolution equations of curves according to Serret-Frenet equations in G_4 also we get the evolution equations for the curvatures κ , τ and σ , that induce the inviscid Burgers' equations.

Then we research the evolution equations for equiformly invariant orthogonal vector and curvatures. We showed that the evolution equation for the equiformly invariant curvature K_1 can be induced the stochastic Burgers' equations. Then we find that the Frenet equations and curvatures of inextensible flows of curves and its equiformly invariant vector fields and intrinsic quantities are independent of time. So we conclude that the motions of curves and its equiform geometry can be defined by the inviscid and stochastic Burgers' equations in 4-dimensional Galilean space G_4 .

Key Words: Galilean space, inextensible flows, motions of curves, Burgers' equations.

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New Characterization of Commutator Curves and Exponential Maps in Lorentzian Heisenberg Group

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ABSTRACT

In this paper, we study commutator curves and exponential maps in the $2n+1$ dimensional Lorentzian Heisenberg group. Furthermore, we give some new characterizations of commutator curves and exponential maps.

Key Words: Commutator curve, exponential map, Lorentzian Heisenberg group.

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Normal Section Curves and Curvatures

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ABSTRACT

Classifications of submanifolds have an important role in studying differential geometry of submanifolds. Many authors categorize the submanifolds by utilizing from distributions on submanifolds. Facilitating the process by given conditions such as being totally umbilical, totally geodesic, as well as integrable, characterizations of submanifolds are investigated. The most basic and the simplest method in examining the geometry of submanifolds is to study on a curve. Chen [3] defined planar normal sections for his purpose and used such curves to study geometry of submanifolds. Chen, Kim, Li [3,4,5,6,10,11,14] classified submanifolds by using curves of normal section.

Let M be an n -dimensional submanifold in an m -dimensional Euclidean space E^m . For any point p in M and any nonzero x at p tangent to M , the vector x and normal space $T_p^\perp M$ determine an $(m - n - 1)$ -dimensional vector space $E(p, x)$ in E^m . The intersection of M and $E(p, x)$ give rise a curve $\gamma(s)$ (in a neighborhood of p), called the normal section of M at p in the direction x , where s denotes the arc length of γ . In general the normal section γ is a twisted space curve in $E(p, x)$. A submanifold M is said to have pointwise k -planar ($2 \leq k \leq m - n$) normal sections if each normal section γ at p satisfies

$$\gamma' \wedge \gamma'' \wedge \dots \wedge \gamma^{(k+1)} = 0,$$

for each p in M .

Motivated by studies in Riemannian and semi-Riemannian manifolds, we investigate curvatures of planar normal section curves. Also we find some necessary and sufficient conditions for a curve in terms of curvatures which is assumed to be a normal section curve and classify such curves.

Key Words: Submanifolds with Planar Normal Section, Normal Section Curve, Curvature.

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On Closedness of the Topological Category of Approach Spaces

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ABSTRACT

The notion of approach space is introduced by Lowen[5] in order to solve the problem of nonmetrizability of arbitrary initial structures of metrizable topological spaces. The main idea behind approach spaces is to axiomatize the concept of distance between points and sets in such a way to generalize both the metric and topological situations.

Approach spaces and contractions form a topological category in the sense of [1] and denoted by AP into which both the categories $p\text{-MET}^\infty$ of extended pseudometric spaces and TOP of topological spaces could be nicely embedded. The product in AP coincides with the topological product in TOP (in fact the embedding of TOP has both a left and right adjoint, i. e. it is bireflective and bicoreflective) but not with the product in $p\text{-MET}^\infty$ (which is only coreflectively embedded). Hence, the arbitrary product of metric spaces in AP is no longer a metric but what we call an approach structure.

The notions of closedness and strong closedness are introduced in topological categories by Baran [2, 3] and also shown that these notions form an appropriate closure operator in the sense of [4] in some well-known topological categories.

In this talk we will characterize the (strongly) closed and (strongly) open subobject of an object in the category of approach spaces using the method given in [2, 3].

Key Words: Approach space, topological category, closedness.

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On Conformal, Conircular And Quasi-Conformal Curvature Tensor of Normal Complex Contact Metric Manifolds

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ABSTRACT

Conformal, concircular and quasi-conformal curvature tensors play an important role in Riemannian geometry. If conformal curvature tensor vanishes then the manifold is called conformally flat. The concircular curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature and if concircular curvature vanishes the manifold is called concircularly flat. Quasi-conformal curvature tensor includes both conformal and concircular curvature tensor as a special case. Also a manifold is called quasi-conformally flat if this tensor is identically zero. Complex contact manifolds was first studied by Kobayashi [7]. In 1970's Ishihara and Konishi [4] studied on Riemannian geometry of complex contact manifolds. They proved existence of almost complex contact structure and developed normality conditions. Their normality is named "IK-Normality" in literature and it get forced the structure to be Kaehlerian. In 2000 IK-Normality was extended by Korkmaz [6]. The definition of Korkmaz is weaker than IK-Normality and it is based on normality of complex Heisenberg group. She also introduced GH -sectional curvature and studied on curvature of normal complex contact metric manifolds. In this article we study on conformal, concircular and quasi conformal curvature tensor of normal complex contact metric manifolds. We obtain some results on flatness of these tensors.

Key Words: Complex contact metric manifold, concircularly flat, conformally flat, quasi-conformal

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On CR-Submanifolds Of S-Manifold With A Semi Symmetric Metric Connection

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ABSTRACT

In this paper we study CR -submanifolds of an S -manifold endowed with a semi-symmetric metric connection. We give an example, investigating integrabilities of horizontal and vertical distributions of CR -submanifolds endowed with a semi-symmetric metric connection. We also consider parallel horizontal distributions of CR -submanifolds.

Key Words: CR -submanifold, S -manifold, Semi-symmetric metric connection.

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On Curvature Tensors of a Normal Paracontact Metricmanifold

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ABSTRACT

Let M a n -dimensional differentiable manifold. If on M we have

$$\left. \begin{aligned} \phi^2 X &= X - \eta(X)\xi, & \phi\xi &= 0, & \eta(\phi X) &= 0, & \eta(\xi) &= 1 \\ \text{and} & & & & & & & \\ g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y), & \eta(X) &= g(X, \xi) \end{aligned} \right\} \quad (1.1)$$

for any vector fields X, Y on M , ξ is a contravariant vector and η is 1-form then M is called almost para contact metric manifold with structure (ϕ, ξ, η, g) defined above (1.1).

An almost paracontact metric manifold M is said to be normal if

$$(\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi$$

and

$$\phi X = \nabla_X \xi.$$

A normal paracontact metric manifold M is said to have a constant c if and only if

$$\begin{aligned} R(X, Y)Z &= \frac{c+3}{4} \{g(Y, Z)X - g(X, Z)Y\} \\ &+ \frac{c-1}{4} \{ \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\ &- g(Y, Z)\eta(X)\xi + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z \} \end{aligned}$$

for any vector fields $\forall X, Y, Z \in \chi(M)$ on M .

The concircular curvature tensor and quasi-conformal curvature tensor of a normal paracontact metric manifold M are, respectively, defined

$$\tilde{Z} = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]$$

and

$$\begin{aligned} \tilde{C}(X, Y)Z = & aR(X, Y)Z + b[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \\ & - \frac{r}{n} \left[\frac{a}{n-1} + 2b \right] [g(Y, Z)X - g(X, Z)Y] \end{aligned}$$

for any $\forall X, Y, Z \in \chi(M)$, where R denotes the Riemannian curvature tensor of M .

The object of the present paper is to study the curvatures satisfying properties of a normal paracontact metric manifold when it is locally symmetric, concircular semi-symmetric, quasi-conformal semi-symmetric, conformal flat and quasi-conformal flat.

Key Words: Paracontact metric manifold, concircular semi-symmetric, conformal flat.

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On Golden Shaped Rotation Surfaces in Minkowski 3-Space

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ABSTRACT

A rotation surface in Euclidean space is generated by rotating of an arbitrary curve (which is generally called profile curve) about an arbitrary axis. If $\alpha: I \subset \mathbb{R} \rightarrow \Pi$ is a curve in a plane Π of Minkowski 3-space E_1^3 equipped with induced non-degenerate metric with 1-index and ξ is a straight line of the plane Π which does not intersect the curve α , then a rotation surface M in E_1^3 is defined as a non-degenerate surface generated by rotating the curve α around the axis ξ . In Minkowski space, however, there are different types of curves (spacelike, timelike or lightlike(null)) as well as different types of rotation axes (spacelike, timelike or lightlike(null)), so that there are different types of rotation surfaces in this context.

Furthermore, the differential geometry of metallic means family on Riemannian manifolds is a popular subject for mathematicians. In [2], some applications of the metallic means family and generalized Fibonacci sequences on Riemannian manifolds have been studied. Also, metallic shaped hypersurfaces in real space forms have been determined and some classifications of these hypersurfaces have been investigated in [3].

In this study, we investigate golden shaped rotation surfaces whose profile curves are timelike and with constant curvature in Minkowski 3-space E_1^3 , write the parametric expressions of these rotation surfaces by finding the curvatures of the profile curves' and draw the graphs of these rotation surfaces with the aid of *Maple 2015*.

Key Words: Rotation surface, Planar curves, Golden shaped hypersurfaces.

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On Legendre Curves of (ε, δ) Trans-Sasakian Manifolds

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ABSTRACT

The concept of (ε) -Sasakian manifolds were introduced by A. Bejancu and K. L. Duggal [1] and X. Xufeng, C. Xiaoli [10] proved that these manifolds are real hypersurfaces of indefinite Kahlerian manifolds. Some curvature conditions of (ε) -Sasakian manifolds were obtained in [6]. In 2010, a new type of manifolds called (ε) -almost paracontact manifolds were defined by M. M. Tripathi et. al. [9]. Also U. C. De and A. Sarkar [3] established (ε) -Kenmotsu manifolds and investigated curvature conditions on such manifolds. As a generalization of both (ε) -Sasakian manifolds and (ε) -Kenmotsu manifolds, H. G. Nagajara et. al. [7] introduced (ε, δ) trans-Sasakian structures.

The notion of Legendre curves in almost contact metric manifolds was introduced in [2]. J. Inoguchi [5] classified the proper biharmonic Legendre curves and Hopf cylinders in 3-dimensional Sasakian space form and in [4] the explicit formulas were obtained. C. Özgür and M. M. Tripathi [8] proved that a Legendre curve in an (α) -Sasakian manifold is biharmonic if and only if its curvature is zero. Moreover, some characterizations of Legendre curves in quasi Sasakian manifolds and almost paracontact metric manifolds were given by J. Welyczko [11, 12].

Motivated by these studies, we study Legendre curves in 3-dimensional (ε, δ) trans-Sasakian manifolds. Explicit formulas for curvature and torsion of such curves are obtained. Also we investigate necessary and sufficient conditions for Legendre curves to be biharmonic.

Key Words: Legendre Curves, Biharmonic curves, (ε, δ) trans-Sasakian manifolds

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On Mannheim Partner Curves Of Constant Precession

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ABSTRACT

Curve theory is one of the fundamental research area who interested in differential geometry because of having many application field from mathematics to engineering.

Among numerous curve type, the curves which are defined by the relations between normal and binormal lines of each other are widely studied ones.

From this point of view, we will concern with a curve couple called Mannheim. These type of curves, simply the principal normal lines of one curve coincides with the binormal lines of the other one.

On the other hand, a constant precession curve is a curve which has property that is transversed with a unit speed, its centrodes (Darboux vector field)

$$w = \tau T + \kappa B$$

revolves about a fixed axis with constant angle and speed. If one describes this Darboux vector field in terms of an alternative moving frame, this vector provides the following conditions

$$D\Delta N = N', \quad D\Delta C = C', \quad D\Delta W = W'$$

Then we call it C-constant precession curve.

In this paper, we consider the properties of Mannheim partner curves with constant precession and obtain some characterizations on it.

Key Words: C-constant precession curve, Darboux vector fields, Mannheim partner curves.

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On Normal Paracontact Metric Manifold Satisfying Some Conditions on the Weyl Projective Curvature Tensor

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ABSTRACT

Let M a n -dimensional differentiable manifold. If on M we have

$$\left. \begin{aligned} \phi^2 X &= X - \eta(X)\xi, & \phi\xi &= 0, & \eta(\phi X) &= 0, & \eta(\xi) &= 1 \\ \text{and} & & & & & & & \\ g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y), & \eta(X) &= g(X, \xi) \end{aligned} \right\} \quad (1.1)$$

for any vector fields X, Y on M , ξ is a contravariant vector and η is 1-form then M is called almost para contact metric manifold with structure (ϕ, ξ, η, g) defined above (1.1).

An almost paracontact metric manifold M is said to be normal if

$$(\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi \quad (1.2)$$

and

$$\phi X = \nabla_X \xi.$$

A normal paracontact metric manifold M is said to have a constant c if and only if

$$\begin{aligned} R(X, Y)Z &= \frac{c+3}{4} \{g(Y, Z)X - g(X, Z)Y\} \\ &+ \frac{c-1}{4} \{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\ &- g(Y, Z)\eta(X)\xi + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z\} \end{aligned} \quad (1.3)$$

for any vector fields $\forall X, Y, Z \in \chi(M)$ on M .

The concircular curvature tensor, Weyl-projective curvature tensor and quasi-conformal curvature tensor of a normal paracontact metric manifold M are, respectively, defined

$$\tilde{Z} = R(X, Y)Z - \frac{r}{n(n-1)} [g(Y, Z)X - g(X, Z)Y], \quad (1.4)$$

$$P = R(X, Y)Z - \frac{1}{n-1} [S(Y, Z)X - S(X, Z)Y] \quad (1.5)$$

and

$$\begin{aligned} \tilde{C}(X, Y)Z = & aR(X, Y)Z + b[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \\ & - \frac{r}{n} \left[\frac{a}{n-1} + 2b \right] [g(Y, Z)X - g(X, Z)Y]. \end{aligned} \quad (1.6)$$

for any $\forall X, Y, Z \in \chi(M)$, where R denotes the Riemannian curvature tensor of M .

For $\{e_1, e_2, \dots, e_{n-1}, \xi\}$ orthonormal basis of M , from (1.3) we obtain

$$S(X, Y) = \left[\frac{c(n-6) + 3n + 2}{4} \right] g(X, Y) + \left[\frac{c(6-n) + n - 10}{4} \right] \eta(X)\eta(Y)$$

which is equivalent to

$$QX = \left[\frac{c(n-6) + 3n + 2}{4} \right] X + \left[\frac{c(6-n) + n - 10}{4} \right] \eta(X)\xi.$$

In the present paper, we have studied the curvature tensors of a normal paracontact metric manifolds satisfying the conditions $P(\xi, X)R = 0$, $P(\xi, X)\tilde{Z} = 0$, $P(\xi, X)P = 0$, $P(\xi, X)S = 0$ and $P(\xi, X)\tilde{C} = 0$. According these cases, we classified normal paracontact metric manifolds.

Key Words: Paracontact metric manifold, Projective curvature tensor, concircular curvature tensor, quasi-conformal curvature tensor.

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On Semi-Invariant Submanifolds Of Almost A-Cosymplectic F-Manifolds Admitting A Semi-Symmetric Non-Metric Connection

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ABSTRACT

In this paper, we study semi-invariant submanifolds of an almost α -cosymplectic f-manifold endowed with a semi-symmetric non-metric connection. We give necessary and sufficient conditions on a submanifold of an almost α -cosymplectic f-manifold to be semi-invariant submanifold with semi-symmetric non-metric connection. Moreover, we obtain the integrability condition of the distribution on semi-invariant submanifolds of an almost α -cosymplectic f-manifold with semi-symmetric non-metric connection.

Key Words: Almost α -Cosymplectic f-Manifolds, Semi-Invariant Submanifolds, Semi-Symmetric Non-Metric Connection, Integrability of Distributions Conditions.

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On Series Solutions of Curves by Using Gaussian Curvature

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ABSTRACT

In differential geometry, the Gaussian curvature κ of a surface at a point is the product of the principal curvatures k_1 and k_2 , at the given point:

$$\kappa = k_1 k_2.$$

The sign of the Gaussian curvature can be used to characterize the surface. At a point p on a regular surface in IR^3 , the Gaussian curvature is also given by

$$\kappa(p) = \det S(p),$$

where S is the shape operator. Gaussian curvature is an intrinsic measure of curvature, depending only on distances that are measured on the surface. In mathematics, the power series method is used to seek a power series solution to certain differential equations. In general, such a solution assumes a power series with unknown coefficients, then substitutes that solution into the differential equation to find a recurrence relation for the coefficients. The method works analogously for higher order equations as well as for systems. From the differential equations viewpoint, Gaussian curvature solve the differential equation to find the main curve.

In this study, we present series solutions to determine the curve and demonstrate our results on some well-known surfaces such as sphere, catenoid and torus. These results are suitable for surface of revolution. Also, the Gaussian curvature can be used to determine constant surface of revolution

Key Words: Series solutions, Gaussian operator, shape operator.

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On The Harmonic Curves On a Semi-Riemannian Manifold

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ABSTRACT

In this study, we characterize the harmonic curves on a semi-Riemannian manifold. We give some properties about such curves and research the relations between biharmonic and harmonic curves. Finally we explore some surface on semi-Riemannian manifolds which we can say they are harmonic.

Key Words: Harmonic curves, harmonic surfaces, semi-Riemannian manifold.

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On The Quaternionic Inextensible Flows of Curves in Euclidean Space E^4

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ABSTRACT

In this paper, we study quaternionic inextensible flows of curves in 4 dimensional Euclidean spaces E^4 . Necessary and sufficient conditions for a quaternionic inextensible flows are expressed as a partial differential equation involving the curvature.

Key Words: Inextensible flows, Euclidean spaces, quaternion algebra.

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On The Solid Angle of Spherical Tetrahedron

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ABSTRACT

In this work, we introduce the solid angle of spherical tetrahedron in 3-dimensional spherical space. We obtain a convenient relation between solid angle and dihedral angle of a spherical tetrahedron. The fundamental relationship between these angles is stated for regular tetrahedron. Initially, the analogue of Cayley-Menger determinant which is related to Gram and edge matrices of the tetrahedron is determined for a regular tetrahedron then it is generalized for any spherical tetrahedron.

In addition, the sine and cosine law deduced from n-dimensional solid angle. We present some applications of the solid angle in spherical geometry for volume computation.

Key Words: Tetrahedron, sphere, solid angle, dihedral angle, Gram matrix.

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On the Tube Curve of the Focal Curve

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ABSTRACT

The analysis of space curves in differential geometry is a classical subject. For any unit speed curve, the focal curve is defined as the centers of the osculating spheres. According to the Frenet frame $\{t(s), n(s), b(s)\}$ of unit speed curve $\gamma(s)$, the focal curve is given as follows,

$$C_\gamma(s) = (\gamma + c_1 n + c_2 b)(s)$$

where the coefficients are smooth functions that are called focal curvatures of γ .

The circle $\cos\theta n(s) + \sin\theta b(s)$ is perpendicular to γ at $\gamma(s)$. As this circle moves along γ it traces out a surface called the tube about γ . This surface is defined as follows

$$X(s, \theta) = \gamma(s) + r(\cos\theta n(s) + \sin\theta b(s)).$$

This paper, we study a surface called the tube about the focal curve. We characterize this surface and give some examples.

Key Words: Focal curve, tube surface, fundamental form

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On The Zero-Dimensional Objects in Category of Local Filter Convergence Spaces

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ABSTRACT

Dimension theory was originated and formulated by Menger [5]. There are different types of dimensions like small inductive dimension, large inductive dimension, covering dimension, Hausdorff dimension and recently fractal dimension. Among these dimensions we will introduce zero-dimension which is smallest dimension in topological spaces. Zero-dimensional spaces were defined by Sierpinski in 1921. A zero-dimensional space was defined as a non-empty space in which there is a basis for the open sets made up of sets which are clopen, i.e. open and closed, by Hurewicz and Wallman [4, 8].

In 1997, Stine [7] has proved that a zero-dimensional topological space can be characterized by using induced structures (initial lift) and discreteness. Stine proved that a topological space (X, τ) is zero-dimensional if and only if there exists a family of discrete spaces (X_i, τ_i) and a family of functions $f_i : X \rightarrow X_i$ such that X is the topology induced on X by (X_i, τ_i) via f_i . Hence, the notion of zero dimensional object can be defined in any topological category. In 2010, Erciyes [3] gave zero-dimensional objects in some well-known topological categories. In 1996, Baran [1] defined four new generalization of Hausdorff spaces in topological categories. Also, Baran gave $PreT_2$ objects in category of local filter convergence spaces [2].

The main objective of this study is to characterize zero dimensional objects in category of local filter convergence spaces.

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Key Words: Topological category, zero-dimensional objects, local filter convergence spaces.

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On 3-Dimensional Quasi-Sasakian Manifolds

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ABSTRACT

In this paper we study 3-dimensional quasi-Sasakian manifolds with the D_a -homothetic deformation structure. Also, we adapte semi-symmetric non-metric connection to D_a -homothetic deformation structure and we study some curvature conditions on a 3-dimensional quasi-Sasakian manifold.

Key Words: Quasi-Sasakian manifold, D_a -homothetic deformation structure.

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Parallel Hasimoto Surfaces in Minkowski 3-Space

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ABSTRACT

In this paper, we study parallel surfaces of Hasimoto surfaces in Minkowski 3-space E_1^3 . We give some characterizations for the parallel surfaces of the Hasimoto surfaces satisfying the smoke ring equation. We also give an explicit example for these surfaces. Moreover some representations of such surfaces are given in order to show their nice shapes. Finally, we investigate κ and τ non-zero curvature and torsion functions of Hasimoto surface whose parallel surface is also Hasimoto one.

Key Words: Hasimoto surface, Parallel surfaces, Timelike surfaces, Minkowski spacetime.

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Pseudosymmetric Lightlike Hypersurfaces*

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ABSTRACT

Pseudosymmetric manifolds have been discovered during the study of totally umbilical submanifolds of semisymmetric manifolds [1]. Then, it is known that a semi-Riemannian manifold is called pseudosymmetric if at every point of the manifold the tensor $R \cdot R$ and Tachibana tensor $Q(g; R)$ are linearly dependent. Also, it is clear that every semisymmetric Riemannian manifold ($R \cdot R = 0$) is a pseudosymmetric manifold but the converse is not true.

On the other hand, lightlike hypersurfaces have metric with vanishing determinants and this degeneracy of these metrics leads to several difficulties. So, in this study, we study lightlike hypersurfaces of a semi-Riemannian manifold satisfying pseudosymmetry conditions. We give sufficient conditions for a lightlike hypersurface to be pseudosymmetric and show that there is a close relationship of the pseudosymmetry condition of a lightlike hypersurface and its integrable screen distribution. We obtain that a pseudosymmetric lightlike hypersurface is a semisymmetric lightlike hypersurface or totally geodesic under certain conditions. Moreover, we give an example for pseudosymmetric lightlike hypersurfaces. Later, we investigate pseudoparallel lightlike hypersurfaces and give characterizations about such hypersurfaces. Furthermore, we introduce Ricci-pseudosymmetric lightlike hypersurfaces, obtain characterizations and give an example for such hypersurfaces. Finally, we define Ricci-generalized pseudoparallel lightlike hypersurface and give a result about this hypersurface.

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Key Words: Pseudosymmetric manifolds, Lightlike hypersurfaces,

Pseudoparallel lightlike hypersurfaces, Ricci-pseudosymmetric
lightlike hypersurfaces.

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Ptolemaean Inequality in Complex Hyperbolic Space $H_{\mathbb{C}}^2$

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ABSTRACT

The theorem of Ptolemaeus in planar Euclidean geometry states that the product of the euclidean lengths of the diagonals of an inscribed quadrilateral equals to the sum of the products of the Euclidean lengths of its opposite sides. When one vertex of the quadrilateral does not lie on the circle passing from the other three vertices, then we have inequality, known as the Ptolemaean inequality[3].

This result has a simple statement but it is of great interest. We just mention here few different proofs given by Buckley, Falk and Wraith [1], Platis [3], [4].

The complex hyperbolic 2-space $H_{\mathbb{C}}^2$ is a natural generalisation of plane hyperbolic geometry which is different from the more familiar generalisation of higher dimensional real hyperbolic space.

The boundary of complex hyperbolic 2-space $\partial H_{\mathbb{C}}^2$ is the one point compactification of the Heisenberg group in the same way that the boundary of real hyperbolic space is the one point compactification of Euclidean space of one dimension lower [2].

In this study we will use the complex cross-ratios to prove the Ptolemaean inequality and the theorem of Ptolemaeus in the setting of the boundary of complex hyperbolic 2-space $H_{\mathbb{C}}^2$.

Key Words: Complex hyperbolic space, Ptolemaean inequality, Cross ratios.

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Schlaflı Differential Formula And Area of De Sitter Triangles

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ABSTRACT

I. Asmus give ten different triangles of de-Sitter plane in "Duality Between Hyperbolic and de-Sitter Geometry". In this study, we obtain parametric equations of lines in de-Sitter plane and give some properties of these lines. By using Schlaflı differential formula, we find the area in terms of dihedral angles of de-Sitter triangles with non-null edges.

KeyWords : de-Sitter Triangles, Schlaflı differential formula

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Some Characterizations of a Spacelike Curve in Minkowski Space

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ABSTRACT

Investigating of special curves is one of the most attractive topic in differential geometry. Some of these special curves are spacelike curves, timelike curves and null curves. Spacelike curves and timelike curves were initially investigated and developed by several authors[1, 2, 3, 6]. Later, this topic drew attention of several authors and they studied different kinds of curves in the Lorentzian manifolds R_1^3 and R_1^4 .

In this paper, we study the position vectors of a spacelike curve in the Minkowski 4-space R_1^4 . We give some characterizations for spacelike curves to lie on some subspaces of R_1^4 and give some theorems for these curves. The curvature functions of a spacelike curve α in R_1^4 are given by k_1, k_2 and k_3 in the paper. We give spacelike curves in terms of their curvature functions. If we consider a spacelike curve α in R_1^4 with the Frenet frame $\{T, N, B_1, B_2\}$ where N is a spacelike vector and B_1 is a timelike vector. The Frenet frame of $\alpha(s)$ satisfies the following Frenet equations:

$$\begin{aligned}\nabla_T T &= k_1 N \\ \nabla_T N &= -k_1 T + k_2 B_1 \\ \nabla_T B_1 &= k_2 N + k_3 B_2 \\ \nabla_T B_2 &= -k_3 B_1\end{aligned}$$

By using these equations we give the position vectors of spacelike curves in terms of their curvature functions k_1, k_2 and k_3 in 4-dimensional Minkowski space.

In the other part of the paper we investigate the conditions for a spacelike curve to be an inclined curve and we give a theorem about the inclined spacelike curve in R_1^4 .

Key Words: Spacelike curve, Minkowski space, Frenet frame.

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Some Characterizations of Bertrand Partner Curves in three-dimensional Lightlike Cone

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ABSTRACT

The notion of Bertrand curves was discovered by J. Bertrand in 1850.

If there exists a corresponding relationship between the space curves x and x^* such that, at the corresponding points of the curves, the principal normal lines of x coincides with the normal lines of x^* , then x is called a Bertrand curve and x^* a Bertrand partner curve of x . The pair $\{x, x^*\}$ is said to be a Bertrand pair.

In this study, we consider the idea of Bertrand curves for spacelike curves lying in the 3-dimensional lightlike cone. We also give some characterizations of a Bertrand pair and we examine these curves in three-dimensional lightlike cone. Considering the asymptotic orthonormal frame field, we define Bertrand curves for spacelike curve in 3- dimensional lightlike cone. We give some characterizations of a Bertrand pair in three-dimensional lightlike cone and we obtain $\{x, x^*\}$ Bertrand pair in terms of their curvature functions and give also some equations. Furthermore, we characterize an arbitrary helix curve in term of their curvature functions and give also some differential equations concerned helix. Also, we give an example for a Bertrand curve in the three-dimensional lightlike cone, In this example, we find the relations between the curvature and the torsion for a helix curve.

Key Words: Bertrand pair, lightlike cone, helix equation, Bertrand equation.

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Some Properties of the Quasicompact-Open Topology

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ABSTRACT

The compact-open topology was introduced by Fox [1] is one of the best known and commonly used topologies on $C(X)$ of all continuous real-valued functions on space X . Another familiar topology on $C(X)$ is the uniform topology (or topology of uniform convergence). It is shown in [3] that the compact-open topology is the proper setting to study sequences of functions converging uniformly on compact subsets. This topology is sometimes called the topology of uniform convergence on compact sets. If space X is compact, then the compact-open topology is equal to the topology of uniform convergence. We know that there have been many topologies that lie between the compact-open topology and the topology of uniform convergence. A space X is said to be quasicompact [2] if every covering of X by cozero-sets admits a finite subcollection which covers X , also known as z -compact space. We recall that any compact space is quasicompact, any quasicompact space is pseudocompact and the continuous image of a quasicompact space is quasicompact.

The purpose of this study is to define quasicompact-open topology on $C(X)$ by using quasicompactness. Also we compare this topology with compact-open topology and the uniform topology, and investigate the properties of the quasicompact-open topology.

Key Words: Function space, compact-open topology, quasicompactness.

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Some Results on Digital Fibrations

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ABSTRACT

Digital image analysis has been a very popular research area in recent times. The most interesting property of this area is to be applied to various fields of science. Researchers use topology and algebraic topology to classify digital objects. From this viewpoint, we think that defining of digital fibrations will be useful. Fibration can be viewed as a generalization of the fiber bundle. It plays a significant role in algebraic topology with homological properties.

We now deal with some important progresses in digital topology. In [1], digital homology groups were defined. This theory was extended in [2] and it was calculated digital simplicial homology groups of some digital images. Karaca and Ege [5] obtained some results on digital homology groups of two dimensional digital images. In [3], some theorems related to digital simplicial homology groups were proved. Moreover, digital relative homology groups and their some properties were given in [4]. Moreover, some results about Euler characteristics and reduced homology groups of digital images were presented in [4]. Digital fiber bundles and digital fiber bundle map were introduced in [7].

In this talk, we present some properties about digital fibrations and give some results on homology properties of digital fibrations and Euler characteristics.

Key Words: Digital image, digital fibration, digital simplicial homology group.

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Some Special Curves In Simple Isotropic Space And In The Equiform Geometry Of The Simple Isotropic Space

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ABSTRACT

The theory of biharmonic functions is an old and rich subject. Biharmonic functions have been studied since 1862 by Maxwell and Airy to describe a mathematical model of elasticity.

The Riemannian generalization of the elastic energy, called the bioenergy, is defined as:

$$E_2(c) = \frac{1}{2} \int \kappa^2 ds,$$

where κ is the geodesic curvature of the curve c . Critical points of E_2 , called biharmonic curves, are described by the equation [6]:

$$\nabla_c^3 c = R(c, \nabla_c^c) c.$$

Many interesting results on biharmonic curves have been obtained by many mathematicians [2 – 7]. Chen and Ishikawa [2] classified biharmonic curves in semi-Euclidean space E_v^n . They showed that every biharmonic curve lies in a 3-dimensional totally geodesic subspace. Thus, it suffices to classify biharmonic curves in semi-Euclidean 3-space.

More recently, besides in semi-Euclidean space, many studies have been made in other space: in Euclidean 3-space [3], in Lorentzian space [4], in Minkowski 3-space [5], in Finsler spaces [6], in 3-dimensional Heisenberg group [7].

In this paper, 1-type curves and biharmonic curves are given by using the curvature vector field in Simple Isotropic Space I_3^1 and in the equiform geometry of the Simple Isotropic Space I_3^1 . In Simple Isotropic Space I_3^1 , we showed that biharmonic curve holds if and only if a Frenet curve is a geodesic. Additionally, in the

equiform geometry of Simple Isotropic Space I_3^1 , we showed that biharmonic curve holds if and only if a Frenet curve is a geodesic and a null cubic.

Key Words: Biharmonic curve, 1-type curve, simple Isotropic space.

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Some Special Smarandache Curves of Non-Degenerate Curves in Anti de Sitter 3-Space

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ABSTRACT

In this paper, we investigate special spacelike Smarandache curves of timelike curves according to Sabban frame in Anti de Sitter 3-Space. Moreover, we give the relationship between the base curve and its Smarandache curve associated with their Sabban Frames. However, we obtain some geometric results with respect to special cases of the base curve. Finally, we give some examples of such curves and draw their images under stereographic projections from Anti de Sitter 3-space to Minkowski 3-space.

Key Words: Anti de Sitter space, Semi Euclidean space, Smarandache curve

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Spacelike Curve Flows and Integrable Systems in the Lorentzian symmetric space $SO(n,1)/SO(n-1,1)$

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ABSTRACT

In this paper, Integrable systems are derived by applying a general moving frame method to non-stretching spacelike curve flows in the Lorentzian symmetric space $SO(n,1)/SO(n-1,1)$. Also, non-stretching spacelike curve flows are shown to yield the defocusing mKdV (the modified Korteweg-de Vries) equation in $SO(n,1)/SO(n-1,1)$.

Key Words: Spacelike curve, Lorentz Space, Curve Flow.

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TC – Bezier Transition Curves

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ABSTRACT

Curves and surfaces design is an important topic of CAGD (Computer Aided Geometric Design) and computer graphics. The parametric representation of curves and surfaces specially in polynomial form is most suitable for design, as the planar curves cannot deal with infinite slopes and are axis dependent too. The theory of Bezier curves uphold a key position in CAGD. In recent year trigonometric polynomial curves like those of Bezier type are considerably in discussion. Many new curves related with Bezier curves are introduced by many authors [1-3].

In this talk, we present a new type curve with shape parameters namely TC – Bezier curves. The curve is constructed based on new cubic trigonometric polynomials. The cubic trigonometric Bezier curve can be made closer to the cubic Bezier curve or nearer to the control polygon than cubic Bezier curve due to shape parameters. On the other hand, transition curves are important in railway design, road design and in the orbit of satellite design. Therefore, it is important to find transition curves between line to line, line to circle or circle to circle in different shapes like C – shape, S – shape or J – shape. Therefore in this talk finally, we find sufficient conditions for such curve due to be transition curves from line to line and circle to circle.

Key Words: Cubic trigonometric basis functions, Cubic trigonometric Bezier curves, Transition curves.

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Totally Paracontact Umbilical Radical Transversal Null Submanifolds of Para-Sasakian Manifolds

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ABSTRACT

The differential geometry of null (lightlike) submanifolds is different from non-degenerate submanifolds because of the fact that the normal vector bundle has non-trivial intersection with the tangent vector bundle. So, one cannot use the classical submanifold theory for null submanifolds. For this problem K. L. Duggal and A. Bejancu were introduced a new method and presented a book about null submanifolds.

In 1985, on a semi-Riemannian manifold M^{2n+1} , S. Kaneyuki and M. Konzai introduced a structure which is called the almost paracontact structure and then they characterized the almost paracomplex structure on $M^{2n+1} \times \mathbb{R}$. Recently, S. Zamkovoy studied paracontact metric manifolds and some subclasses which are known para-Sasakian manifolds.

In the present paper we study the lightlike submanifolds of almost paracontact metric manifolds and obtain many characterizations about these submanifolds. We examine radical transversal lightlike submanifolds and totally paracontact umbilical radical transversal lightlike submanifolds of para-Sasakian manifold. Also, we give some examples of these types submanifolds.

Key Words: Para-Sasakian Manifolds, Null Submanifolds.

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T2 - Objects in the Categories of Cauchy Spaces

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ABSTRACT

There are various generalization of the usual T_2 (Hausdorff) axiom of topology to an arbitrary topological category defined in [1]. In this paper, an explicit characterizations of the generalized separation properties $PreT_2$ and T_2 ([1, 2, 3]) are given in the topological category of Cauchy spaces [4]. Moreover, specific relationships that arise among the various generalized separation properties T_i , $i = 0, 1$, $PreT_2$ and T_2 are examined in this category [4].

Key Words: Topological category, Cauchy space, Cauchy map, separation, connectedness, compactness.

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MATHEMATICS EDUCATION

A Case Study on Mathematics Fears of Secondary School Students

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ABSTRACT

Fear is defined as feeling anxiety in the face of danger or idea of danger (TDK, 2016). A person can be afraid of any situation, object or phenomenon that he/she thinks that it is dangerous or a threat. Students have various anxieties because they worry about their courses, schools and social environment so sometimes these anxieties turn into fear. Mathematics is one of the fear factors for students. According to Ufuktepe (2009) mathematics fear is stress, tense of a person or the interruption of the thinking process while dealing with numbers / figures, thinking about real life problems. Richardson and Suinn (1972) described this fear as the tenseness and anxiety obstructing the manipulation of numbers and solution of mathematical problems. Since mathematics fear is a factor affecting students' mathematics achievement (Keklikçi ve Yılmaz, 2013), in terms of school life it is important to determine sources, reasons of fears and solutions to these fears. In this context, this research aims to investigate secondary school students' mathematics fears, reasons of these fears and students' solutions to their fears. Research has been shaped by case study of qualitative research methods. The participants of this research is 50 secondary school students who are studying in primary education institutions located in central district of a province in the South-eastern Anatolia Region. In the selection of the students' criterion based sampling method has been used. The basic criterion of this research is secondary school student. Two-part open ended questions, prepared by researchers, have been used as the data collection tool. Questions about students' demographic characteristics are located in the first part. And the second part consists of four open ended questions that aimed at revealing students' simulations of fears, reasons of these fears and students' solutions to their fears. In order to determine concepts of fear and relations between these concepts content analysis method has been used. According to the data obtained from the research results, it has been determined that students simulate their fear of mathematics as concrete forms like spider, hyenas, father and abstractions like nightmares, demon and death. According to the coding on the reasons of students' mathematics fears, the results of the analysis shows that these fears are divided into two themes, including environmental and individual. Furthermore, the impacts of fears on school

achievement are grouped into three themes as positive, negative and ineffective. Finally students' solutions to these fears are determined as self-oriented reasons like not getting excited, self-belief , having self-confidence; extraverted reasons like game based teaching, thoughtful teachers, abolition of course and having no solution.

Key Words: Mathematics education, mathematics fear, anxiety.

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A Study upon the Reasoning Abilities of 5th Graders

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ABSTRACT

Mathematical reasoning consists of creating mathematical estimations, developing and evaluating mathematical arguments. It also includes abilities of presenting mathematical information in various ways (NCTM, 1989). Besides, the mathematical reasoning abilities of the children who belong to the primary education level are stated by NCTM.

The aim of this study is to search the reasoning abilities of 5th graders which can be considered as one of the vital skills and the numeracy skills. The research of this study was conducted in Yıldız Middle School located in Dereli, Giresun within the first semester of 2015-2016 school year. 26 5th graders were included in this research. With the participation of these students 6 groups were created which consisted 5 students in four groups and 4 students in two groups. It is known that different types of ideas emerge in heterogeneous groups. From this viewpoint these groups were formed according to their latest mathematical grades so that they can show heterogeneous characteristics and the views of their mathematics teacher for a year. This study is a case study from the qualitative research patterns. In this study four multiple-choice questions were elected from Problem Solving Test developed by Özsoy (2007) and they were used as open ended problems. For these four open ended questions each group was given one-hour lesson in order to solve these questions and it is applied by researchers. The students were expected to explain their solutions, thought and their answers related to the process for each of the questions.

The acquired information was analyzed with the “4-Point Rating Scale” developed by Marzano (2000) and provided validity and reliability. Every problem was evaluated by its own step. This evaluation was conducted by two researchers and the percentage of agreement of each step was calculated. The acquired finds show that students have difficulty in reasoning substeps like “Deciding the accuracy of solution way, Solving nonroutine problems, Developing logical discussion related to the solution, Generalizing, Determining and Adopting the suitable reasoning” Marzano (2000).

Key words: Mathematic education, Numeracy, Reasoning.

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An Analysis of the Daily Life Examples of the Mathematics Teaching Preservice Teachers about the Function Types

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ABSTRACT

During pupilage, one of the most common question that the students ask is “What good will this subject bring to us in our daily lives?” (Özsoy and Yüksel, 2010). It is important to relate the subjects to the daily life in terms of answering this question, overcoming memorizing, catching the attention of the student (Alacacı ve Karakoç, 2012). On the other hand, one of the basic aims of the mathematics education is to relate the learnt mathematical concepts to the daily life and other disciplines according to Mathematics Teaching Curriculum of Ministry of National Education (Milli Eğitim Bakanlığı [MEB], 2013). Within this scope, it is important that the teacher do this relating. On the other hand, using analogy which is defined as explaining a fact which has a teaching method and feels strange with similar and known facts in mathematics education is important for the teachers to be well informed about the daily life examples. In the faculty of education which is a teacher-training institution, the pre-service teachers should be well equipped to maintain these aims with this in the education process they are through. From this point forth, the aim of this study is to analyze the daily life examples of the mathematics education pre-service teachers about the function types. Qualitative research models are used in the study. The participants of the study consist of 42 undergrads who get educated in the department of elementary mathematics education of a state university. In the form which is used as a data collection tool, some function model is given by using set model after describing function and the pre-service teachers are asked to write any daily life examples which describes these function next to them. The functions types which are asked to be exemplified here are one-to-one function, injective function, constant function, surjective function and identity function. The data is analyzed with content analysis. The finding of the study will be shared inclusively during the presentation.

Key Words: Functions, Function Types, Analogy.

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An Analysis of the Middle School Mathematics Curriculum in Terms of Metacognitive Control Skills

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ABSTRACT

Metacognition means an individual's awareness of his own thinking processes and his ability to control these processes (Flavell 1979). At the same time, metacognition is used as a term to cover self-awareness of students in the cognitive processes such as planning the solution of a mathematical problem, following up, and evaluating (Panaoura and Philippou, 2007). One of the components of metacognition, metacognitive control, is described as the skill of the effective use of metacognitive knowledge (Özsoy, 2011), and using this skill to control and regulate cognitive processes (Schraw and Dennison, 1994). Consequently, the metacognitive activities constituting metacognitive control can be considered as metacognitive skills (Deseote, 2001; Lucangeli and Cornaldi, 1997). Related literature focused on four main metacognitive skills as prediction, planning, monitoring, and evaluation (Deseote, 2001; Lucangeli and Cornaldi 1997). To gain a better understanding of successful mathematical performance, metacognition seems to be important (Lucangeli and Cornaldi 1997).

Mathematics curriculum has been revised based on the constructivist learning approach in order to support individuals to become the learners of how to learn. The first applications of the new programs under the "4+4+4" schedule implemented in 2012 has been started in 2013-2014 academic years (Baykul, 2012). Several aspects of this new schedule, such as content, structure, and evaluation methods, have been subjects to several studies. However, there has been no research analyzing the content and gains of the schedule from the perspective of metacognitive control skills. Furthermore, development of students' higher level thinking skills to be used for their problem solving processes in *Middle School Mathematics Curriculum (5,6,7 and 8th Grades)* in Turkey is expected [Ministry of Education's (MoE), 2013].

Accordingly, analyzing the programs based on metacognitive control skills can enrich the existing literature by providing a different and new perspective about the content of the program.

In this context, the purpose of this research was to investigate the *Middle School Mathematics Curriculum (5,6,7 and 8thGrades) (MoE, 2013)* in terms of metacognitive control skills. Qualitative method was used in this study and document review method, one of the qualitative research techniques, was used to obtain qualitative data. The qualitative data that obtained from the *Middle School Mathematics Curriculum (5,6,7 and 8thGrades)* was analyzed descriptively.

In this study, frequency and percentage distributon of metacognitive control skills, which were determined by the learning area based on grade levels and gains from sub-learning area were displayed. Also, direct quotes have been shared to reflect the expressions about the metacognitive control skill gains. The analyses suggest that the most common metacognitive control skill are planning in the fifth grade, planning and monitoring in the sixth grade, planning and monitoring in the seventh grade, and planning in the eighth grade in the order of frequency. For all grades, the gains of prediction skills are at the lowest level. In light of the findings of this study, the researchers have developed suggestions.

Key Words: Mathematics education, program evaluation, mathematics curriculum

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An Assessment of the Studies Based on Computer Assisted Learning in Mathematics Education in Turkey

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ABSTRACT

When the literature is searched, only few studies that were systematically investigating the topic were found (Açıkgül and Aslaner, 2014; Demir, 2013; Liao, 2007). Therefore, holistic investigation of CAME is important as it will guide the future studies in this field. This study by taking these explanations into consideration is aimed at revealing a general outlook and inclinations of the theses in CAME that are registered in YOK database in Turkey by an in-depth analysis. To realise this aim, masters and PhD thesis were analysed based on their distributions of the university and graduate school and the year they were completed, their type, aimed audience, research approach and model, data collection instruments and research topic.

In this study, document analysis was used to investigate CAME studies. “Computer Assisted Learning”, “Computer Assisted Teaching”, “Cabri”, “Geogebra”, “Dynamic Mathematics” and “Dynamic Geometry” were searched in YOK national thesis data base to identify the thesis that could be included in the study. because often new theses are uploaded into the system, on 24.02.2016 YOK database was searched for the last time. At the end of the searching, 99 studies, with full text that were completed between 2002 and 2016, were included in the study. The theses were coded according to the form developed by the researchers and analysed with SPSS 21.0. Frequencies and percentages were used in the descriptive analysis.

Most of the theses prepared in the CAME area were master theses. 51,5 % of the theses were completed in 31 different universities at the graduate schools of education. Gazi and Marmara universities were the first two universities in the number of theses in CAME with the higher preference of audience in primary education. Most of the theses (61,6 %) used quantitative approach and experimental model (f:79). In most of the theses (f:72), SPSS was used to analyse the data.

Key Words: computer aided instruction, mathematics education, computer-aided mathematics.

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An Examination Of Mathematical Thinking Skill Levels Of The Prospective Mathematics Teachers According To The Solo Taxonomy

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ABSTRACT

The importance of mathematical thinking for mathematics education gradually increases. Thus, it is important to determine mathematical thinking skill levels of the teachers and prospective teachers, who will educate students, and take measures for improving these skill levels. The purpose of this study is to examine mathematical thinking skill levels of the prospective mathematics teachers, and to determine whether these levels vary by the grades prospective teachers attend. The study was carried out with 146 prospective teachers attending the Department of Mathematics Education at a state university in the 2013-2014 fall semester. The study was conducted by means of descriptive research method, which is a non-experimental research design. Open-ended questions regarding mathematical thinking skills were administered to prospective students for data collection by taking into consideration the definition of mathematical thinking. Each one of the administered questions was analyzed based on the SOLO (Structure of Observed Learning Outcomes) taxonomy. The research result shows that mathematical thinking skills of the prospective teachers are at uni-structural level according to the SOLO taxonomy. In addition, it is seen that the 2nd and 3rd grade prospective teachers have higher mathematical thinking skill scores in comparison to the 1st and 4th grade prospective teachers.

Key Words: Mathematics education; Mathematical thinking; Prospective mathematics teachers; SOLO taxonomy

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An Investigation of the Mathematics Problem Solving Strategies Developed by the Gifted and Non-Gifted Students

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ABSTRACT

Problem solving has a vital role in mathematics education and developing problem solving skills has an important place in students' mathematics learning at all levels (Schoenfeld, 1985). When students solve mathematics problems, they can use different strategies such as making a list, using guess-and-check, drawing a diagram, finding a correlation, using variables, working backwards and making a table (Altun, 2002; Dhillon, 1998; Hatfield, 1997; İsrail, 2003). The strategies may vary based on both the individual's way of thinking and the structure of the problem. Developing problem-solving skills may enable students to develop appropriate strategies for different problems and facilitates making sense of mathematical information.

This study was conducted to determine problem-solving strategies of fourth-grade students who have been identified as gifted and other fourth-grade students and to examine the differences in these two groups in terms of their problem solving strategies.

This study was carried out with the participation of 21 students (9 of which are gifted) from a Southeastern city and 71 students (41 of which are gifted) from a Central city in Turkey. "Mathematics Achievement Test (MAT)" developed by the researchers was used as the data collection tool. The researchers implemented MAT in one lesson period (40 minutes). The data were analyzed using descriptive analysis and content analysis techniques. The strategies used by the students have been categorized as writing mathematics sentences, using guess-and-check, using logical reasoning, drawing shapes and diagrams, and others. As a result, it was determined that both group of students mostly used logical reasoning strategy. In addition, the

results showed that both group of students developed similar strategies when they solve mathematics problems. More detailed studies should be conducted in order to investigate the differences in problem solving strategies of gifted students and other students.

Key Words: Gifted Students, Mathematics Education, Problem Solving, Strategy Development

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Analysis of The Geometry and Mathematics-Oriented Self—Efficacy Beliefs of Secondary School Students in Terms of Three Variables

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ABSTRACT

The purpose of this study is to examine secondary school students' self-efficacy beliefs about geometry and mathematics by the variables of gender, grade level, and mathematics report card mark. The survey method was used in this study with the adoption of quantitative research approach. The study was conducted with a total of 321 students including 89 sixth-grade students, 132 seventh-grade students, and 100 eighth-grade students. The data of the study were collected using the "*Self-Efficacy Belief Scale Towards Geometry*" developed by Cantürk-Günhan and Başer (2007) and the "*Mathematics Self-Efficacy Perception Scale*" developed by Umay (2001).

The data obtained from the study were analyzed using SPSS 20 Software. In this context, the Independent t-test, One-WayANOVA, and Correlation analysis were performed. The Scheffe test was applied to determine from which groups the differences originated in the cases where the analysis of variance was found to be significant.

As a result of the independent t-test was determined that there was no significant difference between students' geometry and mathematics self-efficacy beliefs according to the variable of gender. On the other hand, as a result of the One Way Anova test performed, it was observed that there was a significant difference between students' geometry and mathematics self-efficacy beliefs according to the variable of grade level. In this context, it was observed that 6th-grade students' self-efficacy beliefs towards both mathematics and geometry were higher than those of

7th and 8th-grade students and that students' self-efficacy beliefs decreased as the grade level increased.

Similarly, it was determined that there was a significant difference between students' mathematics report card marks and mathematics and geometry self-efficacy beliefs. It was found out that there was a significant difference between students with mathematics report card marks of 1, 2, 3 and 4 and students with mathematics report card marks of 5 in terms of mathematics self-efficacy beliefs. Likewise, it was determined that there was a significant difference between students with mathematics report card marks of students with in terms of geometry self-efficacy beliefs. It was observed that this difference was in favor of students with mathematics report card mark of 5 in both scales, and therefore, students' self-efficacy beliefs towards mathematics and geometry increased as the mathematics report card mark increased.

In addition, as a result of the correlation analysis performed, it was determined that there was a significant positive relationship between students' mathematics self-efficacy beliefs and geometry self-efficacy beliefs.

Key Words: Self-efficacy, Gender, Class level, Card mark

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College Students' Collaborative Problem Solving Skills: An Investigation from the Cognitive Domain

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ABSTRACT

Collaborative problem solving has become increasingly popular in mathematics education in recent years due to its important role in collaborative learning [1]. Students have an opportunity to enhance their mathematical knowledge through active engagement while they work on mathematical tasks in pairs [2],[3]. However, previous research on mathematical problem solving indicates that grouping students to work together does not automatically create collaboration [4],[1]. As a multifaceted, complex and coordinated process, collaborative problem solving requires different social and cognitive skills [5]. The purpose of this study was to identify college students' peer collaborative problem solving skills from the cognitive domain. We conducted task-based interviews with ten students who worked in pairs to solve geometric locus problems. Findings revealed that students were able to link the pieces of information that were given in the task or expressed in a shared language. They also completed the subtasks that they created together. However, they had problems in identifying connections between multiple pieces of information that were necessary to reason mathematically and testing alternative hypotheses to resolve the problems. We believe that sharing college students' interaction with each other through the collaborative problem solving process will contribute to the research in mathematics education by helping to understand the nature of collaborative learning.

Key Words: Teacher education, collaborative learning, collaborative problem solving

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Conceptual Analysis of Addition and Subtraction Operation Problems with Fractions Posed by Fifth Grade Students

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ABSTRACT

Cognitive perception of fractions and operations in fractions has a significant importance for learning especially advanced level subjects such as algebra and for the development of problem solving skills (Kar and Işık, 2014). Nonetheless, fractions and operations in fractions have been one of the most challenging subjects for students (Yim, 2010).

Problem posing is an important aspect of mathematics education and learning (Işık and Kar, 2015). The problem posing step, which is the fifth step according to Polya's problem solving process (Gonzales, 1998), is included in the middle school mathematics curriculum for fifth to eighth grades in Turkey. Mathematics curriculum concerns about the problem posing learning about the addition and subtraction operation with fractions for fifth grade students (Ministry of Education's, 2013).

However, the literature research suggests that problem posing in fractions and operations with fractions have been mostly carried out with teachers or prospective teachers. Although there is a limited number of research concerning the challenges experienced by sixth and seventh grades about the addition and subtraction operation with fractions with daily life situations through problem posing, there has been no study focused on the fifth grade students. Thus, the goal is to help students develop their skills to get prepared for advanced subjects in the mathematics curriculum by first strengthening their problem posing skills on fractions as much as possible. Furthermore, the analysis of the problems posed by fifth grade students would enable us to understand the cognitive needs of students in a comprehensive way. In this context, the purpose of the study was to conceptually analyze and to determine potential errors, experienced by fifth grade students related to the problems posed by them about the addition and subtraction operation with fractions.

The research sample were composed of 132 fifth grade students, randomly selected from a middle school in Elazığ in the spring term of 2015-2016 academic year. Descriptive research method which is an appropriate tool for quantitative research approach was used in this study. The Problem Posing Worksheets composed of six items about the addition and subtraction operation with fractions was used as a data collection tool in the research. The Problem Posing Worksheets consist of six problems posing item towards symbolic representations within semi-structured problem posing situations. The answers given by the students were classified in accordance with the categories of problem, not a problem and empty. Then, the errors in the answers within the category of problem were analyzed. Frequency and percentage values of the data are presented in tables.

The results of this study showed that students had conceptual deficiencies in the process of problem posing regarding addition and subtraction operation with fractions. Eight errors were found out in the problems posed by the students. The most significant difficulty was observed in posing problems including subtraction of two mixed fraction to produce a mixed fraction; on the contrary the least significant one was experienced in posing problems including addition of two simple fractions to produce a simple fraction.

Key Words: Problem posing, fractions, mathematics education

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Development of Math Achievement Questionnaire to Investigate the Characteristics of Low Achievers

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ABSTRACT

It is not easy to allow every individual to acquire sufficient mathematics competence for social life. According to the results of standard achievement tests in the United States, it is estimated that 5-8 % of children have some form of learning disability in mathematics (Geary & Hoard, 2001). In the Programme for International Student Assessment (PISA), students perform below the 25th percentile in their countries are named as low achievers. According to the PISA 2012 math achievement results, 42 % of the students from Turkey performed below level 2 which includes fundamental competencies. Top performers in mathematics (level 5 or 6) are 5,9 % of the students. Thus, there should be more studies focusing on low achievers, particularly in Turkey.

There are numerous factors affecting students' achievement and it is found that only 35 % percent of the observed variation is related to students' intelligence. There are many factors, particularly socio-economic and socio-psychological factors, affecting students' achievement. In the literature there are many studies having investigated the effect of various factors and teaching methods on students' math achievement (Acar, 2005; Coşkun, 2007; Konak, 2009; Özturan Sağırlı, 2010; Dikmen, 2015). But the researchers encountered a limited number of studies in the literature directly focusing on low achievers and their characteristics (Savaş et. Al., 2010; Ölçüoğlu, 2015).

This study is a part of a broader study about low achievers. In this study, it is aimed to develop a data gathering tool to investigate the characteristics of low achievers and the factors affecting their math achievement in a negative way. Thus, the researchers developed items for "math achievement questionnaire". For the questionnaire 80 items had been chosen from an item pool that contains 158 items and the reliability of these items were discussed by two experts. There was 89,60 %

agreement between the experts. The researchers edited the questionnaire according to the experts' opinions. They are following the questionnaire development process the procedures of which Büyüköztürk defined in his study as (1) defining the problem, (2) writing the items, (3) asking experts' opinions about the questionnaire and (4) pilot study. The questionnaire has three basic parts: a) Introduction and guideline, b) The items that require demographic information, c) The items related to factors associated with characteristics of the students, students' teachers and students' families.

After the pilot implementation is administered to 350 secondary school students in a state school in İstanbul, the researchers are going to perform an exploratory and confirmatory analysis for validity and reliability checks. At the end of the study, it is expected to develop a reliable and valid questionnaire to focus on low achievers and the researchers will provide a prevalence of low achievers' characteristics in the Eyup district of İstanbul. The findings are also expected to provide important contribution to the literature about low achievers.

Key Words: Low Achiever, Math Achievement, Questionnaire

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Elementary School Mathematics Curricula (Grades 1-5) with respect to Physical Instructional Objects

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ABSTRACT

Most of the people agree that mathematics is very important for everybody. “Every child can learn mathematics” as stated in the elementary school mathematics curricula of 2009 approved by the Republic of Turkey Ministry of National Education (MoNE)[1,p.7]. Unfortunately, the results of several international exams verify that a lot of students in different countries including Turkey have difficulties in mathematics[2]. The findings related to us have been supported with mathematics scores in our national exam results. There is a great need to determine the methods on how to teach and learn mathematics effectively. Thus, there are many research studies on investigating the effects of different teaching methods on students’ mathematics achievement. One of them is related to using the concrete materials in teaching and learning mathematics [3,4].

The use of concrete instructional materials in teaching and learning mathematics has very long history as “Manipulative materials were included in the activity curricula of the 1930s. The mid-1960s began another period of emphasis on using concrete objects and pictorial representations in mathematics instruction.”[5, p.498]. Moreover, according to Bruner and Piaget, young children can have some acquisitions from exploring concepts by using concrete materials[6,7]. “In Germany, the main advocate for the use of concrete models and dynamic instruments was Klein. In many ways, Monge in France and Klein in Germany set the standards for how mathematics was taught in Europe, Northern America and the Far East in the nineteenth and early twentieth centuries(Klein& Riecke,1904;see also

Schubring,1989)”[8,p.21]. We analysed three mathematics textbooks in 1920s and 1930s. There are some tasks to make conjectures such as sum of measure of interior angles of triangles by cutting paper and area of circle by dividing a paper into small sectors and the relationship between the volumes of square prism and square pyramid. There are also instructional materials such as pantograph and clinometer. We also have investigated a book on instructional materials in 1960s. Hence, the purpose of this study is to investigate our elementary school mathematics curricula (grades 1-5) from 1948 through 2015 with respect to utilization of the physical instructional objects.

The present study is a qualitative study. The document analysis technique is used to analyse the data obtained from elementary school mathematics curricula documents approved by the MoNE in 1948, 1968, 1983, 1998, 2009, 2013, and 2015. They are coded by two researchers separately. These codes are compared and discussed to become consensus on them.

The results of the study reveal that there are many explanations about how to use physical instructional objects in teaching and learning numbers, geometry and measurement in all mathematics curricula. They also are used in statistics and probability in mathematics curricula of 2009. They can be grouped as natural objects such as our body and stone; the productions for real life such as balls and clothes; and the objects produced by learners, teachers or companies such as counting sticks and geoboard. They have been utilized to teach and learn mathematics meaningfully, especially by using discovery learning method and explaining the conjectures.

Key Words: Mathematics curriculum, elementary school, instructional material.

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Elementary School Mathematics Teachers' Interpretation of Learning Outcomes*

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ABSTRACT

Effective teaching requires being skillful both inside and outside of the classroom. In other words, effective teaching moderately depends on systematic and intently demonstration of skills outside of the classroom [1]. Accordingly, planning and regulation of teaching mathematics is one of the indispensable teacher competencies. This competence must be demonstrated in the direction of curriculum [2]. Curriculum is only a plan at the beginning and it is limited by itself [3]. In this framework, as a necessary skill for planning teaching activities effectively, teachers should understand learning outcomes (LOs) which are important constituents of curriculum. Along the same line, teachers should interpret outcomes to make students acquire them as they were designed in the curriculum. Above mentioned skill is called as *learning outcome literacy* [4].

The purpose of this study is to investigate teachers' interpreting LOs which is one of the main components of learning outcome literacy. Interpreting LOs incorporates four sub-components [2]:

- i. revealing the aims of LO
- ii. analyzing knowledge and skills in the LO to make students acquire it
- iii. knowledge about outcomes' instructional relation with other learning domains and other outcomes in the curriculum
- iv. evaluating whether** LO is acquired by students or not

It is a case study which is based on the qualitative design. Participants were five elementary mathematics teachers working at different elementary schools in Turkey.

*This study was developed from the first author's M.Sc. research at Gazi University, Institute of Educational Sciences, Department of Primary Science and Mathematics Teaching; the other author supervised.

The data were collected through semi-structured interviews and observations which were based on ten learning outcomes selected from Algebra and Number Learning Domain in Mathematics Curriculum [5].

The results show that teachers' interpretation of LOs- especially the subcomponent of analyzing knowledge and skills to make students acquire it- were not sufficient as expected. Additionally, although teachers were able to reveal aims of LO, they were not sufficient enough to interpret outcome to make students acquire it. Teachers' knowledge about instructional relation between LOs and curriculum influences their revealing aims of and analyzing knowledge and skills of LOs. Considering the interrelation between its components, LOs literacy should be developed with all sub-components to make effective teaching of LOs possible.

Key Words: Teacher Competence, Learning Outcome Literacy, Mathematics Education

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Evaluating Difficulties on the Basic Terms in Geometry Through Didactic Triangle at Secondary School Level

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ABSTRACT

The studies conducted on the education of geometry have pointed out that students are suffering to comprehend the basic geometrical concepts (Cunningham and Roberts, 2010). Especially the undefined basic terms of the geometry like point, line, and plane affects the perception of the other geometrical subjects, adversely (Marjanović, 2007). It has been observed that students have problems to consider the point as non-dimensional, to comprehend the infinity of line, ray and plane and to build a relationship between geometrical terms(Doyuran, 2014). In this study, the basic geometrical terms will be investigated in the light of didactical transposition theory. The message given by the textbook, the way that the teachers receive and deliver the message and the perception of the message by the students will be evaluated. Therefore, it is aimed to reveal the reasons and the sources of the problems by focusing the internal and external didactic transposition processes.

The qualitative case study design was used to evaluate the transition between the knowledge to be taught and the knowledge to be learned in this study. The participants were seven secondary students at fifth grade who had difficulty in basic geometric terms and their mathematics teacher. The study lasted five-weeks and during this process, usage of the textbook by the teacher and interpretation of the knowledge to be taught by students were followed. The observation and semi-structured interview techniques were utilized during the data acquisition process. The interview data were analysed with content analysis method and presented using direct quotation.

The initial findings of the study revealed that the message given by textbook and the interpreting of the message by the teacher is different from each other.

While, the terms like line, line segment, and ray are explained based on the point termin the textbook,the teacher preferred more intuitive definitions. According to the teacher, the textbook definitions of line and ray concepts are hard to understand even for mathematicians. The interview findings of students showed that the teacher's over usage of the daily life examples, when the geometrical concepts are explained, may cause the misconception. It is also discussed that some examples on the textbook (the similarity between train tracks and line) can lead to misconceptions.The sources of the learning difficulties, which have been identified in this study,can be tested in future research on larger samples.

Key Words: Geometry, difficulties, didactic triangle.

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Evaluating the Mathematical Reasoning Abilities of 5 Graders in Secondary School

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ABSTRACT

The purpose of this research is to evaluate the mathematical reasoning abilities of 5th graders. Quantitative research method was used for this research. The 5th graders of Samsun Salıpazarı İmam Hatip Secondary School formed the treatment group of the research which was performed in the first semester of 2015-2016 academic year. As the data collection tool, 6 questions were used which had been selected from the 2011 TIMSS questions. Students were given one hour to answer the questions. The collected data was analysed with SPSS 22.0. The analysed data was rated by "4-Point Rating Scale" developed by Marzano(2000).

The criterion which was used to find the levels of students' reasoning abilities was calculated for each phase. Score intervals were determined by dividing the difference between maximum point and minimum point for each phase to the numbers of levels. The levels were specified as "beginner", "pre-intermediate", "intermediate", "upper intermediate" and "advanced".

As a result of the research, it is stated that students are intermediate in the phase of "Choosing and Using the Appropriate Reasoning", intermediate in the phase of "Developing Logical Arguments Suitable for the Solving", intermediate in the phase of "Generalising", intermediate in the phase of "Deciding the Precision of the Solving Way/Result", pre-intermediate in the phase of "Solving the No routine Problems". It is evaluated that students are intermediate in mathematical reasoning skills in general. Also, the effect of the gender on mathematical reasoning ability was analysed. As a result of the analyses, it is come through that gender doesn't have much effect on mathematical reasoning ability.

Key Words: Mathematical Reasoning, Evaluation of Reasoning

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Examination of the Relation Between Computer Literacy and Mathematics Literacy among the Secondary School Students

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ABSTRACT

With respect to literacy, new definitions are being made every day in the literature. Every new interpretation leads to the idea that it can change depending on the current environment, tools used and/or purpose intended and there may be different types of literacy including computer literacy, media literacy and visual literacy (Reinking, McKenna, Labbo and Kieffer, 1997; Tuman, 1994). This is also valid for computer literacy and mathematics literacy. Therefore, studies on computer literacy and mathematics literacy are always needed. In the present study, the objective was to determine the computer and mathematics literacy levels of secondary school 6th, 7th and 8th grade students and to reveal the relation between these levels. In this scope, sample consisted of province, district and village schools as well as private schools. The relation among the computer and mathematics literacy levels of the students selected from various regions was determined and the difference among the levels of those schools was detected as well. Since the relation between computer literacy and mathematics literacy among secondary school students was determined, the method of the study was relational screening model. Relational (correlative) studies aim at determining whether there are relations among two or more variables (Karasar, 2009). Population of the research was composed of the 6th, 7th and 8th grade students of a secondary school found in Elazığ while the sample consisted of 130 6th grade, 135 7th grade and 128 8th grade students. In the study, "Mathematics Literacy Scale" and "Computer Literacy Scale" which were developed by the researcher and tested for validity and reliability were used. The data obtained were analyzed by means of specific statistical techniques. After the findings were analyzed, results were discussed in line with the literature and necessary recommendations were made.

Key Words: Computer Literacy, Mathematics Literacy, Secondary School Students

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Identifying the Situations of the 5th Grade Students within the 4+4+4 (Years of) Education System in the Scope of the Views of Math Teachers

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ABSTRACT

Change is one of the leading phenomena that effect the man of the 21th century. The doubling of the knowledge in each four years, and the transforming the societies from the era of industry to information give a more explicit view about the speed and effect of the change (Özer, 2011). Today, new improvements occur continuously in the fields of economy, social and cultural life, political and social order, and technological structure (Erdoğan, 2012). The paradigmatic developments that start with the midst, but intensify through the end of the twentieth century and that still go on force the education system to change, and they seem to continue forcing the system (Özden, 2010).

As the results of the decisions taken at the 18th National Education Council and the causes which can partly be the measures of the government, The Law With the Number 6287 and Date 30/03/2012, Including Replacements in the Laws of Primary Schools and Education, and Some Other Legislations, was adopted by the Turkish National Assembly. With this law there has been a fundamental change especially for the 5th grades. According to the legislation, the 5th grades were added to the secondary education, and they weren't assigned class teachers any more. Therefore this caused the 5th graders to face with the teachers of various branches one year earlier. Regarding this, the problem sentence of our study is determined as 'what are the opinions of the math teachers about the positive and negative aspects of the integrating of 5th grades to the secondary education within the 4+4+4 education system'.

The aim of the research is to reveal and evaluate the opinions of math teachers about the integrating of 5th grades to the secondary education within the 4+4+4 compulsory education system. It is a subject worth researching when the opinions of math teachers about the law no.6287, the problems they face, the system and its area of influence considered. This is a qualitative research that identifies the results of integrating the 5th graders to secondary education within the 4+4+4 education system. Of the qualitative research designs, the phenomenological design was used for this research. The phenomenal deign

focuses on the cases which we are aware of, but not know in depth and in detail (Çepni, 2010). The sampling of the study consists of 8 math teachers working in Tokat Province in 2015-2016 educational year. The semi structured interview technique, preparation and the validity process of which was done by the researchers, was used as the data gathering tool. The NVIVO program was used in analysing the interviews. The positive and negative opinions of math teachers about the 4+4+4 education system will be discussed by using the data gathered from the study, and suggestions will be proposed about our education system and the education policies of our country with regard to the findings of our study.

Key Words: 4+4+4 education system, math teachers, 5th grade students

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Investigating Mathematical Modeling Processes Of Prospective Science and Mathematics Teachers

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ABSTRACT

In general we can define mathematical modelling as cyclical process in which real life situations and its relations are expressed and analysed by using mathematical methods (Verschaffel & De Corte, 1997). In recent times, It has been emphasized that mathematics curriculums should involve mathematical modelling as an educational approach from elementary levels to higher education (Erbaş et. al., 2014). However, teachers have not used modelling applications enough in the classroom. Blum and Ferri (2009) states that the main reason of the gap between educational goals about modelling and classroom applications is that the modelling is difficult for teachers and students.

This research is a case study which aims to determine the modelling processes of prospective science and elementary mathematics teachers. The researchers also made some comparisons between prospective science teachers and elementary mathematics teachers. Data was collected from 8 prospective science and mathematics teachers via observations, focus group interviews and document analysis. The researchers used model eliciting activities as a data gathering tool and they will share the results obtained from the Ferris Wheel activity in the presentation. The qualitative data was analysed by descriptive analysis according to the stages Strauss and Corbin (1990) defined. For the coding process, the researchers considered the phases in the modelling cycle of Blum and Leiss: (1) understanding the task, (2) simplifying/structuring, (3) mathematizing, (4) working mathematically, (5) interpretation, (6) validation, (7) presenting (Ferri, 2006).

The results showed that prospective teachers tried to solve the problem according to the some phases in the modelling cycle of Blum and Leiss, but they did not demonstrate any effort for the interpretation of results. Where prospective science

teachers' modelling process did not feature any phase about validation, prospective mathematics teachers tried to make some attempts for validation of their result. Only two prospective mathematics teachers validated their results.

Key Words: Modelling, Mathematical Modelling, Mathematical Modelling Processes

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Investigating of Primary School Mathematics Teacher Candidates' Computational Estimation Skills

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ABSTRACT

The aim of this study is to investigate primary school mathematics teacher candidates' computational estimation skills. For this purpose, primary school mathematics teacher candidates' computational skills was investigated based on class levels and gender. Furthermore, primary school mathematics teacher candidates' opinions about computational estimation, their conceptual knowledge about computational estimation and its strategies, and which computational estimation strategies they use in problems have been also studied.

A computational estimation skill test was prepared by the researcher to measure computational estimation skills of primary school mathematics teacher candidates. A pilot study was made by the researcher to measure validity and reliability of the test.

Totally 171 teacher candidates, 25 boys and 146 girls took part in the study. Computational estimation test were applied to the all participants. After the test, 13 of the participants were chosen for study group through purposeful sampling. Semi-structured interviews prepared and made by the researcher with this purposeful sampling to understand which computational estimation strategies they used while solving questions of computational estimation test.

The aim of the semi-structured interview was to learn about teacher candidates' opinions about computational estimation, their conceptual knowledge of computational estimation and its strategies. The questions in the interview form prepared by the researcher and asked for expert opinion.

Cronbach Alpha reliability coefficient of the computational estimation skill test was measured as 0.72. There were no meaningful difference each of between 1st,

2nd, and 3rd classes, however there was a difference between 4th classes with other classes on behalf of 4th classes. There was also a significant difference between girl and boy groups on behalf of boy groups. This finding is parallel to some research studies (Munakata, 2002; Tekinkır, 2008), however it was found in some other research studies that there was no meaningful differences between boys and girls about computational estimation skills (Boz, 2004; Aytekin, 2012). In semi-structured interviews, it was seen that teacher candidates with high points in computational estimation test had flexible thinking abilities while making estimation. This finding shows parallelism with literature (Bestgen, Reys, Rybolt & Wyatt, 1980; Reys, Rybolt, Bestgen & Wyatt, 1982; Levine 1982; Behr, Wachsmuth & Post, 1985; Reys, 1986; Reys, Reys, Penafiel, 1991). Teacher candidates' conceptual knowledge about computational estimation was observed as insufficient. Many of them did not know theoretically any strategy other than rounding strategy. However during the interviews, it was observed that each of the candidates used some of the existing estimation strategies.

Key Words: Estimation, Estimation Skill, Computational Estimation, Computational Estimation Strategies

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Investigation of Attitudes of Classroom Teacher Towards Mathematics Laboratory and Laboratory Practices

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ABSTRACT

In 2003, in an international study that was conducted in 46 countries to measure the success of 4th and 8th grade students by standard tests, it was seen that their average achievement was fewer than 50%. This can be considered sufficient to some educationists who claim that it is not a requirement for everybody to learn mathematics. Those who make this claim base their argument on Gardner's multiple intelligences theory (1993). In contrast to these ideas, Poisson (1781-1840) pointed out the importance of mathematics and told about two things which are worth living in life; discovering mathematics and teaching mathematics. Parallel views to this idea have now started to dominate. Therefore; mathematical education, should be given to every student, remains valid opinion. The reason for the continuation of dizzying technological development and the need for them is the need to use mathematics knowledge. Therefore, the basic mathematical knowledge should be given to our children.

Today, mathematics laboratory studies enable to students to understand the conceptual nature of learning and are used extensively all over the world. Most laboratory activities provide information recognition of knowledge about lesson and it is based on conceptual learning.

The aim of this study is determine the attitudes of candidates' primary school teachers of mathematics laboratory and its applications. The study sample consisted of 156 teachers. In the study data was conducted with survey model which consists of eight subscales 69-point Likert-type and the opinions of candidates primary school teachers collected through "a questionnaire composed of open-ended questions" and analyzed through the "content analysis "technique (t-test). Their attitude toward laboratory practices was generally positive (86%) However, the preparation of laboratory practices, guides, and attitudes towards the handling and physical conditions were significantly lower.

Key Words: mathematics, laboratory, material, attitude, achievement and retention

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Investigation of the Pre-Service Mathematics Teachers Ability of Constructing Appropriate Graphics to Real Life Situations

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ABSTRACT

Graphics which help organization, sharing and interpretation of numerous data and understanding of verbal expression easier by visualizing emerge in mathematics as well as many areas. This case requires all students to have adequate knowledge and skills related to graphics [1, 3]. One of the skills required to effectively use graphics by students is 'constructing a graph' [2, 4]. Also, to be gained constructing images in mathematics such as chart, table to students is among the targeted skills in middle school mathematics curriculum [5]. As some of the goals of the mathematics education are creating graphs, sense-making and making an association between daily life situations by students correctly, the quality of teachers in achieving those can be clearly seen to be quite significant [1]. In this regard, the purpose of this study is to investigate the ability of constructing appropriate graphics to verbal real life situations.

The present study was conducted with 36 junior pre-service teachers, studied in Elementary Mathematics Education Program in a state university. While choosing candidates, it was considered that they have taken Statistics required to be familiar with the basic knowledge about usage of graphics. An achievement test whose some parts of supported by literature was devised by the researchers as a data collection tool. The achievement test consists of seven open ended questions. In addition, the pre-service teachers were asked to explain their answers for the whole test with reasons. In data analysis, frequency of the answers given by the pre-service teachers for each question was examined and the most commonly given wrong answers were

specified. Also, the explanations of them were analyzed with content analysis a method of qualitative analysis order to obtain concepts and relations to clarify collected data. The results of the study indicated that pre-service teachers more often have difficulty in constructing appropriate graphics to verbal real life situations. They tend to draw the form of an increasing function under all conditions, to take incorrect reference point, to interpret variables inexactly.

Key Words: Graphing skill, constructing graph, pre-service mathematics teachers.

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Investigation of Students' Theoretical Awareness on Continuity, Differentiability and Integrability in Analysis Course

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ABSTRACT

Many students who can successfully use rules and formulas in the subject of Analysis course have difficulty in conceptualizing and linking with different contexts these operations. The main reasons of these difficulties are students' inappropriate and weak mental links between calculus concepts such as limit, continuity, derivative and integral. The aim of this study is to evaluate students' theoretical awareness on the concepts having key importance in the Analysis course. Within the scope of research, question sought to answer is that "What is the theoretical awareness of undergraduate students on the concept of continuity, differentiability and integrability?"

The case study which is one of the qualitative research design has been used to examine thoroughly the basic concepts in the Analysis course within the scope of their own life frame. The participants of the study are forty three freshman in the mathematics department of a state university. The participants consist of students attending Calculus I and II courses and the applications have been performed at the end of these courses. Concept maps have been used to as a data collection tool. A set of propositions has been presented for the relations between the concept of continuity, differentiability and integrability. The participants have been expected to transform these propositions into the concept maps at the end of the course. For example, two propositions are expressed for the concept of continuity and differentiability as follow: "(1) If f has a derivative at $x = c$, then f is continuous at $x = c$, (2) If f is continuous at $x = c$, then f has a derivative at $x = c$ ". While data has been evaluated in the concept maps, accuracy order, hierarchy of the propositions, examples given extra and other associations have been considered and graded.

The findings of study have showed that only half of the participants have mapped the propositions containing the relation of continuity and differentiability with a correct order. It has been observed that the participants have more difficulty in the proposition containing the relations of differentiability–integrability. The rate of participant misinterpreting the proposition between these two concepts is 60%. The proposition in which the students' experience the most complexity in the relation between derivative and integral is as below: "*If f is a Riemann integrable function on (a, b) , then f is differentiable on (a, b)* ". The majority of the participants (74%) have given answer "yes" to this proposition and have reached to a wrong conceptualism. The other findings and the factors to cause to misconceptualization will be shared in the conference presentation.

Key Words: Theoretical awareness, analysis course, concept map.

Mathematica Applications in Educational Technologies and Scientific Studies

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ABSTRACT

Knowledge is the most valuable treasure for humankind. This case increases the importance of access to knowledge and use knowledge effectively.

In educational field, the development in computer technology emerges as educational technologies. This subject is rapidly developing in our country as in the world. The use of this technology in parallel to this development have become necessary in training methods and techniques. There are many programs and mobile applications in the world. In our country, it can be accessed to these programs and applications with EBA (Education Information Network) [1]. Also, the development of educational technologies effects the developments in science and technology. It is clear that the maximum benefit can not be provided without interdisciplinary interaction.

With the slogan “The world's definitive system for modern technical computing” [2] Mathematica is a computer program that can support at the highest level both educational technologies and the studies of science and technology. The program has options for different usage; desktop, online and mobile. Mathematica has an advantage compared to similar programs. Mathematica is not used only a discipline, it is used for many field based on mathematic [3]. This situation also seem to provide a solution to problems in science and technology production.

Mathematica is quite widely used in educational Technologies and scientific studies in the world, unfortunately, it is not sufficiently recognized and used in our country. In this study we aim to introduce the program of Mathematica with model practises in our country and to encourage the using of the program.

Key Words: Mathematica, Educational Technology, Scientific Study

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Mathematics Teachers' Beliefs About Teaching as predictor of Their Approaches to Measurement and Assessment

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ABSTRACT

Teachers have certain beliefs about teaching which characterize their educational philosophy, teaching strategies, methods and techniques etc. (Gürbültürk & Şad, 2009). While traditional beliefs advocate teacher-based approach valuing rote learning, constructivist beliefs about teaching puts learners in the center of learning process who construct knowledge through experience guided by their teachers (Anderson, 1998; Arslan, 2007; Cansız Aktaş & Baki, 2013). Moreover, such a constructivist approach also advocates alternative, contemporary or complementary measurement and assessment approaches as against traditional ones (Gelbal & Kelecioğlu, 2007). Mathematics curriculum is one of the fields where considerable emphasis has been made on constructivist teaching beliefs and contemporary measurements and assessment approaches. The purpose of the present study is to test the predictive value of mathematic teachers' constructivist vs. traditional teacher beliefs in addition to several other variables (e.g., professional seniority, weekly teaching hours, gender etc.) on the alternative/contemporary vs. traditional measurement and assessment approaches they adopt. Designed as a quantitative correlational research (Fraenkel, Wallen & Hyun, 2012), present study was conducted on 174 mathematics teachers accessed online using Google forms application. Data were collected using the adapted versions of *Measurement and Evaluation Approaches Scale* originally developed by Şad and Göktaş (2013) and *Teachers Belief Survey* originally developed by Woolley, Benjamin & Woolley (2004) and adapted into Turkish by Gürbültürk and Şad (2009). The multiple linear regression analysis (using enter method) revealed that mathematics teachers' constructivist and traditional beliefs, weekly teaching hours, professional seniorities, and other properties defined as dummy variables, i.e. gender, faculty of graduation (education vs. other), type of school (state vs. private), and stage of school (middle vs. high) are altogether associated at a moderate level ($R=.588$, $R^2=.345$) with their alternative/contemporary measurement and assessment approaches ($F_{(8-165)}=10.877$; $p<0.05$). However, the analysis regarding the significance of regression coefficients suggested that among all eight variables only constructivist teacher beliefs was a significant predictor of alternative/contemporary assessment approaches adopted by math teachers ($\beta=.559$; $t=8.648$, $p<0.05$), with a partial correlation of .558. This suggests that the more math teachers have constructivist beliefs about teaching, the more alternative/contemporary assessment approaches they adopt. A subsequent analysis on the prediction of math teachers' traditional assessment approaches by the same eight variables revealed that again all variables were moderately ($R=.435$; $R^2=.189$) and significantly associated with math teachers' traditional assessment

approaches ($F_{(8-165)}=4.814$; $p<0,05$). However, the analysis regarding the significance of regression coefficients suggested that among these eight variables only traditional teacher beliefs ($\beta=.326$; $t=4.551$, $p<.05$) and constructivist teacher beliefs ($\beta=-.261$; $t=-3.629$, $p<.05$) were significant predictors of traditional assessment approaches adopted by math teachers. The moderate level of positive partial correlation ($r=.334$) indicated that an increase in math teachers' traditional teacher beliefs also gives rise in their traditional assessment approaches. Similarly rather low negative partial correlation ($r=-.272$) suggests that an increase in math teachers' constructivist beliefs causes a slight decrease in their traditional assessment approaches. In concurrence with the previous research, the results proved that both traditional and contemporary approaches of mathematics teachers to measurement and assessment are nourished from traditional and constructivist teacher beliefs, respectively.

Key Words: Teacher beliefs, constructivism, alternative vs. traditional measurement and assessment.

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On Refinement of the Archimedes' Method of Calculation of Areas

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ABSTRACT

One of the mathematical topic of seasonal lessons and meetings of the International Science-Education school of "Step by Step to Science" (SSS) in the Tbilisi State University aiming to connect pupils with scientists [1], with its original results will be presented.

It is well known that one of the biggest stimulation in the development of mathematical analysis was the problems related to calculation of surface areas. How can we calculate area of a region having a curved boundary? Mankind has made several trials to determine the area of such kind of regions exactly but of course in general case it was impossible without limit operation. It is enough just to remember the trials made to calculate the area enclosed by a circle. This process continued with finding more accurate value of π progressively. The most important step till the foundation of Analysis by Newton-Leibnitz was taken by Archimedes one of the greatest mathematicians of Antiquity. He developed a practical method to calculate exactly the area under the parabola. This represents ideological side of our presentation (see for example [2]).

We should admit that Archimedes' method represents a preface of definition of definite integral and in this field there are many steps covered by scientists. It is enough to remember determination of Riemann integral by Darboux with so called Darboux sums. However we remark, in a philosophical manner, Andrei Kolmogorov's handling of Archimedes' method. Kolmogorov considers limit of sequences and takes a similar manner to Archimedes' construction, where in the beginning he takes straight lines and computes the areas under these lines basically [3]. We should state that this example of Kolmogorov is a philosophical view to understand Riemann's

definite integral. In our lessons of together with SSS pupils, we got more close to the concept of integral. First we take, in the Archimedes example, approximate values as trapezoids consisting of chords and tangents. By using these regions the area under the parabola is determined more exactly. Remainder term is found as $O(1/n)$ in the Archimedes' case and $O(1/n^2)$ in our case. However this is not that we want to essentially indicate. Of course, this method of simultaneous use of chords and tangents is in fact a strong way of approximation to determine Riemann integral for functions having jump discontinuity. Additionally, it is indeed Gauss and trapezoidal rules that is well known in numerical analysis. With this example, intuitive existence of Riemann integral is shown by pupils. Thus, in our opinion, approximate calculation of area under a curve can be remarkably realised constructually and geometrically for pupils having ages between 12-16.

Here as a remark, area enclosed by a circle and a new method of approximate determination of perimeter of a circle depending on the method of Diana Vashakmadze's division of circle for arbitrary natural number will also shown [4].

Key Words: Integral concept, Archimedes method, approximate calculation of areas under a curve.

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On The Studies Related to The Mathematics Education of the Gifted Between 2005-2014*

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ABSTRACT

The purpose of this study was to analyze the studies about mathematics education of gifted students between 2004-2015 both in Turkey and the World in terms of sub-aims of the studies. For this purpose, 12 sub-aim categories were formed and literature was scanned according to those sub-aims. In this research document scanning method, a scanning pattern from quantitative research methods, was used.

The sample of the study included 3 databases such as educational studies from Inonu University e-Library, Higher Education Council National Thesis Center and Google Scholar. 101 studies were chosen from those sources and evaluated.

All 101 studies were analyzed according to those criteria including the studies' type, sample, method, data analysis techniques used, language, country, year, region, aims, designs, journal types, data bases and the levels of the thesis.

At the end of data analysis, it was observed that there were more foreign studies than national ones and more foreign articles were produced. Quantitative methods were preferred, *t* test and qualitative analysis was used as data analysis method. The most preferred language was English and America was the country where most of the studies were done. In Turkey, most of the studies were done in Central Anatolia and Marmara regions. The most productive year was 2010. Most of the studies were obtained from doctorate thesis and published in refereed journals. The most frequently used data base was ERIC. Additionally, case studies were reported to be the mostly preferred qualitative study, and screening studies were the mostly encountered quantitative studies. In mixed studies convergent parallel methods were mostly used and the most frequent aim was mathematics education.

In general, the number of research about mathematics education of gifted students in Turkey is very low. Researchers should be directed to this area to obtain more product and avoid wasting that potential. Researchers mostly prefer qualitative methods in their studies. A study could be planned regarding the reason for method choice. Studies in Turkey are generally in Turkish language. If English language is preferred, the studies will gain international identity, be reachable for foreign researchers and have more impact on international area. National studies were rarely done in Eastern Anatolia and Black Sea regions of Turkey. Researchers could be encouraged with economical or academical benefits to increase the interest towards those regions.

Key Words: Gifted, Mathematics Education, Mathematics education of gifted students.

Problem Solving Skills in Mathematics Attitude Scale; Development Study

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ABSTRACT

Indecisiveness bothering the individual physically and mentally and any situation seeming to have a possibility of more than one solution are a problem (Karasar, 2008). Mathematically, though, is defined as a problem which is required to be found or shown but not obvious at first sight as how to be found or shown with available information (Grouws, 1996; quoted by Kayan and Çakıroğlu, 2008). Problem solving skill is the level of obtaining the rules which would take a person to solution and combining them to a degree for use in solution of the problem (Bilen, 2006). In primary education programs, it was stated that problem solving skill includes the necessary skills to solve the problems that a student would face in life (MEB, 2009, 16).

In this study, it was aimed to develop a valid, reliable and useful scale that can measure the attitude of students towards problem solving skills in mathematics. MPSSS (Mathematical Problem Solving Skill Scale) was developed in this survey type study to determine the attitudes of primary school students towards problem solving skills in mathematics. The study group consisted of 437 students from fourth and fifth grades of Atatürk Primary School and İnönü Primary School, associated with Siirt Directorate of National Education. During the creation of draft scale, the stages of creating scale items, consulting an expert opinion, pretesting, validity and reliability works were followed, which are used in development of measuring tools defined in the literature. For the creation of scale items, 10 open ended questions were asked to 76 students by the researcher in writing to obtain their opinions. In accordance with the responses, a draft scale of 65 items was created, 15 items were taken out by an

expert opinion and a draft scale of 50 items were obtained. To determine the factoring structure of the scale, a basic components factor analysis was made. By the factor analysis applied on the data related to the scale, KMO coefficient 0.928 and Barlett coefficient ($\chi^2= 8161.195$; $p<0.05$) were found meaningful. The KMO value shows that data structure is excellently sufficient to perform a factor analysis (Tavşancıl, 2010: 50). The data obtained show that factor analysis is applicable to our data.

In the first factor analysis applied on the scale data without restricting the number of factors, it was determined that the items in the scale were gathered under 8 factors with eigenvalue of greater than 1 and that these factors explain 51,230% of the variance related to the scale. Due to number of factors related to scale being too many, it is considered to decrease this number. Accordingly, when the factors of the scale are examined, it was determined that the first three factors with high eigenvalue explained 39,790% of total variance. This variance ratio shows that the scale can be evaluated as a scale consisting of three factors. When the other method of determining number of factors of the scale, scree-plot is examined, it was decided that optimal number of factors were to be three. In the process after determining the number of factors of the scale, number of factors were coded as three and varimax vertical rotation method was used in analysis of factors. In the analysis performed, 18 items with factor load value lower than 0,45 and 6 items with the difference between factor load values less than 0,1 were taken out of the scale.

It is seen that the developed scale of 26 items consists of three sub factors as “understanding the problem and trust”, “worry” and “avoidance”. These factors explain 46,193% of total variance. This coefficient shows that the items in the scale are consistent with each other and consists of items measuring same properties.

Key Words: Problem Solving Skills in mathematics, scale development, elementary school students.

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Prospective Elementary Mathematics and Classroom Teachers' Beliefs towards Mathematics

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ABSTRACT

Prospective Elementary Mathematics And Classroom Teachers' Beliefs Towards Mathematics

Beliefs have important place in teaching mathematics. Students especially build the basis of their beliefs in elementary schools. The aim of this quantitative research design is to determine prospective elementary mathematics and classroom teachers' beliefs towards mathematics. For that aim prospective elementary mathematics and classroom teachers' beliefs towards mathematics were compared in terms of gender, department, university and grade level variables. Data of the study were collected by using "Mathematical Belief Scale" adapted into Turkish by Hacıömeroğlu (2012). The study was conducted during 2015-2016 academic year. The scale consists of four factors named 'beliefs about constructing students' mathematics knowledge', 'beliefs about teaching mathematical concepts', 'beliefs about teaching mathematical skill', 'beliefs about designing instruction according to students' mathematical development', and 'beliefs about the developments of students' mathematical skills. Study group includes prospective mathematics and classroom teachers enrolled at Education Faculties of Fırat, Kastamonu, Erciyes, Cumhuriyet and Ağrı İbrahim Çeçen Universities. No specific sample was chosen as the whole participants were included in the study. Independent groups t test was used to see if there were any statistically significant differences among prospective teachers' views based on gender, department variables. The non-parametric statistical technique Mann-Whitney U was used instead of Independent groups t test when the distribution of the data was found to be non-normal. One way ANOVA was used to determine the differences among prospective teachers' views based on university variable. Findings of the study showed statistically significant differences among the views of prospective Classroom and Mathematics teachers. Several recommendations are offered based on the study results.

Key words: Prospective mathematics teacher, prospective classroom teacher, belief about mathematics

Prospective Elementary Teachers' Knowledge of Mathematics Materials

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ABSTRACT

Elementary teachers are teaching mathematics courses at grades 1-4 in Turkish Education system. According to MoNE, elementary teachers should use concrete materials at the mathematics courses. The aim of the study is to reveal the prospective elementary teachers' knowledge about the concrete materials and usage of concrete materials. Ball (1992) questions related to the use of manipulatives in elementary

classrooms: How can educators sort through the variety of concrete materials to distinguish "fruitful from flat"? What are the relative merits of using one type of manipulatives vs. another to teach a particular content? Before starting to seek answers of these questions, knowledge of materials and knowledge of materials usage is an asset. To answer these questions, 72 of 3rd year prospective elementary teachers who are students of state university at the Central Anatolia; filled the concrete materials survey at the beginning of the Methods of Mathematics teaching II. The Preservice Mathematics Teachers Efficacy Beliefs about Using Manipulatives (EBMU) which was adopted by Bakkaloğlu (2007) used to collect data. The survey consisted of three parts; (1) Demographic Information; (2) knowledge about the manipulatives; (3) The Instrument of Preservice Mathematics Teachers' Efficacy Beliefs about Using Manipulatives' (EBMU) but only first and second part was used for this study. First part was asking, age, grade, university and gender, second part was designed to ask knowledge about 14 manipulative materials which were chosen from the Ministry of National Education's elementary mathematics curriculum document for grades 1 to 5 and a question whether they had used these materials before. The sample has 14 male, and 58 female prospective elementary teachers. Their age range was 30-32. According to results, majority of prospective elementary teachers did not know geometry board, pattern blocks, symmetry mirror, tangram,

fraction strips, tens paper, hundreds paper, however, they have knowledge cubes, tens blocks, geometric solids, isometric paper, and dot paper. Teachers did not use the all these materials at the courses. Usage and knowledge of these materials should be increased by the methods courses and school experiences courses.

Key Words: Prospective Elementary Teachers, Mathematics Teaching, Material usage, Concrete Materials.

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Prospective Mathematics Teachers' Views About Teaching Material Exhibitions in The Mathematics Project And Material Olympiads

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ABSTRACT

According to Dale's Cone of Experience (1969), purposeful and direct learning experiences should be used to establish an effective learning process. Fairs and exhibitions are suitable environments that provide using and extending knowledge throughout this kind of experiences. This study aimed to introduce Mathematics Project and Material Olympiads (PROMAT) conducted by MATDER-Association of Mathematicians and to investigate the attitudes of prospective math teachers towards teaching material exhibitions in the Project and Material Olympiads. Project and Material Olympiads is an innovative combination of various exhibitions involving math projects, concrete and digital teaching materials, manipulatives and mathematical shapes. There are three main contest categories in the Olympiad: mathematics project contest, concrete teaching material contest and digital teaching material contest. Since it provides opportunities to visitors in manipulating materials, discussing and solving problems, communicating with their peers and families, it is more like a mathematical fair as Ahuja (1996) defined in his article about popularizing math. He claimed that "a mathematical fair might popularize mathematics if it is enjoyed by the students, public and by mathematics teachers." In the first Project and Material Olympiad in 2014, concrete teaching materials in the exhibition were original materials produced by teachers or prospective math teachers participated in the contest. Most of them were manipulatives; effective tools that can be manipulated by students and promote academic achievement (Post, 1981). Besides providing visitors and students a transition from concrete to abstract level, teaching material exhibitions also refer and promote teachers' and prospective teachers' 'knowledge of curriculum and media' which is accepted as one of the main components of pedagogical content knowledge in the literature. This knowledge includes planning the media for instruction, connecting the setting with a particular mathematical topic, selecting and using suitable teaching materials (Depaepe et. al., 2013).

Various survey methods were used in this qualitative study. The researcher developed an open ended questionnaire to measure the attitudes of prospective math teachers who attended the teaching material exhibitions in the Olympiads. Since the participants were 84 prospective mathematics teachers taking the instructional design course from the researcher, the researcher was able to observe the material preparation process of the participants. Also he interviewed with some of the participants. The researcher followed the stages of qualitative data analysis described by Miles and Huberman (1994). In an effort to provide data triangulation and trustworthiness, the researcher analyzed and compare various data sources included the answers of questionnaire, observations and interview data.

The views of the participants were analyzed according to four main categories; advantages and disadvantages of material preparation, different aspects of communication with students, transferring the experience, pedagogical benefits. It is found that, generally prospective math teachers developed a positive attitude towards teaching material exhibitions. Generally, the prospective math teachers agreed that the experience of preparing a teaching material and presenting it in an exhibition enhanced their professional developments and communication competencies. The researcher proposed that teaching material exhibitions should be a part of instructional design lessons in educational faculties.

Key Words: Math Fair, Teaching Material Exhibition, Mathematics Project And Material Olympiads

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Secondary School Math Teachers' Attitude Towards Supplementary Evaluation Techniques and Reasons For Choice Of

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ABSTRACT

In the period of education and training, behaviors that a person has or has not in targeted behaviors change's direction, level and the deficiencies in this subject are converted to understandable level with assessment and evaluation. This characteristic of assessment and evaluation makes it the most important part of education and training. According to results in accordance with the explanations; teachers are need to know every skills and abilities about assessment and evaluation. (Aşık, 2009).

According to some results that were obtained from different studies; because of unwillingness of teachers, lack of information of methods and techniques, crowded class sizes, lack of time, previous beliefs about assessment and evaluation, lack of sources and equipment, applications of assessment and evaluation techniques were not able to be used in an effective way. (Carnevale, 2006). This situation shows that assesment and evaluation process which is the most important part of curriculums couldn't be applied effectively. When it is analyzed, studies that have been done in Turkey for secondary school Math teaching programs in assessment and evaluation, it has been observed that assessment and evaluation is one of the most difficult dimensions of teaching. (Bal and Doğanay, 2010).

At this study, specifying secondary school mathematics teachers' knowledge level about supplementary evaluation techniques and which of these techniques they prefer, and the reasons of preference of these techniques have been intended for. In this context, "What are the levels of secondary school mathematics teachers' knowledge about supplementary evaluation techniques and reasons of choice for these techniques?", this question has been the focus of this study.

Participants of the study consists of teachers of public and private schools, 50 from each. To gather data a questionnaire with three parts was formed. First and second parts of the questionnaire is about frequency of teachers' using assessment and measurement techniques and whether they have sufficient knowledge to use these techniques respectively. Third part is about the reason why the teachers prefer supplementary assessment and measurement techniques. Data collection tools were formed according to the literature. Then, these tools were modified in accordance with the comments of two experts. Lastly, final changes were applied based on the data obtained from pilot study. Moreover, at the last part of the study semi-constructed interviews were made with six of the teachers.

Acquired data has been analyzed by content analysis. While the content analysis has been done data acquired from questionnaires and reports towards the aims of the study have been categorized by the help of some codes specified before. After that, codes connected with each other have been analyzed under different theme. (Yıldırım ve Şimşek, 1999).

Evaluation of the data has shown that about the supplementary evaluation approaches primary school mathematics teachers can make objective evaluations using these techniques and involve students in the evaluation process. The teachers had positive thoughts about the effectiveness of using these techniques. On the other hand, they had negative thoughts as well such as their being time consuming and difficult to apply in each classroom. Furthermore, it has been observed that these techniques are not used for their purposes. It has been observed that secondary school math teachers often prefer project assignments, multiple choice questions, and open-ended questions, whereas they rarely used math diaries and check lists. As the reason of rare use of these techniques factors such as time problem, intensive curriculum, crowded classrooms, excessive involvement of parents, difficulties in archiving evaluation tools have come forward.

Key Words:Mathematics teachers, supplementary evaluation and assessment

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Square and Rectangular Concepts For Hearing-Impaired Students

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ABSTRACT

In this study, it was investigated how hearing-impaired students defined square, rectangular concepts and their properties and which semiotic resources these students used in process of description. Semiotic, beyond focus on traditional psychological processes in mind function and structure, is a science dealing signs occurred by people. Semiotics is a powerful tool used to interpret how knowledge was taught. Also, semiotics, beyond the behavioral performance, is interested in underlying social norms, reflecting individual creativity, meaning, producing and using of signs [1]. Semiotic resources include words (oral or written forms), extra-linguistic representations (gesture, gaze, glance), different type of inscriptions (figures, graphics) and various other instruments (all technologies from pencil to computer) [2]. This research was carried out by three students selected from deaf students high school in Ankara during 2013-2014 academic year. Grounded theory techniques (open coding, axial coding and selective coding) were used to analyze the data collected via interviews, observations, and documentation reviews. It was determined that hearing-impaired students mentioned from square and rectangular as polygon and utilized language (speech, sign language), gesture and inscriptions as semiotic resources in the process of description. All of the students stated that square and rectangular had angle as basic element, some students explained that these polygons had edge and corner. But the students used "line segment" concept instead of edge and "point" concept instead of corner. Also, students mentioned what these polygons had different properties out of basic elements.

Key Words: Semiotic, gesture, hearing-impaired students, geometric concepts.

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Teaching Trigonometric Functions of an Acute Angle Through Dynamic Geometry Software Cabrılı Plus

ABSTRACT

With the introduction of computers into our daily lives in the second half of the 20th century, their use in education became unavoidable. Thanks to the computers geometric shape which were almost impossible to draw using a compass and a ruler were easily drawn and difficult problems could easily be solved. With the employment of technology in mathematics classes innovations have been in the classroom environments. The dynamic software found in these environments enables students to perform presumptions on geometric structures; test their presumptions and perform generalizations. Dynamic Geometry Software (DGS), with its features such as leaving trace, producing animation, rotating and reflecting help students discover the topological properties of a shape. This discovery enables students to put forward presumptions and students can also verify or refute their presumption through several examples. Several relations and features which cannot be seen or constructed in conventional media can be studied in these dynamic media and students can easily investigate generalizations. In this study we have provided two examples which could be conducive to the teachers' teaching activities and students' struggles to learn. In the first activity the aim was to define trigonometric functions through a right triangle and the second was to help students discover the values of defined functions on the unit circle.

Key Words: Unit Circle, Dynamic Geometry Software, Trigonometric Functions

The Determination Of Secondary School Math Teachers' Educational Beliefs

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ABSTRACT

The beliefs; there are the first condition of all systematic knowledge, of all specified actions and of a life in an ethical framework. The beliefs are located at the origin of all organized ideas (Huxley, 2014: 340). Therefore, we can say that people's beliefs affect their behaviours and they live according to their beliefs. So, the importance of beliefs cannot be denied in education which is also a science of behaviour. Also called educational beliefs, beliefs, propositions and estimates regarding education are actually made up of educational philosophy (Rideout, 2006). The educational philosophy of someone determines its roles, attitudes, values and decisions in the learning environment (Ornstein and Hunkins, 2014; Tozlu, 2012; Yılmaz, Altinkurt and Çokluk, 2011).

In order to give a sense to teachers' behaviours, it is important to detect their beliefs. While planning the educational process, it is important to take advantage of teachers' beliefs that are the most important factors of education. For the preparation of the learning environment, it is necessary to identify mathematics teachers' beliefs, because mathematics contains an important number of applications compared to the other subjects. Thanks to the augmentation of studies on this matter, we will understand mathematics teachers' educational beliefs better and regarding the detecting of education policies, this situation can be better taken into account. That is why; we tried in this study to find responses to the following question: "What are the educational beliefs of secondary school math teachers?"

In this study, the descriptive survey model is used. The group on which we studied in this study is constituted by 77 math teachers employed at Elazığ in 21 different secondary schools, during 2015-2016 academic year. In the study, in order to determine the secondary school mathematics teachers' beliefs about learning, the Education Belief Scale developed by Yılmaz, Altinkurt and Çokluk (2011) was used. This scale consists of sub-scales according to the educational philosophies such as

Progressivism, Existentialism, Reconstructivism, Essentialism and Perennialism. Data analysis was done through SPSS. Gender, seniority in the profession, the time spent in school and educational status are the independent variables of the study. As a result of the data analysis, the conviction of educational philosophies are observed for the existentialism ($\bar{X}=4,42$), for the progressivism($\bar{X}=4,31$), for the perennialism ($\bar{X}=4,05$), for the reconstructivism($\bar{X}=4,04$), for the essentialism ($\bar{X}=2,86$). This shows that math teachers' educational beliefs are very high level at existentialism and progressivism, high level at perennialism and reconstructivism, moderate level at essentialism.

Key Words: Educational beliefs, math teachers, philosophy of education.

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The Effect of Cooperative Learning Method Enhanced with Metacognitive Strategies on Mathematics Self-Concept

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ABSTRACT

A person's mathematics self-concept is defined as learning mathematics concepts, learning the topics easily, being successful, liking the lectures or not, self-evaluation of students in terms of their interest and skills at lectures, and their perceptions about these skills (Marsh, 1992). Self-concept is not an innate feature, it is shaped with the social and physical environment in time. School environment and social groups contribute a lot to the development of self-concept of the children during their primary and secondary schools (Kossowka, 2002). In this context, cooperative learning can be used as a beneficial method in improvement of self-concept. To date, however, research has provided relatively little insight into the role of cooperative learning method on learners mathematics self-concept.

Recently, on one hand, the effects of cooperative learning on several learning outcomes have been discussed (Lucas, 1999); on the other hand, there have been different views on increasing the effectiveness of the cooperative learning method. Some researchers focus on structuring group interactions through cognitive or metacognitive instruction (Mevarech ve Kramarski, 1997). Furthermore, some researches emphasize that the relation between metacognition and social interaction (Jbeili, 2012). Studies conducted on the relation between cooperative learning method and metacognition, however, it is appointed that the changings in learners' affective skills are not discussed enough on these researches. In this context, there is a need for the research of the topic. Using different strategies in teaching activities seem to be important aspects to the teachers since these help them in the activities. Thus, the purpose of this study was to investigate the effect of cooperative learning method supported by metacognitive strategies on mathematics self-concept of sixth grade student.

The research participants were composed of 106 sixth grade students attending a middle school in Elazig in the autumn term of 2015-2016 academic year. The

research has been designed as the pre-test post-test control group quasi-experimental design. There were two experimental groups and a control group. Cooperative learning method supported by metacognitive strategies was used in first experimental group (n=35) and cooperative learning method with no support of metacognitive strategies was used in second experimental group (n=36). There was no intervention for the control group (n=35). Data were collected using the “Self-Description Questionnaire-I” developed by Marsh (1992) and adapted into Turkish by Yıldız and Fer (2008). The quantitative data was analyzed by performing the dependent group t-test and One-way ANOVA.

At the end of the interventions, no significant differences were found in the mathematics self-concept of the first and second experimental groups students. However, mathematics self-concept of the first and second experimental group students were significantly higher than the mathematics self-concept of the control group students.

The results of the research have shown that of cooperative learning method and cooperative learning method supported by metacognitive strategies have a significant effect on the students’ mathematics self-concept. In the light of the findings of this study, the researchers have developed suggestions for those who will conduct further researches on metacognitive strategies and cooperative learning.

Key Words: Mathematics education, cooperative learning method, metacognition, mathematics self-concept

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The Effect on Student Achievement of Drawing Graphs of Functions of the Different Technological Applications In General Maths

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ABSTRACT

In the changes in mathematics education, the fact that computers and technologies that come with them day by day are becoming a part of our lives is a significant factor. In particular, positive effects that were brought into learning environments by the computer software which has been developed for math education have been appreciated by many researchers who work in this field (Aktümen, 2007; Bulut, 2009). There are many studies investigated the effects of different technologies on math achievement (Bulut, 2009; Kutluca, 2009; Aksoy, Çalik and Çinar, 2012).

In this study mixed method research design was carried out in a triangulation pattern. In this pattern; by using quantitative and qualitative methods together, weaknesses of a method are aimed to be completed with the strengths of other method (Creswell & Plano Clark, 2011; cited by Yıldırım and Simsek, 2013). The study has been carried out on teaching of the graphs of functions in the following format in general maths: $y=a$, $y=ax + b$, $y=ax^2+bx+c$, $y=x^a$, $y=a^x$ and inverse functions. It is aimed to determine the effect of different technological applications on the students' achievement in drawing graphs of functions in mathematics teaching.

Research has been conducted with 1st grade students at state university in 2015-2016 academic year-fall term. Students were randomly divided into three groups. Application has been carried out at the first group with Excel program, at the second group with AGC (Advanced Graphing Calculator) and at the last group with a graphic drawing of mobile application that has been downloaded to smart phones by the students. In the study, before and after the application pre-test and post-test parallel to the pre-test which developed by the researchers, have been applied to each group in order to determine the success of groups. After the training by

performing a group interview, students also have been consulted on their views about the effect on their achievement.

As a result of the study; it has been found that groups, independent from one another, performed an increase in a statistically significant level in their post-test achievement point averages each group in itself, according to the pre-test achievement point averages. Furthermore, with applications which downloaded on individual smartphones as a result of the training, the increase obtained in success, has been found higher when compared with other groups. In interviews with students, they declared that the using technology during the training provides convenience to make sense of graphic drawings. With AGC and mobile application that has been downloaded, the groups that carried out the training with devices and with learning about themselves through trial and error, they have expressed that it makes a positive impact on class participation and that they feel more active.

In conclusion, especially for classroom applications of mobile applications, due to the presence of the smartphone opportunity in every individual, easily accessible, free and that it is appropriate to individual learning speed, reveals the development of mobile applications requirements for mathematics education.

Key Words: Graphing calculator, Excel, mobile application, mathematics education, Technology.

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The Effects of 4mat Method to Rate-Ratio Subject

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ABSTRACT

Most of the students have difficulty in learning Mathematics. Many teaching methods have been improved to simplify understanding of Mathematics and ensure permanence of its teaching. In this work, it is being researched effect of the 4MAT teaching method. 4MAT (4 Mode Application Techniques) is on eight steps learning circle which students use mostly while learning. It is based on students brain hemispheres [1]. 4MAT method is a way that takes data to centre and while doing it, organizes the notion which is aimed [2]. Rate-Ratio is one of the most important subjects of Maths. We are always face to face with Maths in our daily lives. Because of this it has a special part for us. We should improve students guessing skills to facilitate understanding of the subject. Rate-Ratio have been examined which is one of the topics of the 7th class to see effect and permanence on student academic success with 4MAT. According to primary tests marks, one group is chosen as an experiment group and the other one is control group. All groups were equal. 4MAT is applied to experiment group. Other group studied with traditional method. Then, last tests marks are calculated. According to result, average success of the experiment is 11,71 and control groups is 9,50. There is significant difference between the groups and it is observed with t-test's results. Teaching method is compared which is given to experiment and control groups. When traditional teaching was applied to control group, although the last test's average success is higher than primary test, this can't be thought as a suitable result statistically. When 4Mat was applied to experiment group, average of the last test marks are higher than average of primary tests. This difference shows, there is a positive difference between two groups statistically. Geometry of Transformation which is one of the subjects of Maths, was taught with suitable activities with 4MAT method. At the end of the activity it was seen that experiment group was more successful and more

permanent than control group[3]. While experiment group was being taught with 4MAT, control group was taught with traditional teaching method, about the fractions subject. So, experiment group's success and permanence was higher than control group's[4]. Our work contains contents of 4MAT method.

At the end of the research, rate-ratio is studied with 4MAT, experiment group is more successful than control group which is studied with a teaching method based on student book. This work is produced of the thesis called 4MAT method's effects to academic success and permanence about rate and ratio.

Key Words: Rate-Ratio, 4Mat Teaching Model, Mathematic Education

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The Effects of Using Mathematical Model for Teaching Decimal Fractions on Academic Achievement

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ABSTRACT

In this study, it is aimed to investigate the effects of mathematical models used for teaching decimal fractions in 6th grade on academic achievement. Data for the study were collected from sixth-grade students in Tokat Market Üzümör secondary school in 2015-2016 academic year. The research design of the study was based on experimental pretest–posttest control group design. The sample of the study consists of 23 6th grade students (12 in experimental group and 11 in control group). During the 3 weeks application process (15 hours), the approach of “teaching decimal fractions with mathematical models” was adapted in the experimental group while the constructivist approach involved in mathematics curriculum is used for teaching decimal fractions in control group.

Line, set and field models used for teaching fractions which were formed rendering models more specific by Bingölbali and Özmantar (2014) were adapted during the application in the experimental group. After having been taught how to model the decimal fractions, they were asked to benefit from the models for problem solving. It is observed that students avoided using the models in the first three hours at the application. The students who easily reached the solution as they used the model for multiplication, division, problem solving and reasoning in decimal fractions have begun to use models of their own volition.

A Mathematics Achievement Test on Decimal Fractions applied to the groups before starting study was applied again at the end of the study. The data were analyzed using SPSS software. The results from analyzing the data show that there are significant difference between pretest and posttest scores after the application of the students in the experimental and control groups. However, when comparing two groups, it is concluded that the learning in experimental group is significantly higher than the learning in the control groups. The result is consistent with the results of the studies carried out by Çiltaş and Işık (2012); Çiltaş and Yılmaz (2013); Eraslan (2011); Gümüş et al (2008).

Key Words: Mathematic Education, Mathematical Model, Decimal Fractions.

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The Evaluation of Eighth Grade Students' Opinions about the Flipped Classroom

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ABSTRACT

Especially in recent years, in order to improve the effectiveness of the learning process, very serious researches have been invested. As a result of these researches, one of the most important educational systems which are so much debated in recent years-is called the flipped classroom. The flipped classroom in the world and in Turkey is on the agenda, though the number of studies on this subject is quite insignificant (Filiz and Kurt, 2015; Gençer, Gürbulak and Adıgüzel, 2014). In this study, we tried to determine the opinions of eighth grade students about the application of the flipped classroom on mathematics courses.

In this study, the case study method is used. The group on whom we worked in this study is constituted by 34 secondary school students in the province of Elazığ who are in 8th grade during the 2015-2016 academic years. Every practice has done by math teacher who has master degree in educational sciences. In total, the study lasted eight class hours, except presentation of flipped classroom. The study is limited by the topic of the equations. First of all, in this study, flipped classroom is presented in detail to the students while two class hours. Then, all e-mail addresses and Facebook accounts of students were recorded. Course content was sent to students via the Internet as videos and in the form of worksheets. Thus, content is taught through video and worksheets in an individual environment during out-of-class assignments; activities and assignments are made on the issue with the teacher in the classroom. Students have commented about sharing at Facebook every day. Courses were done on Monday and Thursday. A questionnaire prepared by the researcher is used in the study as a data collection tool. Every time it is needed, survey questions in the form were asked to the students before and after practice. Generally, it is asked that "What do you think about flipped classroom?"

before and after practice. Data were calculated as frequencies and percentages by researchers.

According to the results obtained by the analysis of the findings, significant differences on the opinions of students about the flipped classroom were found when we compare their opinions during the introduction of this system and after its applications in the class. At the beginning, students think negatively about the flipped classroom; and they also indicated that the flipped classroom experience was a positive experience. For example; before practice, 22 students think that flipped classroom is difficult and complex, 3 students think that flipped classroom is enjoyable. After practice, 5 students think that flipped classroom is difficult and complex, 24 students think that flipped classroom is enjoyable. According to the results obtained at the end of practices, we see that students adopted a more positive thinking attitude. These results are consistent with the results obtained in the studies of Gilboy, Heinerichs and Pazzaglia (2014) and also in the studies of Turan and Göktaş (2015) about the differences of students' opinions before and after practice of the flipped classroom.

Key Words: Flipped classroom, math lesson, learning environment.

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The Examination of Situations Where a Derivative Fails to Exist in GeoGebra Software

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ABSTRACT

The purpose of this study is to examine the situations where a derivative fails to exist in GeoGebra software. Some of these situations (a corner, a cusp, a vertical tangent, and a discontinuity) [1] were moved to a dynamic learning environment by means of GeoGebra. Since the derivative is difficult for students [2] this concept should be visualized using the mathematics software. If a dynamic learning environment is designed for explaining these situations students may learn these concepts and relationships with derivative concepts. For explaining these situations in detail, we developed five dynamic materials with GeoGebra software. This software is one of the free open source software which provides a dynamic learning environment [3]. It was found that this software had a positive effect on students' achievement [4] and motivation [5] in mathematics. For that reason, the materials and worksheets about these situations were developed by the researchers. It is believed that when these situations are moved to a dynamic environment using the dynamic mathematics software GeoGebra, students may learn these concepts easily in an enjoyable classroom environment. Therefore teachers may overcome these difficulties in a calculus class. It is suggested that calculus concepts should be visualized for better understanding. This study is an attempt to promote a framework for better learning these situations via GeoGebra.

Key Words: Derivative, GeoGebra, dynamic learning environment.

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The Examination of the Effectiveness of The In-Service Education Given to the Math Teachers About the Use of Computer in Mathematics Education

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ABSTRACT

Introduction

Technological developments have become a crucial factor in several aspects of life. Education is also affected by those developments in similar way. Therefore, integration of technology to education has become compulsory since it has an important role for improving the quality of education (Cuman, 2001; Aktümen & Kaçar, 2003). Although there is a widespread view that the use of technology creates a huge reform in education, technology is not used in education adequately and it cannot be integrated with mathematics education (Koçak-Usluel, Mumcu-Kuşkaya & Demiraslan, 2007; Monaghan, 2004). The teachers' inadequate background on understanding, adoption, and use of new educational technologies and approaches can be showed as a reason for this. (Baki, 2002). Also, instruction of teachers is one of the prerequisites for the integration of technology to schools (Ersoy, 2005). So, teachers should be educated by in-service education courses related to effective integration of technology and should be supported in this respect (Yılmaz, 2012). In this context, the purpose of the study is to investigate the effectiveness of instruction given to mathematics teachers about technology and software.

Method

Case study technique was used and 9 elementary and high school mathematics teachers were participated in the study. The study was designed as an in-service education program and it lasted totally 10 weeks, 3 hours each week. The data was collected with interview forms including open-ended questions, which are prepared by taking an expert opinion.

Result

Teachers stated that they have become aware of the technological software which they had not know before the study. Also, they expressed that they learned how to integrate technology in mathematics education and how to use technological software which they had known only their names of them but never used before. Additionally, they explained that in-service education about the use of technology in teaching mathematics was helpful for them.

Key Words: Mathematics Education, In-service Teacher Training, Mathematics Education in Technology

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The Geometric Interpretation of the Newton Method with a Dynamic Mathematics Software

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ABSTRACT

The purpose of this study is to visualize the Newton method with a dynamic mathematics software. This method is a well-known method for finding better approximations to the roots of a real valued function successively in calculus. The method starts with a function defined on real line x , derivative of the function and an initial guess for the root of the function [1]. The learning of the Newton method may be difficult for students in calculus lectures. With the development of technology, several tools have been used in the teaching and learning of calculus to overcome such difficulties. Recently, free open source software has been used widely in mathematics teaching. One of the free open source mathematics software is GeoGebra which can be obtained easily and can be used on multiple platforms [2]. With the aid of this software, one can easily visualize the calculus concepts. The features of GeoGebra provide multiple representations of mathematics concepts. Therefore, learners can see both geometric and algebraic representations of these concepts simultaneously. With this software, learners study mathematics concepts in a dynamic environment. In this context, the Newton method, a significant subject of calculus, is transferred to a dynamic mathematics environment. In this way students may learn the Newton method easily in an enjoyable environment.

Key Words: Newton method, dynamic mathematics software, visualizing.

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The Image and Understanding of Undergraduate Students in the Department of Primary Mathematics Teacher Training Related to the Definition of Derivative

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ABSTRACT

One of the biggest targets of analysis, which is one of the main learning areas of mathematics, is to comprehend and interpret the changing quantities and facts, as well as to make predictions about the future [1]. One of the cornerstones used in reaching these targets is derivative, which is one of the basic dynamics of analysis. Nowadays, the concept of derivative is generally considered as the instantaneous change rate, limit of average change rates, the slope of the tangent line at a certain point of a function [2, 3]. The purpose of this study is to reveal the understanding of the students studying in the department of Primary Mathematics Teacher Training related to the definition of derivative. So as to achieve this target, the students' understanding related to the concept of derivative was examined in various dimensions.

The study was based on the case study model of qualitative research approach. This study applied at the state university in the large-scale city of Eastern Anatolia Region of Turkey was carried out at the beginning of the spring semester of 2014-2015 academic terms. The participants of the study are 60 students, including 31 of whom are second grade and 29 of whom are fourth grade, studying in the department of Primary Mathematics Teacher Training. The data of the study were collected with the help of the Derivative Definition Comprehension Form (DDCF) developed by the researchers. The formal definition of derivative and seven open-ended questions associated to this definition are included in the DDCF. Content analysis was used in analyzing the data obtained from the study.

As a result of the study, it was detected that it was difficult for the students to express with their own sentences what they understand from the formal definition of derivative. The reason for the difficulty in understanding the definition to derivative of the students was revealed to be the fact that they were not able to make sense of the mathematical expressions in the definition of derivative. It was revealed that the perception related to the concept of the differentiable function at a point was, the fact that the concerned function was defined, limited, and continuous at that point frequently. Parallel to this finding, it was found that the concept images of the students have in their minds for this concept is, mainly, either a continuous function or a tangent line drawn with a continuous function. When the solutions of the students offered related to the problems about the definition of derivative were examined, it was detected that the students ignored the other conceptual features in the definition while focusing on the process of taking the limit in the definition of derivative. Also, the results obtained from the study were separately evaluated according to the grade levels of the students educated in.

Key Words: Derivative concept, concept image, understanding the definition of derivative, undergraduate students.

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The Influence of 4MAT Method on Academic Achievement and Retention of Learning in Transformation Geometry

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ABSTRACT

As modern educational mentality has emerged, innovations have been introduced to teaching, and many models and methods have started to be used in order to be more productive in education. One of these models is McCarthy learning style model and 4MAT method which is used in the framework of this model (McCarthy, 1982).

4MAT method is based on perceiving and processing knowledge. It defends developing student-centered learning environments based on the learning styles of students and making students discover knowledge by themselves. It enables students to use both hemispheres of their brains effectively (McCarthy, 1990).

Transformation geometry is an abstract subject included in the mathematics curriculum with an amendment introduced by the Ministry of National Education in 2005 (NEM, 2006). Research on transformation geometry has mostly focused on the computer-supported teaching of this subject and compared traditional teaching with computer-supported teaching. However, attention should also be focused on the teaching of transformation geometry through other contemporary learning approaches, and 4MAT method should be used for teaching transformation geometry, too.

This study investigates the influence of 4MAT method in the teaching of "Transformation Geometry" – a subject included in secondary school 7th grade mathematics curriculum – on students' academic achievement and retention of learning. In this period, transformation geometry was taught to one group with activities appropriate to 4MAT method which took into consideration learning style and brain hemisphere and to another group as based on textbook. The pretest-posttest control group quasi-experimental design was used in the study. The Transformation Geometry Knowledge Test developed by the researcher was used as data collection tool. ANCOVA test was used to determine if there is any significant difference between the influence of teaching via 4MAT method and teaching based on the textbook on students' academic achievement and retention of learning in transformation geometry subject. It was seen that 4MAT method was more effective in the teaching of transformation geometry in comparison to textbook-based teaching.

The research result is similar to those of many studies in literature. As in this study, in many of studies, subjects taught experimental groups via 4MAT method and taught control groups by use of textbooks based on traditional lecture and question and answer teaching (Appell, 1991; Ursin, 1995; Tsai, 2004; Dikkartın, 2006; Tatar,

2006; Öztürk, 2007; Uysal, 2009; Aktaş, 2011; Ergin, 2011). All of these studies found out that experimental groups were significantly more successful than control groups.

Key Words: 4MAT method, learning style, brain hemisphere, transformation geometry

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The Investigation of Euler's Formula on Platonic Solids by Origami

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ABSTRACT

Leonard Euler (1707-1783) is the founder of the Graph Theory. He solved the Königsberg seven bridge problem and developed a formula named after him. In the problem there are seven bridges on the Pregel River and these bridges connect the lands. So can a person visit all the lands by passing only once from each bridge? According to Euler this is not possible because there are odd numbers of edges connected by vertices (Figure 1). While this investigation Euler found a formula $V-F+E=2$. These paths are called Euler's path and the graphs that contain Euler's paths are called as Euler's graphs.

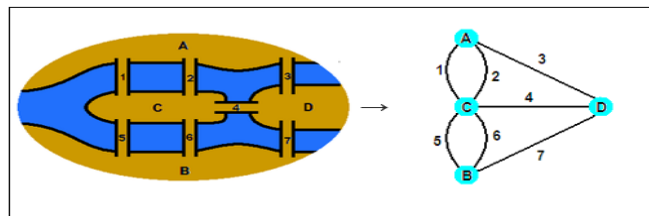


Figure 1

This formula satisfies for the regular polyhedrons, also. These polyhedrons are cube, tetrahedron, octahedron, dodecahedron and icosahedrons which are called as Platonic Solids. Understanding the properties of these solids is important for high school students for both solving problems and for improving their spatial abilities. Both problem solving and mathematical communication are important skills for mathematics education. Students should communicate through mathematics and speak about mathematics [1, 2, 3]. In education to improve these abilities concrete materials are helpful for teachers. According to research results Origami is one of the tools that helps students to communicate mathematically and to comprehend geometrical relationships easily [4, 5]. Nowadays, Origami has been becoming an educational

tool in mathematics classrooms [1, 2, 3, 6]. Our elementary school mathematics curricula of 2009 and secondary school geometry curriculum emphasized on using the Origami in mathematics and geometry classes [1, 2, 3]. In some of them there are detailed explanations about how to use origami in teaching and learning geometry including some hints for the activities. Moreover, its advantages are explained as acquisitions in terms of behaviour, psychomotor skills, language, social and affective characteristics. For improving geometrical abilities of students Origami is very efficient method that can be used in the classrooms [6]. Therefore the aim of the current study is to provide a perspective for mathematics teachers for teaching and learning Euler's formula on graphs and platonic solids through Origami.

An action research is going to be conducted in the study. Action research is a process in which participants examine their own educational practice systematically and carefully, using the techniques of research [7]. The participants of the study is a group of eleventh grade students.

5E learning cycle will be applied to prepare lesson plans on Euler's formula for solids. Origami tasks on platonic solids will be integrated into the instruction. The instruction will last 4 hours. The interviews will be conducted at the beginning and at the end of the treatment to obtain data about their opinions about using origami in teaching and learning Euler's Formula and determine their perceptions about using origami in mathematics classes. The participants will also observed. The instructor will take field notes especially for determining difficulties and strengths of the instruction.

The lesson plans and the findings of the present study will be explained in the conference.

Key Words: Euler's formula, Graph theory, Platonic solid, 5E lesson plan

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The Understanding of the Undergraduate Students in the Department of Primary Mathematics Teacher Training Related to the Definition of Limit

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ABSTRACT

The concept of limit is defined as the approached value by a function in response to the value to which a variable in the function approaches [1]. As one of the basic concepts of analysis; limit is connected to the subjects of function, derivative, integral, continuity, and differential [2]. Over the formal definition of limit, which was first introduced by Cauchy and is also known as ϵ - δ definition, the definitions of the concepts, such as derivative, integral, and Taylor series were built [3]. Limit plays a key role in the learning of its subsequent concepts, such as continuity, derivability, and definite integral [4]. The purpose of this study is to reveal the understanding of the students studying in the department of Primary Mathematics Teacher Training related to the definition of limit. For this purpose, the understanding of students related to the formal definition of limit was examined in detail.

The study was based on the case study model of qualitative research approach. This study applied at the state university in the large-scale city of Eastern Anatolia Region of Turkey was carried out at the beginning of the spring semester of 2014-2015 academic terms. The participants of the study are 60 students, including 31 of whom are second grade and 29 of whom are fourth grade, studying in the department of Primary Mathematics Teacher Training.

The data of the study were collected with the help of the Limit Definition Comprehension Form (LDCF) developed by the researchers. The formal definition of limit (epsilon-delta definition) and seven open-ended questions are included in the LDCF. During the development of LDCF, the help of an academician who is an

expert on analysis was received. The comprehension form used in the study were filled in writing by the students. Content analysis was used in analyzing the data obtained.

As a result of the study, it was determined that most of the students had difficulty in comprehending the formal definition of limit. It is possible to suggest that most of the students are not aware of the intuitive meanings underlying the formal definition of limit. It is reckoned that this is the result of the difficulties in the process of interpreting and comprehending for the meaning of the topological concepts and symbols (epsilon-delta) in the definition of limit. It is revealed that the concept of the existence of limit of a function at a certain point is mostly perceived by the students as “the equality of the right and left limits” and “that the point searched is defined in the function”. Upon the analysis of the data obtained from the problems asked to students, it was understood that the students ignore the fact that the point whose limit is searched for is the accumulation point and misuse an important theorem about limit. The findings obtained from the study were separately evaluated according to grade levels of the students educated in.

Key Words: Limit concept, limit of a function at a point, understanding the definition of limit, undergraduate students.

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The Perceptions of School Administrators, Teachers and Students of the Effect of Mathematics Street Application on Learning Mathematics

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ABSTRACT

One of the most effective learning way is self-learning realized through learners efforts. Students' learning process should not be limited to the form of learning in the classroom. While classroom is one of the most important places for students to learn, the places outside the classroom also hold an important role at this point. One of them is school corridor. This study investigates the contribution of school corridors enriched with mathematical visual elements and called Mathematics Street on students learning.

The aim of this study is to determine the views of administrators', teachers' and students' about the contribution of mathematics streets application on learning mathematics. For that aim, interviews were held at a secondary school of which corridors were enriched with mathematical images and mathematical formulas. Research was based on qualitative research method. Qualitative research method covers both events and perceptions in an integrated manner (Yıldırım and Şimşek, 2013). The phenomenology pattern was used in this qualitative research. Studies based on phenomenology, reveal individual perceptions and perspectives on a particular subject (Yıldırım and Şimşek, 2005). Study group was selected by using maximum variety sampling. In this context, school administrators, teachers and

students were interviewed. Research data was collected via a semi-structured interview form created by the researchers. Descriptive analysis was used to analyse the data. Study results revealed school administrators, teachers and students expressed that mathematics street application was useful, effective, beneficial and contributed them learn mathematics better. Several recommendations are offered based on the study findings.

Key Words: Teaching of Mathematics, mathematics street, mathematical images.

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The Role of Technology in Math Education

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ABSTRACT

In today's world the use of Web 2.0 is becoming more common and the technology regarding the educational apps is developing rapidly. Thus the number of teachers and students using these apps as part of their teaching process is on the rise. The use of Web 2.0 applications not only contributes to the efficiency of the teaching/learning process but also helps enhance student motivation and encourages in-depth learning and also enables students to acquire the sense of "learn how to learn". This is why transforming traditional educational tools into new generation interactive tools has become a necessity in order to make the learning permanent.

The aim of this presentation is to demonstrate how we use the Web 2.0 applications and educational technologies in our school both in the in-class activities and out with a range of classes from 4th grade to 9th grade maths lessons. The main reason for us using such technology in our classes is to conform to the rapidly changing world and technology and at the same time to enable the active participation of our students in our lessons, get them to like maths and have fun during the process.

We would like to share with you some of the lessons and activities planned and executed using apps such as Learning Apps, Plickers and QR Code. The examples we will share include specific mathematical topics in interactive teaching applications, in-class assessment and evaluation, assigning homework and extracurricular activities and geometry software. The example activities shared in this

presentation are in a nature that allows them to be actively used in math lessons for all levels and they would give ideas to the attendees how they use Web 2.0 tools and technology in math education.

Key Words: Math Education, Web 2.0 Tools, Educational Apps.

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Using Creative Drama Method in Fraction Teaching

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ABSTRACT

In almost every part of our daily lives we might come across the concept of fraction which is one of the main subjects of Mathematics. Progressivity relationship takes an important role in mathematics; therefore, it is important to teach and learn the main concepts of mathematics.

In this study abstract concepts brought to concrete concepts by practicing creative drama method actively with students in a classroom. In this way the subject of fraction was taught to students and examples from the application and students' views were presented within the results of the study.

This study is actualised with third grade students in a public elementary school in Aydın and these students were selected randomly. In a classroom with 19 students in the fraction lower learning area the following concepts were covered by using creative drama techniques; the concepts of quarter, half, total, numerator and denominator, fraction-bar, fraction, and unit fraction. In another classroom traditional methods were used in order to teach the same subjects. In total 38 students participated in the study.

At the end of the process an evaluation test, which consists of three questions, applied to all students and the results were analyzed with descriptive analysis method. At the same time semi-structured interviews were conducted with randomly selected 10 students (5 girls and 5 boys) from the classroom where creative drama methods were applied, in order to understand the students' experiences of creative techniques.

At the end of the study it is found that the students in creative drama classroom could comprehend the concepts of fraction faster than the students from the classroom where traditional methods were applied. In addition to this, the students from creative drama classroom reported that during the activities they had

fun, and they could stimulate the concept of fraction in their minds. They also created some practical application of performances in the classroom by themselves to better understand the concept of fraction.

Key Words: The concept of fraction, mathematics teaching, creative drama.

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What is the Meaning of “Mathematics” for the Prospective Preschool Teachers?

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ABSTRACT

Mathematics which is described as the abstracted form of the life [1] is defined as analyzing all the possible patterns [2]. Mathematics which processes information, infers results from them and which is a key for problem solving is a useful tool for people to understand the world and our environment [3]. For this reason, searching understanding by starting from pre-school period can be seen as the first steps of understanding mathematics.

In this sense, the purpose of this research is to determine the meaning of mathematics according to prospective pre-school teachers. With this regard, it is aimed to present the concepts which are visualized by the pre-school prospective teachers when it is said mathematics and the type of relationship that they construct between them. Within the scope of this aim, the study was carried out together with 63 second year students from pre-school teaching department in an education faculty. The students were asked to create mind maps about mathematics and to produce metaphors about mathematics. A mind map is a visual representation of relationships between concepts and thoughts [4, 5]. As the concepts, graphics and figures which come to mind of an individual about the subject at first create mind maps, it is seen as a useful data collection tool for this research. Metaphors are thinking substances and a form of acquisition [6]. At this point, data was collected by making students complete the following expression; “*Mathematics is like*; *Because*” Students were first asked to write a metaphor to the first blank and then they were asked to write why they wrote this metaphor.

The data obtained will be evaluated by using content analysis. The main purpose of this analyze is to find concepts and relationships which can explain the

data collected by researchers. The next thing within the scope of this purpose is to gather similar data around specific concepts and then to interpret them by bringing all together [7]. The findings obtained as a result of data analyses will be discussed and suggestions will be presented.

Key Words: Prospective pre-school teachers, mathematics, mind maps, metaphors.

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POSTER

A Survey on Researches Related To The Teaching of Abstract Algebra

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ABSTRACT

Abstract Algebra is one of the important subjects of mathematics that the mathematically educated person should know at least at the introductory level. Indeed, a degree in mathematics always contains a course covering these concepts. Unfortunately, abstract algebra is also seen as an extremely difficult to learn since it is so abstract.

The aim of this study is to investigate researches that have been done related to the abstract algebra.

Leron and Dubinsky (1995) contend that students' difficulties in learning abstract algebra are largely due to their inability to understand fundamental processes and objects. They further argue that the lecturing method may be insufficient to overcome these difficulties, because for most students, simply "telling students about mathematical processes, objects, and relations is not sufficient to induce meaningful mathematical learning."

Researches shows that constructivist methods have been successful for student learning in abstract algebra.

Using computer applications to teach abstract algebra can promote meaningful learning of the concepts that can be difficult to grasp otherwise. Abstract algebra software combined with proper teaching techniques can offer prosperous learning environments where students engage in discovery, control their learning, and work collaboratively in groups.

According to Leron and Dubinsky, (1995) comments that are often heard from both teachers and students can be summarized in two different statements: (1) the teaching of abstract algebra is a disaster, and this remains true almost independently of the quality of the lectures and (2) the stuff is too hard for most students but students are not well-prepared and they are unwilling to make the effort to learn this very difficult material.

Finally, it has been concluded that for most prospective teachers, undergraduate mathematics that they study and the mathematics that they will teach are completely different.

Key Words: Abstract Algebra Teaching, Teaching Methods, Group Theory.

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Developing School Project and an Example for Elementary Mathematics Class

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ABSTRACT

It is well known that there is no sufficiently interaction and cooperation between Education Faculties which are educated students to become a teacher and Ministry of National Education (Karacay,2005). As a good practices for school and university collaboration; Gazi University Foundation Private Schools, Gazi University Gazi Education Faculty and Gazi University Institute of Education Sciences developed the "Developing Schools Project (DSP)" project in 2013. The purposes of the project was below.

- a. Support the school education objectives,
- b. Be paired with the theory and practice,
- c. Based process and intend to include all partners (DSP, 2013).

In this study, we designed an activity cover of this project for mathematics lesson. This activity was designed by six - fourth grade primary school teachers from Gazi University Foundation Private Schools, and a lecturer from Gazi University Gazi Education Faculty. We took data from 148 students.

At this activities students tried to explain some mathematical concepts or skills determined by the researchers with their own sentences. These concepts that have been specifically chosen because of the key concepts in the fourth grade level.

Concepts list was composed as follows: Zero, Angle, Abacus, Arithmetic, Edge, Unit, Plane, Geometry, Interior Angles, Diagonal, Mathematics, Model, Point, Rhythmic Counting, Symbol, Symmetry, Figure, Prism, Comparison, Predict, Decimal Fraction, Whole, Probably

The reason for doing such an activity; to understand how to construct these concepts in students' minds, to develop new materials or activities for removing misconceptions noticed at this process, to force the students when define concepts by the way of combine words as senseful, and lastly to determine their affective views on the these concepts.

The findings are discussed in this study.

Key Words: Cooperation of University-School, Mathematical Concepts, Views of Students.

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Evaluation of the Applicability of Concept Maps When Teaching the Subject of Fractions

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ABSTRACT

Many researches, in which the positive effects of mind maps were put forward, have been carried out. There are many researches available on the effects that are improving success. However, in literature, there isn't any research in which mind maps are designed, and teachers' and students' thoughts are evaluated. The purpose of this study is to evaluate students' thoughts on the applicability of concept maps when teaching the subject of fractions.

Because an attempt was made to analyze an issue deeply in this research, "the qualitative research approach" was adopted. Within this framework, it was attempted to describe the teachers' and students' thoughts on the applicability of concept maps created for teaching the subject of fractions in math class. The study group consisted of 20 students of 6th grade. In this study, descriptive analysis method was used in order to reveal the teachers' and students' thoughts. Concept map activities were applied on the students, and a survey consisting of open-ended questions was conducted in order to find out the students' thoughts on these activities and the relevant teaching method. The data collected in order to determine the students' thoughts, were analyzed by descriptive analysis method. In this context, the data collected were analyzed, categories were created in accordance with the research objective, and several samples were encoded, then presented with citations.

When the students' opinions were taken into consideration at the end of the study, it was determined that the students had positive opinion on the learning process conducted by means of concept maps. However, while the majority of the students were of the opinion that the learning environment of this type would be of much help for them; it was observed that a few students, who had displayed a high performance in a lesson thought by the old system, showed a low performance in this lesson and got ill-tempered.

Key Words: mathematics education, concept map, fractions

Exact Solutions of Unstable Nonlinear Schrödinger Equation by use of a New Version of Generalized Kudryashov Method

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ABSTRACT

Kudryashov suggest a Q function method to solve nonlinear differential equations. In this study, we use different Q function to determine the exact solution of unstable nonlinear Schrödinger equation. As a result, we find some new function classes of solitary wave solutions by use of proposed method. This method is straightforward which can be applied to other nonlinear partial differential equations.

Key Words: A new version generalized Kudryashov method, unstable nonlinear Schrödinger equation, solitary wave solutions.

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Journey in Mathematics World with Polyominoes and Polycubes

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ABSTRACT

The school subjects curricula have been approved by The Republic of Turkey Ministry of National Education (MoNE). The psychomotor skills have been given emphasis on mathematics curricula for long years [i.e. 1, 2, 3, 4, 5, 6]. Physical instructional objects are used to develop these skills in all these mathematics curricula. They also can contribute to the development of students' spatial abilities [5]. Moreover, at the end of meta-analysis including the participants from kindergarten to university Sowell stated that "mathematics achievement is increased through the long-term use of concrete instructional materials and that students' attitudes toward mathematics are improved when they have instruction with concrete materials provided by teachers knowledgeable about their use" [7, p.489].

The elementary school mathematics curriculum of the 2009 explains how to use polyominoes and polycubes in teaching and learning process. There are two objectives related to them: "Students should be able to use polyominoes effectively" and "students should be able use polycubes effectively" [5, p.22]. One of the project topics is about preparing a game with these instructional materials [5, p.108]. While polyominoes consist of one, two, three, four or five squares, the polycubes are constructed with one through five cubes. Some of the polyominoes are monomino, domino, tromino and tetromino. For example, while a pentomino is one of the polyominoes (polysquares) and a set of material composed of five squares by joining unit squares edge to edge, the corresponding polycubes called a pentacube is a set of material formed with five cubes by connecting the unit cubes face to face. The MoNE has been producing and giving physical manipulatives including polyominoes (called as polysquares) and polycubes to many elementary schools in our country.

There are some hints on how to use polyominoes or polycubes to reach these objectives: “Students should be able to solve problems and pose problems on the areas of plane regions [5 p.183], to solve problems and pose problems on rectangular prism, square prism and cubes” [5, p.186], to explain the relationship between perimeter and area” [5, p.258], and to construct buildings according to given drawings and to draw the buildings constructed with polycubes” [5, p.315]. There is a statement about the pentominoes in the assessment of the objective: “Students should be able to determine diagonals, interior and exterior angles of polygons [5, p.235] such as constructing a square with pentominoes. Moreover, polyominoes and polycubes can be used to reach some of the objectives in every school mathematics curricula.

In the poster there will be various hints for the activities with polyominoes and polycubes including given examples in the school mathematics and secondary school geometry curricula such as a game and some real life figures.

Key Words: Polyominoes, polycubes, instructional material

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Mathematica Applications In Educational Technologies And Scientific Studies

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ABSTRACT

Knowledge is the most valuable treasure for humankind. This case increases the importance of access to knowledge and use knowledge effectively.

In educational field, the development in computer technology emerges as educational technologies. This subject is rapidly developing in our country as in the world. The use of this technology in parallel to this development have become necessary in training methods and techniques. There are many programs and mobile applications in the world. In our country, it can be accessed to these programs and applications with EBA (Education Information Network) [1]. Also, the development of educational technologies effects the developments in science and technology. It is clear that the maximum benefit can not be provided without interdisciplinary interaction.

With the slogan “The world's definitive system for modern technical computing” [2] Mathematica is a computer program that can support at the highest level both educational technologies and the studies of science and technology. The program has options for different usage; desktop, online and mobile. Mathematica has an advantage compared to similar programs. Mathematica is not used only a discipline, it is used for many field based on mathematic [3]. This situation also seem to provide a solution to problems in science and technology production.

Mathematica is quite widely used in educational Technologies and scientific studies in the world, unfortunately, it is not sufficiently recognized and used in our country. In this study we aim to introduce the program of Mathematica with model practises in our country and to encourage the using of the program.

Key Words: Mathematica, Educational Technology, Scientific Study

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Monomorphism and Epimorphism Properties of Soft Categories

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ABSTRACT

The definition of soft sets was given by Molodtsov in 1999 and the concept of soft sets became important in computer science, modelling problems in engineering, economics, medical and social science. On the other hand, basic notions of soft category theory were introduced by Sardar and Gupta [5]. They gave some introductory introductory results for soft category theory.

In this paper, firstly we recall some definitions and basic properties of soft set theory, category theory and soft category theory. We study on monomorphism, epimorphism, split monomorphism, split epimorphism, and isomorphism for soft categories. We give some connections for these morphisms. Also we define the section an retraction for soft categories. Finally we obtain some results for morphism properties of soft categories.

Key Words: Soft Set, Soft Category, Monomorphism, Epimorphism.

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Morphism Properties of Digital Categories

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ABSTRACT

Category Theory is a branch of mathematics which formalizes the mathematical structure in terms of the collection of objects and arrows. It plays multidisciplinary roles in different subject of mathematics. Also Category Theory may be considered as study of (abstract) algebras of functions. On the other hand, the concept of digital category was introduced in [3]. They investigated the connections between epic-
monic and commutativity for cubic diagram of digital objects.

In this paper we study properties of monomorphism, epimorphism and isomorphism for digital categories which is defined in [3]. Also initial and terminal objects in digital categories is defined by using κ - adjacency relation. Hence we determined the initial and terminal objects of digital categories which has digital image with κ - adjacency as objects. In addition to this we proved that the objects of the same type in a digital category are isomorphic.

Key Words: Digital Image, Digital Category, Initial Object, Terminal Object.

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Periodic Solutions for Nonlinear Boundary Value Problems

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ABSTRACT

In this study, we show that, the periodic boundary value problem

$$\begin{aligned}y''(t) + \lambda f(t, y(t)) &= 0, & (0, T) \\ y(0) = y(T), & \quad y'(0) = y'(T),\end{aligned}$$

has at least one periodic solutions for λ in a compatible interval under suitable conditions on $f(t, y)$.

Key Words: Periodic solution, Boundary value problem, Existence result.

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Rectifying Curves in the 4-Dimensional Galilean Space

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ABSTRACT

The geometry of curves has long captivated the interests of mathematicians, from the ancient Greeks through to the era of Isaac Newton (1643-1727) and the invention of the calculus. It is a branch of geometry that deals with smooth curves in the plane and in the space by methods of differential and integral calculus. The theory of curves is more popular in its area, because a regular curve in a Euclidean space has no intrinsic geometry. One of the most important notions used to analyze curve is the Frenet frame, a moving frame that provides a coordinate system at each point of curve that is "best adopted" to the curve near the point.

At first, rectifying curves are introduced by B. Y. Chen in [2] as space curves whose position vector always lies in its rectifying plane, spanned by the tangent and the binormal vector fields T and B of the curve in the Euclidean 3-space. Accordingly, the position vector with respect to some chosen origin of a rectifying curve α in E^3 , satisfies the equation $\alpha(s) = \lambda(s)T(s) + \mu(s)B(s)$, where $\lambda(s)$ and $\mu(s)$ are arbitrary differentiable functions in arclength parameter $s \in I \subset \mathbb{R}$.

Many interesting results about rectifying curves in different space, such as in arbitrary dimensional Euclidean space, Lorentzian space and also Galilean 3-space, have been studied by many mathematicians (see for example [1],[3]-[5],[7],[8]).

When one investigated the literatur, rectifying curves have not been presented in the Galilean four space G_4 . We considered this paper because of such a need. In these regards, we defined and obtained of characterization relation with rectifying curves in four dimensional Galilean space G_4 .

Key Words: Rectifying curve, Galilean space.

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Some Generalized Fibonacci Difference Spaces Defined by a Sequence of Modulus Functions

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ABSTRACT

Construction of new sequence spaces and defining their topological and algebraic properties have an important role in summability theory. Until now, many sequence spaces were defined by many different ways. One of them is to use the matrix domain of a special triangle which was recently studied by many researchers.

Difference sequence spaces were introduced by Kizmaz [9]. This notion was generalized by Et and Çolak [7]. For $0 < p < 1$, the difference space bv_p was studied by Altay and Başar [1] and in the case $1 \leq p \leq \infty$ by Başar and Altay [4], and Çolak et al. [6]. Kirişçi and Başar [10] introduced and studied the generalized difference sequence spaces

$$\hat{X} = \{x = (x_k) \in w : B(r, s)x \in X\},$$

where the generalized difference matrix is

$$b_{nk}(r, s) = \begin{cases} r, & k = n, \\ s & k = n - 1, \\ 0, & 0 \leq k < n - 1 \text{ or } k > n, \end{cases}$$

and X is any of the spaces $l_\infty, l_p (1 \leq p < \infty), c, c_0$ and $B(r, s)x = (s \cdot x_{k-1} + rx_k)$ with $r, s \in \mathbb{R} \setminus \{0\}$.

The paranormed difference sequence space

$$\Delta\lambda(p) = \{x = (x_k) \in w : (x_k - x_{k+1}) \in \lambda(p)\}$$

was studied by Ahmad and Mursaleen [3] and Malkowsky [12], where $\lambda(p)$ is any of the paranormed spaces $l_\infty(p), C(p), C_0(p)$ defined by Simons [14] and Maddox [11]. Altay et al. [2] defined the sequence spaces $bv(u, p)$ and $bv_\infty(u, p)$ as follows:

$$bv(u, p) = \left\{x = (x_k) \in w : \sum |u_k(x_k - x_{k+1})|^{p_k} < \infty\right\}$$

$$bv_\infty(u, p) = \{x = (x_k) \in w : \sup_k |u_k(x_k - x_{k+1})|^{p_k} < \infty\},$$

where $u = (u_k)$ is an arbitrary fixed sequence and $0 < p_k \leq H < \infty$ for all $k \in \mathbb{N}$.

The famous Fibonacci sequence is obtained by the recursive formula, for $n \geq 2$,

$$f_n = f_{n-1} + f_{n-2},$$

where $f_0 = f_1 = 1$. Then we define the infinite matrix $\hat{F} = (\hat{f}_{nk})$, for each $n, k \in \mathbb{N}$ by [8]

$$\hat{f}_{nk} = \begin{cases} -\frac{f_{n+1}}{f_n}, & k = n - 1, \\ \frac{f_n}{f_{n+1}}, & k = n, \\ 0, & 0 \leq k < n - 1 \text{ or } k > n, \end{cases}$$

Kara [8] introduced the Fibonacci difference sequence spaces, for $1 \leq p < \infty$, $l_p(\hat{F})$ and $l_\infty(\hat{F})$ as

$$l_p(\hat{F}) = \left\{ x = (x_n) \in w : \sum_n \left| \frac{f_n}{f_{n+1}} x_n - \frac{f_{n+1}}{f_n} \cdot x_{n-1} \right|^p < \infty \right\}$$

$$l_\infty(\hat{F}) = \left\{ x = (x_n) \in w : \sup_{n \in \mathbb{N}} \left| \frac{f_n}{f_{n+1}} x_n - \frac{f_{n+1}}{f_n} \cdot x_{n-1} \right| < \infty \right\}.$$

Now, let's recall definition of modulus function.

A modulus function is defined by a function $f: [0, \infty[\rightarrow [0, \infty[$ as satisfying the following conditions:

- i) $f(x) = 0 \Leftrightarrow x = 0$,
- ii) $f(x + y) \leq f(x) + f(y)$,
- iii) $f: [0, \infty[\rightarrow [0, \infty[$,
- iv) f is increasing,
- v) f is continuous from right at 0.

Raj et al. [13] introduced the Fibonacci difference sequence spaces $l(\hat{F}, \mathcal{F}, p, u)$ and $l_\infty(\hat{F}, \mathcal{F}, p, u)$ by using a sequence of modulus functions and Fibonacci Band matrix as mentioned above. Motivated by these studies we define the Fibonacci difference sequence spaces $l(\hat{F}(r, s), \mathcal{F}, p, u)$ and $l_\infty(\hat{F}(r, s), \mathcal{F}, p, u)$ under the domain of the matrix $\hat{F}(r, s)$ constituted by using Fibonacci sequence and non-zero real numbers r, s and investigate some properties of them.

Key Words: Fibonacci numbers, modulus function, sequence spaces.

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WORKSHOP

Innovative Learning and Teaching in Math Education

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ABSTRACT

SAMR is a model designed to help educators infuse technology into teaching and learning. Popularized by Dr. Ruben Puentedura, the model supports and enables teachers to design, develop and infuse digital learning experiences that utilize technology. The goal is to transform learning experiences so they result in higher levels of achievement for students. Teachers need to both create tasks that target the higher-order cognitive skills (Bloom's) as well as design tasks that have a significant impact on student outcomes (SAMR).

Teachers adopt new ways of integrating technology is crucial when it comes to mathematics education. The flipped classroom is a pedagogical model in which the typical lecture and homework elements of a course are reversed. Online lesson content from networks such as TED-Ed, the Khan Academy are making math education mobile and accessible inside and outside the school. These platforms also allow students to work at their own pace. The classroom becomes collaborative areas where students solve problems together while teacher focuses on their individual needs. By using Web 2.0 tools such as Google Drive, Pinterest etc. and educational Ipad applications such as Desmos, Quickgraph etc, we can enrich the learning environment for learners. It is not about how are we going to fit technology in to the curriculum. The question is which technology will best fit the curriculum so SAMR must be main goal in this way. In the workshop, we will share our experiences and educational vision. In addition, some of our formative projects will be applied to participant educators. At the same time, we will try to perform some of the

applications we use at our school and try to comment effective ways and disadvantages of these programs as a result of our experiences.

Key Words: Web 2.0, Ipad applications, Flipped classroom

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